International Journal of Engineering & Technology, 7 (4.7) (2018) 306-310

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Upper Limit Superior and Lower Limit Inferior of Soft Sequences

Ahmed Hasan Hameed[1], Ekhlas Annon Mousa[2], Abdulsattar Abdullah hamad[3]*

[1] *Department of Mathematics Science, Ministry of Education, Babylon*
Email:amqa92@yahoo.com
[2] *Department of Mathematics Science, Ministry of Education, Babylon*
Email: ekhlasanoon@yahoo.com
[3] *Department of Mathematics, College of Science, Tikrit University*
**Corresponding author E-mail: satar198700@gmail.com*

Abstract

In this paper we have provided some of evidence work of the authors R.N.Hasan, O.A Tantawy in 2016 [1] thy given proof of the concept between two bound soft sets & subsets of soft elements real numbers also was concluded an upper bound and lower bound by using two sequences of soft element real numbers which is

$$(I)\sup(\tilde{r}_n/A \oplus \tilde{\iota}_n/A) \tilde{\leq} \sup \tilde{r}_n/A \oplus \sup \tilde{\iota}_n/A)$$

$$(II)\inf(\tilde{r}_n/A \oplus \inf \tilde{\iota}_n/A) \tilde{\leq} \inf(\tilde{r}_n/A \oplus \tilde{\iota}_n/A)$$

In this paper, supposed to extend R.N.Hasan, O.A Tantawy work but here we are given new notion and proof for upper limit superior and lower limit inferior with two sequences and subsequences for the conclude new proof after recalled that, which is upper limit superior and lower limit inferior By this we have proved above new two theorems and one proposition & strengthen the example.
AMS Subject Classification: 06D72, 40A05, 54A40

Keywords: *upper limit superior, lower limit inferior, limit superior, limit inferior , sequences and subsequences, supremum & infimum*

1. Introduction

In recent paper R.N.Hasan, O.A.Tantawy 2016 [1] pointed at a concept between two sequences and subsequences clarify the importance and prove the upper bound and lower bound

$$(I)\sup(\tilde{r}_n/A \oplus \tilde{\iota}_n/A) \tilde{\leq} \sup(\tilde{r}_n/A \oplus \sup \tilde{\iota}_n/A)$$

$$(II)\inf(\tilde{r}_n/A \oplus \inf \tilde{\iota}_n/A) \tilde{\leq} \inf(\tilde{r}_n/A \oplus \tilde{\iota}_n/A)$$

Bu using supremum & infimum in [2] 2012 Sujoy Dos and Syamal Kumar Samanta used the concepts soft real numbers & soft real sets also studied convergence theory of sequences of soft real sets which is limit of soft theory and sequence also given many important notation in same time and in same paper he's studies limit theorem of soft sequences and a generalization soft similarity of two soft real numbers the soft set theoretic approach for dimensionality reduction and supremum & infimum has potent applications in many different fields which is include the smoothens of functions , game theory , operations research, and in [3] soft set theory is applied to commutatives ideals in BCK-algebra it was suggest the notion of soft limit for function F at the point see definition (3.3.1) the upper (ε,τ)-soft limit of function F at a point X $\overline{soft} \lim it$ and The lower (ε,τ)-soft limit of function F at a point X <u>soft</u> limit, the collection of all these soft limits forms the notions of upper soft limit of the function F is studied . The notions of commutatives soft ideals and commutative idealistic soft BCK-algebras are introduced and the notion of positive implicative ideals and introduced their basic properties are derived in [4] M. Irfan Ali Introduce some notion such as the restricted intersection, the restricted difference and the extended intersection of two soft sets also given notion of complement of a soft set, and proved that certain De Morgn's lows hold in soft set theory with respect new definitions.

2. Preliminaries

2.1 Least Upper Bound and Greatest Lower Bound in soft Real Analysis

Definition 1.1.[1] A soft subset of soft elements Real numbers $F_A \subset (R,E)$ is called soft bounded

From above iff there exists a soft Real number

$\tilde{r} = \{(e,\{r_e\}), e \in E\}$. Such that

$\tilde{\alpha} = \{(e,\{x_e\}), e \in E\} \tilde{\leq} \tilde{r} \forall \tilde{\alpha} \tilde{\in} F_A$.

i.e. $\forall e \in E F_A(e) \leq r_e$

A soft subset of set elements Real numbers $F_A \subset (R,E)$ is called soft bounded from below if and only if .

There exists a soft Real number $\tilde{r} = \{(e,\{r_e\}), e \in E\}$ such that

$\beta = \{(e,\{y_e\}), e \in E\} \tilde{\geq} \tilde{r} \forall \tilde{\beta} \tilde{\in} F_A$.

i.e. $\forall e \in E F_A(e) \geq \iota_e$

A soft subset of soft elements Real Numbers $F_A \subset (R,E)$ is called soft bounded if it is both soft bounded From above and below .

Definition 1.2. [1] Let $F_A \subset (R,E)$ be a soft subset of soft element real numbers which is bounded from above.

The soft least upper bound $r/A = \{(e,\{r_e\})\}, of\ , F_A$ which is denoted by $\sup F_A$ is a soft element real number satisfying the following Two condition:

$1 - \alpha/A \leq r/A \ \forall \alpha/A \in F_A$

$2 - If\ \ \alpha/A \leq \iota/A \ \forall \alpha/A \in F_A, then\ r/A \leq \iota/A$

These two condition can be formulated in another equivalent form to give another equivalent definition For $\sup F_A$

Proposition 1.3. [1] For a bounded from above soft subset F_A of soft element real number,

If and only if: $r/A = \sup F_A, Iff$

$1 - \tilde{r}/A$ is an upper bound for F_A

$2 - \forall \in/A = \{(e_n,\{\in_{en}\})\} \tilde{\succ} \tilde{0}$ the soft element real number $\tilde{r}/A \oplus \tilde{\varepsilon}/A$ is not upper bound for F_A .

Proof. Straightforward ,Another equivalent form of the definition

Proposition 1.4. [1] For a bounded from above soft subset F_A of soft element real number,

$\tilde{r}/A \sup F_A$ If and only if .

$1 - \tilde{r}/A$ is an upper bound for F_A .

$2 - \forall \tilde{\varepsilon}/A \tilde{\succ} \tilde{0} \exists$ a soft element real number $\tilde{\beta}/A \in F_A$ such that $\tilde{\beta}/A \tilde{>} \tilde{r}/A \ominus \varepsilon`/A$ is not an upper bound of F_A .

Proof. Straightforward

Definition 1.5. [1] For a bounded from above soft subset F_A of soft element real number $\sup F_A \tilde{\in} F_A$

Then we write: $\sup F_A = max F_A$

In this case the maximum of the soft real subset F_A exist

Remark 1.6. [1] We notice that the soft least upper bound of bounded form above soft subset is one

Its upper bounds and therefore we can say that it is the minimum of its upper bounds and write.

$\sup F_A = \min\{\tilde{\iota}/A : \tilde{\alpha}/A \tilde{\leq} \tilde{\iota}/A \ \forall \tilde{\alpha}/A \tilde{\in} F_A\}$

As a dual of the soft least upper bound of soft subset of soft real number ,we have the concept of the greatest soft lower bound.

Definition 1.7. [1] Let $F_A \subset (R,E)$ be a soft subset of soft element real numbers with is bounded from below, the soft greatest lower bound $\tilde{m}/A = \{e_a,\{m_{e_a}\})\}, of\ , F_A$.

which is denoted by : $\tilde{m}/A = \inf F_A$.

is a soft element real number satisfying tow conditions:

$1 - \tilde{\alpha}/A \tilde{\geq} \tilde{m}/A, \forall \tilde{\alpha}/A \tilde{\in} F_A,$

$2 - If\ \ \tilde{\alpha}/A \tilde{\geq} \tilde{n}/A, \forall \tilde{\alpha}/A \tilde{\in} F_A, Then\ \tilde{m}/A \tilde{\geq} \tilde{n}/A.$

These two condition can be formulated in another equivalent form to give another equivalent definition For $\inf F_A$.

Proposition 1.8. [1] For a bounded from below soft subset F_A of soft element real number,

$r/A = \sup F_A$, If and only if

$1 - \tilde{m}/A, is$ a lower bound for F_A ,

$2 - \forall \in/A = \{(e_n,\{\in_{en}\})\} \tilde{\succ} \tilde{0}$ the soft element real number $m/A \oplus \tilde{\varepsilon}/A$ is not a lower bound for F_A ,

Proof. Straightforward Another equivalent form of the definition

Proposition 1.9.[1] For a bounded from below soft subset of soft element real number,

$\tilde{r}/A \sup F_A$ If and only if .

$1 - \tilde{r}/A$ is a lower bound for F_A .

$2 - \forall \tilde{\varepsilon}/A \tilde{\succ} \tilde{0} \exists$ a soft element real number $\tilde{\beta}/A \in F_A$ such that $\tilde{\beta}/A \tilde{>} \tilde{m}/A \oplus \varepsilon`/A$ is not an upper bound of F_A .

Proof. Straightforward

Definition 1.10. [1] For any soft set of soft elements real numbers F_A define $\ominus F_A$.

$\ominus F_A = \{-\tilde{\alpha}/A : -\tilde{\alpha}/A \tilde{\in} F_A\}.$

Proposition 1.12. [1] Let $F_A \subset (R,E)$ be a soft subset of soft real numbers which is bounded then $\ominus\ F_A$ is also Bounded and For which we have :

$(1) \sup \ominus F_A = -\inf F_A$

$(2) \inf\ \ominus F_A = -\sup F_A.$

Proof: (1) Let $\inf F_A = \tilde{m}/A$ them from the equivalent definition of infimum

we get:

$(i)\tilde{m}/A \tilde{\le} \tilde{\alpha}/A \forall \tilde{\alpha}/A \in F_A, And$

$(ii)\forall \tilde{\varepsilon}/A > 0, \exists, a soft, real, number, \tilde{\beta}/A \in F_A, such, that\ \tilde{\beta}/A \tilde{<} \tilde{m}/A \ \Theta \ \tilde{\varepsilon}/A,$

Multiplying by -1 we get :

$(i) -\tilde{\alpha}/A \tilde{<} -\tilde{m}/A \forall -\tilde{\alpha}/A \in_{\Theta} F_A, And$

$(ii)\forall \tilde{\varepsilon}/A > \tilde{0}, \exists, a soft, real, number\ -\tilde{\beta}/A \in \oplus F_A, such that\ -\tilde{\beta}/A \tilde{>} -m/A \ \Theta\ \tilde{\varepsilon}/A,$ **(2)** is similar to (**1**).

1.13. [1] **Properties of Supremum and Infimum in soft Real Numbers**

O.A.Tantawy and R.M. Hassan is given some Pasic properties of supremum and infimum in soft Real Numbers are given in details.

Proposition 1.15. [1] Suppose that $F_A.G_B$ are non-empty soft sets of soft element real numbers such that $\tilde{\alpha} \tilde{\le} \tilde{\beta}$ for all $\tilde{\alpha} \tilde{\in} F_A, And\ \tilde{\beta} \tilde{\in} G_B$. Then $F_A \tilde{\le} \inf G_B$.

Proof: Fix $\tilde{\beta} \tilde{\in} G_B$. since $\tilde{\alpha} \tilde{\le} \tilde{\beta}$ for all $\tilde{\alpha} \tilde{\in} F_A$. it follows that $\tilde{\beta}$ is an upper bound of F_A so, sup F_A is finite and $\sup F_A \tilde{\le} \tilde{\beta}$. Hence sup F_A is a lower bound of, G_A ,so inf G_A Is finite and $\sup F_A \tilde{\le} \inf G_A$.

Definition 1.16. [1] If $F_A \tilde{\subset} (R,E)$ and $\tilde{r} \tilde{\in} [R]_A^E$, then we define

$\tilde{r}\ _{\Theta} F_A = \{\tilde{\iota} \tilde{\in} [R]_A^E : \tilde{\iota} = \tilde{r}\ _{\Theta} \tilde{\gamma}, for\ every, \tilde{\gamma} \tilde{\in} F_A\}$.

Proposition 1.17. [1] If $\tilde{r} \tilde{\ge} \tilde{0}$, then

$\sup(\tilde{r}\ _{\Theta} F_A) = \tilde{r} \sup F_A, \inf(\tilde{r}\ _{\Theta} F_A = \tilde{r} \inf F_A.$

If $\tilde{r} \tilde{\le} \tilde{0},$ then

$\sup(\tilde{r}\ _{\Theta} F_A) = \tilde{r} \inf F_A, \inf(\tilde{r}\ _{\Theta} F_A = \tilde{r} \sup F_A.$

Proof: the result is obvious if $\tilde{r} = \tilde{0}.$ if $\tilde{r} \tilde{>} \tilde{0},$ then $(\tilde{r}\ \Theta\ \tilde{\gamma}) \tilde{\le} \tilde{\alpha}$ if and only if $\tilde{\gamma} \tilde{\le} \tilde{\alpha}/\tilde{r},$ which show that $\tilde{\alpha}$ is an upper bound of $\tilde{r}\ \Theta F_A$, if and only if $\tilde{\alpha} = \tilde{r}$ is an upper bound of F_A so $\sup(\tilde{r}\ \Theta\ F_A) = \tilde{r} \sup F_A .If\ , \tilde{r} \tilde{<} \tilde{0}, then (\tilde{r}\ _{\Theta} \tilde{\gamma}\ ) \tilde{\le} \tilde{\alpha}$.
if then if and only if $\tilde{\gamma} \tilde{\ge} \tilde{\alpha}/\tilde{r},$ so $\tilde{\alpha}$ is an upper bound of $\tilde{r}\ _{\Theta} F_A,$
if and only if $\tilde{\alpha} = \tilde{r}$ is a lower bound of F_A , so

$\sup(\tilde{r}\ _{\Theta} F_A) = \tilde{r} \inf F_A.$

the remaining results follow similarly,

Definition 1.18. [1] If $F_A, G_B \tilde{\subset} (\tilde{R}, E),$,then we define

$F_A \oplus G_B = \{\tilde{r} \tilde{\in} [R]_{\le E}^E : \tilde{r} = \tilde{\alpha} \oplus \tilde{\beta}$ for some $\tilde{\alpha} \in F_A, \tilde{\beta} \in G_A\},$

$F_A\ _{\Theta}\ G_B = \{\tilde{\iota} \tilde{\in} [R]_{\le E}^E : \tilde{\iota} = \tilde{\alpha}\ _{\Theta} \tilde{\beta}, for\ some.$
$\tilde{\alpha} \in F_A, \tilde{\beta} \in G_A\},$

Proposition 1.19. [1] if F_A, G_B , are non-empty soft real sets then,

$\sup(F_A, G_B) = \sup F_A \oplus \sup G_B, \inf(F_A, G_B) = \inf F_A \oplus \inf G_B.$

$\sup(F_A, G_B) = \sup F_A \ \Theta\ \sup G_B,$

$\inf(F_A, G_B) = \inf F_A\ _{\Theta} \sup G_B.$

Proof; the soft set $F_A \oplus G_B$ is bounded from above if and only if F_A and G_B are bounded from above ,

So, $\sup(F_A \oplus G_B)$, exists if and only if Both $Sup F_A$ and $\sup G_B$ exist, in that case $\tilde{\alpha} \tilde{\in} F_A, And, \tilde{\beta} \in G_A$, Then,

$\tilde{\alpha} \oplus \tilde{\beta} \sup F_A \oplus \sup G_B,$

So, $\sup F_A \oplus \sup G_B$, is an upper bound of $F_A \oplus G_B$, and therefore $\sup(F_A \oplus G_B) \tilde{\le} \sup F_A \oplus \sup G_B.$

To get the inequality in the opposite directing, suppose that $\tilde{\varepsilon} \tilde{>} \tilde{0}.$ then there exist $\tilde{\alpha} \tilde{\in} F_A$ .and $\tilde{\beta} \tilde{\in} G_B$ such that

$\tilde{\alpha} > \sup F_A \,\Theta\, \frac{\tilde{\varepsilon}}{\tilde{2}}, \tilde{\beta} \tilde{>} \sup G_B \,\Theta\, \frac{\tilde{\varepsilon}}{\tilde{2}}.$

It follows that

$\tilde{\alpha} \oplus \tilde{\beta} \tilde{>} \sup F_A \oplus \sup G_B\ _{\Theta} \tilde{\varepsilon}.$

For every $\tilde{\varepsilon} \tilde{>} \tilde{0},$ which implies that

$\sup(F_A \oplus G_B) \tilde{\ge} \sup F_A \oplus \sup G_B.$

Thus ,

$\sup(F_A \oplus G_B) = \sup F_A \oplus \sup G_B.$

It follows that

$\sup(F_A\ \Theta\ G_B) = \sup F_A \oplus \sup(\ \Theta\ G_B) = \sup F_A\ \Theta\ \inf G_B.$

The proof of the results for $\inf(F_A \oplus G_B), And, \inf(F_A \Theta G_B)$ and is similar, or we can apply the results for the supermom to ΘF_A and G_B

Example 1.20. [1] Let $\{\tilde{r}_n/A\}, And \{\tilde{\iota}_n/A\}, n \in N$ be a sequence of soft element real numbers.

Then

$(1) \sup(\tilde{r}_n/A \oplus \tilde{\iota}_n/A) \tilde{\le} \sup \tilde{r}_n/A \oplus \sup \tilde{r}_n/A,$

$(2) \inf \tilde{r}_n/A \oplus \inf \tilde{r}_n/A \le \inf(\tilde{r}_n/A \oplus \tilde{\iota}_n/A).$

Proof: in fact ,let

$\tilde{m}/A = \sup \tilde{r}_n/A, n/A = \sup \tilde{\iota}_n/A, and, \varepsilon/A = \sup(\tilde{r}_n/A \oplus \tilde{\iota}_n/A).$

and it is then required to show that:

$\tilde{\varepsilon}/A \le \tilde{m}/A \oplus \tilde{n}/A.$

From the definition we get

$\tilde{r}_n/A \tilde{\leq} \tilde{m}/A, \forall n, and, \tilde{\iota}_n/A \tilde{\leq} n/A, \forall n$

Thus, $\tilde{r}_n/A \oplus \tilde{\iota}_n/A \oplus \tilde{n}/A, \forall n \in N$

Hence $\tilde{m}/A \oplus \tilde{n}/A$ is an upper bound for the soft real numbers $\tilde{r}_n/A \oplus \tilde{\iota}_n/A$ and consequently is greater than or equal to the least Upper bound $\tilde{\varepsilon}/A$ t he proof of (2) is similarly

Proposition 1.21. [1] let F_A, and, G_B bounded soft subset of soft elements real numbers

Such that $F_A \tilde{\subseteq} G_B$,then we get;

$\inf G_B \tilde{\leq} \inf F_A \tilde{\leq} \sup F_A \tilde{\leq} \sup G_B$

Proof: let $G_B = \tilde{m}$ then form the definition we get

$\tilde{m} \tilde{\leq} \tilde{\alpha}, \forall \tilde{\alpha} \tilde{\in} G_B$

Consequently,

$\tilde{m} \tilde{\leq} \alpha, \forall \tilde{\alpha} \tilde{\in} F_A$

Hence is a lower bound for the soft subset. It follows then that

Clearly, $\inf F_A \tilde{\leq} \tilde{\alpha} \tilde{\leq} \sup F_A . \forall \tilde{\alpha} \in F_A$

Finally, let $\sup G_B = \tilde{r}, then$

$\tilde{r} \tilde{\geq} \tilde{\alpha}, \forall\ \tilde{\alpha} \tilde{\in} G_B$ therefore

$\tilde{r} \tilde{\geq} \tilde{\alpha}, \forall\ \ \tilde{\alpha} \tilde{\in} F_A$

Hence $\tilde{r}$ is an upper bound for the soft subset F_A, and then

$\tilde{r} \tilde{\geq} \sup F_A$.

3. Main Result

Theorem: let $\{X_{n+1}\}$ be a soft convergent subsequences of soft real sets converging to soft real sets X, then

$\underline{\overline{\lim}} \inf |< x_{n+1}(\lambda) - < x(\lambda) >| = 0$ for every $\lambda \in A$.

Proof : since $X_n \sim X$ for each $\lambda \in A$ then we can Express that

$\underline{\lim} \sup < x_{n+1}(\lambda) >= \sup < x(\lambda) >$ and also can say

that $\underline{\overline{\lim}} \inf < x_{n+1}(\lambda) >= \inf < x(\lambda) >$ for every

$\lambda \in A$, $\underline{\lim} |\sup < x_{n+1}(\lambda) > - \sup < x(\lambda) >| = 0$

suppose that $\lambda \in A$, $\sup < x_{n+1}(\lambda) >= M_{n+1}(\lambda)$ for all $n \in N$ and $\sup < x(\lambda) >= M(\lambda)$

$\therefore \underline{\overline{\lim}} |M_{n+1}(\lambda) - M(\lambda)| = 0$ for arbitrary some $\in > 0$

there is a + ve integers m such that

$- \in M_{n+1}(\lambda) - M(\lambda) < \in$ for all $n \geq m$,in same way we can choose $x'_{n+1} < x_{n+1}(\lambda) > and, x' \in < x_n(\lambda) >$

Such that

$M_{n+1}(\lambda) - \in /4 \ < x'_{n+1} \leq M_n(\lambda)$ and

$M(\lambda) - \in /4 < x' \leq M(\lambda) >$

$\therefore - \in - \in /4 [M_{n+1}(\lambda) - M(\lambda)]$

$- \in /4 \leq [x'_{n+1} - x' \leq M_{n+1}(\lambda)] - M(\lambda) + \in /4 < \in + \in /4.$

For all $n \geq m$ since $\in > 0$ is arbitrary, thus we have

$\underline{\overline{\lim}} \inf |< x_{n+1}(\lambda) - < x(\lambda) >| = 0$, $\forall \lambda \in A$

Remark: if $\{x_{n+1}\}$ be subsequence of soft real numbers such that $x_{n+1} \to x$ and $x(\lambda) \neq o, \forall \lambda \in A$ for any $\lambda \in A$ then

$1 l x_{n+1} \sim 1/x$.

Theorem: suppose $H_A . G_B$ are non-empty soft sets of soft element real numbers such that

$\tilde{\alpha} \in \beta \ \ \forall \tilde{\alpha} \in H_A$ and $\tilde{\beta} \in G_B$

Let $\tilde{\alpha} \geq \beta$, $1 - \sup\ H_A \geq \inf G_B$ is upper bound,

$2 - \inf H_A \leq \sup G_B$ is lower bound

Proof: clearly $\tilde{\alpha} \tilde{\in} H_A$ and also $\tilde{\alpha} \geq \tilde{\beta} \ \ \forall \tilde{\beta} \in F_A$ it is follows that $\tilde{\alpha}$ is upper bound of G_B then

sup G_B is finite and $sup G_B \leq \tilde{\alpha}$,

hence sup G_B is a lower bound of H_B , so inf H_A is finite and we get that $sup\ H_A \geq \inf G_A$.

Proposition: if a H_A, G_B are non-empty soft real sets then

$\underline{\lim} \sup\{H_A, G_B\}(\lambda) = \underline{\lim} \sup\{H_A(\lambda)\} + \underline{\lim} \sup\{G_B(\lambda)\}$

$= \underline{\lim} \sup\{H_A(\lambda) + \underline{\lim} \sup\{G_B(\lambda)\}$

$= [\underline{\lim} \sup\{H_A(\lambda) = \sup_n \inf_{k \geq n} x_k(\lambda)] + [\underline{\lim}\{G_B(\lambda)\} = [\sup_n \inf_{k \geq n} x_k(\lambda)]$

And

$\underline{\overline{\lim}} \inf\{H_A, G_B\}(\lambda) = \underline{\overline{\lim}} \inf\{H_A(\lambda)\} + \underline{\overline{\lim}} \inf\{G_B(\lambda)\}$

$= \underline{\overline{\lim}} \inf\{H_A(\lambda)\} + \underline{\overline{\lim}} \inf\{G_B(\lambda)\}$

$= [\underline{\overline{\lim}} \inf\{H_A(\lambda) = \inf_n \sup_{k \geq n} x_k(\lambda)] + [\underline{\overline{\lim}}\{G_B(\lambda)\} = \inf_n \sup_{k \geq n} x_k(\lambda)]$

Proof: in a soft sets

$\underline{\lim} \sup\{H_A(\lambda)\} + \underline{\lim} \sup\{G_B(\lambda)\}$ is a bound from above if and only if $\underline{\lim} \sup\{H_A(\lambda)\}$ and

$\underline{\lim} \sup\{G_B(\lambda)\}$ is bound from above it is also should be

$\underline{\lim} \sup\{H_A, G_B(\lambda)\}$ exists if and only if both

$\underline{\lim} \sup\{H_A(\lambda)\}$ and $\underline{\lim} \sup\{G_B(\lambda)\}$ exists

Here we can say that the case $\tilde{\alpha} \in H_A$ And $\tilde{\beta} \tilde{\in} G_B$ then

$\tilde{\alpha} + \tilde{\beta}$ $\underline{\lim} \sup\{H_A(\lambda) + \underline{\lim} \sup\{G_B(\lambda)\}$, so

$\underline{\lim} \sup\{H_A(\lambda) + \underline{\lim} \sup\{G_B(\lambda)\}$ is an upper bound of

$(H_A + G_B)$ and therefore $\underline{\lim} \sup\{H_A + G_B\}(\lambda) \tilde{\leq}$

$\underline{\lim} \sup\{H_A(\lambda)\} + \underline{\lim} \sup\{G_B(\lambda)\}$

Will be getting that inequality in the opposite direct in suppose that $\tilde{\varepsilon} \tilde{\geq} o$, then there exists $\tilde{\alpha} \tilde{\in} H_A$ and $\tilde{\beta} \tilde{\in} G_B$ such that

$\tilde{\alpha} \tilde{>} \underline{\lim}\sup H_A - \frac{\tilde{\varepsilon}}{2}$, $\tilde{\beta} \tilde{>} \underline{\lim}\sup G_B - \frac{\tilde{\varepsilon}}{2}$ thus follows that every $\tilde{\varepsilon} \tilde{\geq} o$, it is implies that $\tilde{\alpha} + \tilde{\beta}$

$\underline{\lim}\sup\{H_A(\lambda) + \underline{\lim}\sup\{G_B(\lambda)\} - \tilde{\varepsilon}$ for every

$\tilde{\varepsilon} \tilde{\geq} o$, that is implies

$$\underline{\lim}\sup\{H_A + G_B\}(\lambda) \geq$$

$$\underline{\lim}\sup\{H_A(\lambda)\} + \underline{\lim}\sup\{G_B(\lambda)\}$$

Thus

$$\underline{\lim}\sup\{H_A + G_B\}(\lambda) = \underline{\lim}\sup\{H_A(\lambda)\} + \underline{\lim}\sup\{G_B(\lambda)\}$$

We get that $\underline{\lim}\sup\{H_A - G_B\}(\lambda)$

$$= \underline{\lim}\sup\{H_A(\lambda)\} + \underline{\lim}\sup\{G_B(\lambda)\}$$

$$= \sup H_A - \inf G_B$$

Example: let $\{x_n/a\},\{y_n/b\}$ $n \in N$ be a two sequence of soft element real numbers then

1- $\underline{\lim}\sup\{x_n/a(\lambda)\} + \{y_n/b(\lambda)\} \leq \underline{\lim}\sup < \{x_n/a(\lambda)\} + \underline{\lim}\sup\{y_n/b(\lambda)\}$

2- $\overline{\lim}\inf\{x_n/a(\lambda)\} + \overline{\lim}\inf\{y_n/b(\lambda)\} \leq \overline{\lim}\inf < \{x_n/a(\lambda)\} + \{y_n/b(\lambda)\} >$

Proof: (1) let $\tilde{\imath}/a = \underline{\lim}\sup\{x_n/a(\lambda)\}$ and

$\tilde{\varphi}/a = \underline{\lim}\sup\{y_n/b(\lambda)\}$ and

Let $\tilde{\varepsilon}/a = \underline{\lim}\sup\{x_n/a\} + \{y_n/b\}(\lambda)$ and it is that the required to prove that $\tilde{\varepsilon}/a \tilde{\leq} \tilde{\imath}/a + \tilde{\varphi}/a$ from the definition we get $\{x_n/a(\lambda)\} \tilde{\leq} \tilde{\imath}/a$, $\forall n$, and $\{y_n/b(\lambda)\} \tilde{\leq} \varphi/a$, $\forall n$

Thus is $\{x_n/a(\lambda)\} + \{y_n/b(\lambda)\} + \tilde{\varphi}/a$

$\forall n \in N$ .Hence the $\tilde{\imath}/a + \tilde{\varphi}/a$ is an upper bound for the soft real numbers $\{x_n/a(\lambda)\} + \{y_n/b(\lambda)\}$ also and consequently is greater than or equal the least upper bound $\tilde{\varepsilon}/a$ prove (2) is similarly.

References

[1] Hassan.R.N , Tantawy.OA (2016), Soft Real Analysis, *Journal of Progressive Research in Mathematics*. vol 8, Issue 1, 1207-1219

[2] Das.S, samanta,SK, (2012) soft real sets, soft real numbers and their properties, *Journal of fuzzy mathematics* vol.20,NO.3, P 551-576.

[3] Molodtsov.D, (1999), Soft set theory-first results, Compu. Mathematics Applications,Vol.37,issue (4/5), 19-31 Available online https://doi.org/10.1016/S0898-1221(99)00056-5 last visited 16.01. 2018

[4] Jun YB. Kimand HS. Park CH (2011), Positive implicative ideals of BCK-algebras based on soft set theory, *Bull.Malays.Math.Sci-Soc*, 34 (2), 345-354

[5] Irfan Ali M, Feng.F, Liu.X, Min WK, and Shabir.M (2009), On some new operations in soft set theory, *Computers and Mathematics with Applications*, 57(9), 1547-1553 available onlin ehttps://doi.org/10.1016/j.camwa.2008.11.009 last visited 02.02. 2018