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Research paper

Some Difference Double Sequences Spaces with Respect to Double Orlicz Function

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Abstract

This work concerned with studying of new difference double sequence spaces concerning with lacunary sequences with respect to double Orlicz function. Moreover, from this work we proved some inclusion relations involving these spaces.

1. Introduction and Preliminaries

The Banach spaces of real bounded and convergent double sequences$(u,v)=(u_{n,m},v_{n,m})$are denoted by$2\ell_\infty$, and$2c$, respectively. Whichdefined the normed by $\|u,v\|=\sup_{n,m}\{|u_{n,m}|,|v_{n,m}|\}$ where$\|u\|=\sup_{n,m}\{|u_{n,m}|\}$,$\|v\|=\sup_{n,m}\{|v_{n,m}|\}$

In 1994, Parashar and Choudhary [9] have been constructed asequence spaces defined by Orlicz functions. The idea of double sequence spaces has been introduced in 1999 by M. Basarir and O. Sonalcan [3], also by [1,6,11]and other authors have been studied anewdifference double sequence spaces. The notion of double sequence spaces defined by Orlicz function was structured by [2],[10],[12] and [13], et al.

Definition 1.1

A doublesequence of positive integers $2\vartheta=(\mathcal{K}_{r,s})$ is called lacunary if $\mathcal{K}_0=0,\ 0<\mathcal{K}_{r,s}<\mathcal{K}_{r+1,s+1}$ and $h_{r,s}=\mathcal{K}_{r,s}-\mathcal{K}_{r-1,s-1}\to\infty$as $r,s\to\infty$. The intervals determined by2ϑwill be denoted by $\mathbb{I}_{r,s}=(\mathcal{K}_{r-1,s-1},\mathcal{K}_{r,s})$and $z_{r,s}=\mathcal{K}_{r,s}/\mathcal{K}_{r-1,s-1}$. According to Freedman et al [5] , we can define the space of lacunary strongly convergent double sequence $2\mathcal{N}_\vartheta$ by

$$2\mathcal{N}_\vartheta=\{(u,v):\lim_{r,s\to\infty}h_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}[|u_{n,m}-t|\vee|v_{n,m}-t|]\text{ for some }t\}$$

Definition 1.2: [4]

A double Orlicz function is a function $\mathcal{F}:[0,\infty)\times[0,\infty)\to[0,\infty)\times[0,\infty)$defined by $\mathcal{F}(u,v)=(\mathcal{F}_1(u),\mathcal{F}_2(v))$such that $\mathcal{F}_1:[0,\infty)\to[0,\infty)\&\mathcal{F}_2:[0,\infty)\to[0,\infty)$ are Orliczfunctions which are continuous, non-decreasing , even, convex and satisfies the following conditions:

i) $\mathcal{F}_1(0)=0$, $\mathcal{F}_2(0)=0$implies that$\mathcal{F}(0,0)=(\mathcal{F}_1(0),\mathcal{F}_2(0))=(0,0)$

ii) $\mathcal{F}_1(u)>0,\mathcal{F}_2(v)>0$implies that$\mathcal{F}(u,v)=(\mathcal{F}_1(u),\mathcal{F}_2(v))>(0,0)$ for $u>0,v>0$, we mean by $\mathcal{F}(u,v)>(0,0)$ that $\mathcal{F}_1(u)>0,\mathcal{F}_2(v)>0$

iii) $\mathcal{F}_1(u)\to\infty,\mathcal{F}_2(v)\to\infty$as $u\to\infty,v\to\infty$, then $\mathcal{F}(u,v)\to(\infty,\infty)$.

Definition1.3:[4]

Let $2w$ be the spaces of all real or complex double sequence $(u,v)=(u_{nm},v_{nm})$.

We can define a double Orlicz function on double sequence spaces by means of Lindenstrauss and Tzafriri [8]

$$2\ell_{\mathcal{F}}=\{(u,v):\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\mathcal{F}_1\left(\frac{|u_{n,m}|}{\varrho}\right)\vee\mathcal{F}_2\left(\frac{|v_{n,m}|}{\varrho}\right)\right]<\infty,\varrho>0\}$$

which is called a double Orlicz double sequences spaces $2\ell_{\mathcal{F}}$ is a Banach space with a norm:

$$\|(u,v)\|=\inf\{\varrho>0:\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\mathcal{F}_1\left(\frac{|u_{n,m}|}{\varrho}\right)\vee\mathcal{F}_2\left(\frac{|v_{n,m}|}{\varrho}\right)\right]\le 1\}$$

According to Kizmaz [6], we define a double sequence spaces as

$$2\ell_\infty(\Delta)=\{(u,v)=(u_{n,m},v_{n,m}):\sup[|\Delta u_{n,m}|,|v_{n,m}|]<\infty\}$$

$$2c(\Delta)=\{(u,v)=(u_{n,m},v_{n,m}):\lim_{n,m}[[|\Delta u_{n,m}-t|\vee|v_{n,m}-t|]]=0,\text{ for some }t\}$$

$$2c_0(\Delta)=\{(u,v)=(u_{n,m},v_{n,m}):\lim_{n,m}[[|\Delta u_{n,m}|\vee|v_{n,m}|]]=0\}$$

where$(\Delta u_{n,m},\Delta v_{n,m})=(u_{n,m}-u_{n+1,m+1},v_{n,m}-v_{n+1,m+1})$

2. Main Results

In this part, we introduce a double sequence spaces andconstruct inclusion relations between double sequence spaces.

Definition 2.1

Let $\mathcal{F}=(\mathcal{F}_1,\mathcal{F}_2)$ be a double Orlicz function and $\mathbb{p}=\mathbb{p}_{n,m}$ be any bounded double sequence of strictly positive real numbers. Then

$$2w_0^{2\vartheta}(\mathcal{F},\mathbb{p})_\Delta=\{(u,v):\lim_{r,s\to\infty}h_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]=0,\varrho>0\}$$

$$2w^{2\vartheta}(\mathcal{F},\mathbb{p})_\Delta=\{(u,v):\lim_{r,s\to\infty}h_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]=0,\text{ for some }t,\varrho>0\}$$

$$2\mathbb{w}_{\infty}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}=\{(u,v):\sup_{n,m}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]<\infty,\varrho>0\}$$

If $(u,v)\in 2\mathbb{w}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$, we say that (u,v) is lacunaryΔ-convergence to t with respect to the double Orlicz function $\mathcal{F}=(\mathcal{F}_1,\mathcal{F}_2)$. When $\mathcal{F}(u,v)=(\mathcal{F}_1(u),\mathcal{F}_2(v))=(u,v)$, then we write $2\mathbb{w}_0^{2\vartheta}(\mathbb{p})_{\Delta}$, $2\mathbb{w}^{2\vartheta}(\mathbb{p})_{\Delta}$ and $2\mathbb{w}_{\infty}^{2\vartheta}(\mathbb{p})_{\Delta}$ for the spaces $2\mathbb{w}_0^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$, $2\mathbb{w}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$ and $2\mathbb{w}_{\infty}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$, respectively. If $\mathbb{p}_{n,m}=1$ for all n,m, then $2\mathbb{w}_0^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$, $2\mathbb{w}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$ and $2\mathbb{w}_{\infty}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$ reduce to $2\mathbb{w}_0^{2\vartheta}(\mathcal{F})_{\Delta}$, $2\mathbb{w}^{2\vartheta}(\mathcal{F})_{\Delta}$ and $2\mathbb{w}_{\infty}^{2\vartheta}(\mathcal{F})_{\Delta}$, respectively.
We need the following inequality in this paper,

$$|e_{n,m}+d_{n,m}|^{\mathbb{p}_{n,m}}\le\mathcal{C}\left(|e_{n,m}|^{\mathbb{p}_{n,m}}+|d_{n,m}|^{\mathbb{p}_{n,m}}\right)\quad(1)$$

Where$e_{n,m}$ and $d_{n,m}$ are complex numbers, $\mathcal{C}=\max(1,2^{\mathcal{H}-1})$ and $\mathcal{H}=\sup\mathbb{p}_{n,m}<\infty$

Theorem 2.2

Let $\mathcal{F}=(\mathcal{F}_1,\mathcal{F}_2)$ be a double Orlicz function and $\mathbb{p}=\mathbb{p}_{n,m}$ be a bounded double sequence of strictly positive real numbers. Then$2\mathbb{w}_0^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$, $2\mathbb{w}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$ and $2\mathbb{w}_{\infty}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$ are vector spaces over the set of complex numbers.

Proof

Let $(u,v),(p,q)\in 2\mathbb{w}_0^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$ and let $\alpha,\beta\in\mathbb{C}$. Then

$$\lim_{r,s\to\infty}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}|}{\varrho_1}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}|}{\varrho_1}\right)^{\mathbb{p}_{n,m}}\right]=0,\varrho_1>0$$

and

$$\lim_{r,s\to\infty}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta p_{n,m}|}{\varrho_2}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta q_{n,m}|}{\varrho_2}\right)^{\mathbb{p}_{n,m}}\right]=0,\varrho_2>0$$

Take $\varrho=\max\{2|\alpha|\varrho_1,2|\beta|\varrho_2\}$. We have

$$\lim_{r,s\to\infty}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\alpha\Delta u_{n,m}+\beta\Delta p_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\alpha\Delta v_{n,m}+\beta\Delta q_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]\le$$

$$\le\lim_{r,s\to\infty}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}|}{\varrho_1}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}|}{\varrho_1}\right)^{\mathbb{p}_{n,m}}\right]+$$

$$+\lim_{r,s\to\infty}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta p_{n,m}|}{\varrho_2}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta q_{n,m}|}{\varrho_2}\right)^{\mathbb{p}_{n,m}}\right]=0.$$

So, $\alpha(u,v)+\beta(p,q)\in 2\mathbb{w}_0^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$. Therefore $2\mathbb{w}_0^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$ is a vector space.
Similarly, we can prove the other spaces.

Theorem 2.3

Let $\mathcal{F}=(\mathcal{F}_1,\mathcal{F}_2)$ be a double Orlicz function. If $\sup_{n,m}(\mathcal{F}_1(u),\mathcal{F}_2(v))^{\mathbb{p}_{n,m}}<\infty$ for all$u>0,v>0$, then

$$2\mathbb{w}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}\subset 2\mathbb{w}_{\infty}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$$

Proof

Let$(u,v)\in 2\mathbb{w}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$. There exists some positive ϱ_1,ϱ_2 such that

$$\lim_{r,s\to\infty}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}-t|}{\varrho_1}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}-t|}{\varrho_2}\right)^{\mathbb{q}_{n,m}}\right]=0$$

Set$\varrho=(2\varrho_1,2\varrho_2)$. Since $\mathcal{F}=(\mathcal{F}_1,\mathcal{F}_2)$is non-decreasing and convex ,by using (1.1), wehave

$$\sup_{r,s}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]=$$

$$=\sup_{r,s}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}-t+t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}-t+t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]$$

$$\le\mathcal{C}\left\{\sup_{r,s}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\frac{1}{2^{\mathbb{p}_{n,m}}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}-t|}{\varrho_1}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}-t|}{\varrho_2}\right)^{\mathbb{p}_{n,m}}\right]+\sup_{r,s}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\frac{1}{2^{\mathbb{p}_{n,m}}}\left[\mathcal{F}_1\left(\frac{|t|}{\varrho_1}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|t|}{\varrho_2}\right)^{\mathbb{p}_{n,m}}\right]\right\}$$

$$<\mathcal{C}\left\{\sup_{r,s}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}-t|}{\varrho_1}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}-t|}{\varrho_2}\right)^{\mathbb{p}_{n,m}}\right]+\right.$$

$$\left.+\sup_{r,s}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|t|}{\varrho_1}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|t|}{\varrho_2}\right)^{\mathbb{p}_{n,m}}\right]\right\}<\infty$$

Hence$(u,v)\in 2\mathbb{w}_{\infty}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$.This completes the proof.

Theorem 2.4

Let $\mathcal{F}=(\mathcal{F}_1,\mathcal{F}_2)$ be a double Orlicz function and let$0<\hbar=\inf\mathbb{p}_{n,m}$Then
$2\mathbb{w}_{\infty}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}\subset 2\mathbb{w}_0^{2\vartheta}(\mathbb{p})_{\Delta}$if and only if

$$\lim_{r,s\to\infty}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}[\mathcal{F}_1(t)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2(t)^{\mathbb{p}_{n,m}}]=\infty\quad(2)$$

For some $t>0$.

Proof

Let$2\mathbb{w}_{\infty}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}\subset 2\mathbb{w}_0^{2\vartheta}(\mathbb{p})_{\Delta}$.Suppose that (2) does not hold. Therefore there are a subinterval$\mathbb{I}_{(r,s)(m)}$of the set of interval $\mathbb{I}_{r,s}$and a number$t_0>0$, where $t_0=\frac{|\Delta u_{n,m},\Delta v_{n,m}|}{\varrho}$for all n,m, such that

$$\hbar_{(r,s)(m)}^{-1}\sum_{n\in\mathbb{I}_{(r,s)(m)}}\sum_{m\in\mathbb{I}_{(r,s)(m)}}[\mathcal{F}_1(t_0)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2(t_0)^{\mathbb{p}_{n,m}}]\le\mathcal{K}<\infty\quad(3)$$

Such that $m=1,2,3,\ldots$
Let us define $(u,v)=(u_{n,m},v_{n,m})$as following

$$(\Delta u_{n,m},\Delta v_{n,m})=\begin{cases}(\varrho t_0,\varrho t_0), & n,m\in\mathbb{I}_{(r,s)(m)}\\ (0,0), & n,m\notin\mathbb{I}_{(r,s)(m)}\end{cases}$$

Thus by (3),$(u,v)\in 2\mathbb{w}_{\infty}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$.But$(u,v)\notin 2\mathbb{w}_0^{2\vartheta}(\mathbb{p})_{\Delta}$. Hence (2) must be hold.
Conversely, suppose that (2) holds and that$(u,v)\in 2\mathbb{w}_{\infty}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$. Then, for each r,s

$$\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]\le\mathcal{K}<\infty \quad (4)$$

Suppose that$(u,v)\notin 2w_0^{2\vartheta}(\mathbb{p})_\Delta$.Then, for some number $0<\varepsilon<1$, there is anumbern_0, m_0 such that , for a subinterval $\mathbb{I}_{r_1,s_1}$of the set of interval $\mathbb{I}_{r,s}$, $\varepsilon<\frac{|\Delta u_{n,m},\Delta v_{n,m}|}{\varrho}$for $n_0\le n$, $m_0\le m$. From properties of the double Orlicz function, we can write

$$[\mathcal{F}_1(\varepsilon)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2(\varepsilon)^{\mathbb{p}_{n,m}}]\le\mathcal{F}_1\left(\frac{|\Delta u_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}$$

Whichcontradicts (2), by using (4). Hence we get $2w_\infty^{2\vartheta}(\mathcal{F},\mathbb{p})_\Delta\subset 2w_0^{2\vartheta}(\mathbb{p})_\Delta$.This completes the proof.

Definition 2.5:[7]

The Orlicz function $\mathcal{F}$ is said to satisfy the Δ_2-condition for all values of x, y, if there exists a constant $L>0$ such that[15 and 16]

$$\mathcal{F}(2x,2y)\le L\mathcal{F}(x,y), x\ge 0, y\ge 0$$

Theorem 2.6

Let$0<\hbar=\inf\mathbb{p}_{n,m}\le\mathbb{p}_{n,m}\le\sup\mathbb{p}_{n,m}=\mathcal{H}<\infty$. For a double Orlicz function $\mathcal{F}=(\mathcal{F}_1,\mathcal{F}_2)$ which satisfies Δ_2-condition, we have $2w_0^{2\vartheta}(\mathbb{p})_\Delta\subset 2w_0^{2\vartheta}(\mathcal{F},\mathbb{p})_\Delta$, $2w^{2\vartheta}(\mathbb{p})_\Delta\subset 2w^{2\vartheta}(\mathcal{F},\mathbb{p})_\Delta$ and $2w_\infty^{2\vartheta}(\mathbb{p})_\Delta\subset 2w_\infty^{2\vartheta}(\mathcal{F},\mathbb{p})_\Delta$.

Proof

Let$(u,v)\in 2w^{2\vartheta}(\mathbb{p})_\Delta$, then we have

$$\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]\to 0 \text{ as } r,s\to\infty$$

for some t.

Let $0<\varepsilon$ and chooseδ with $1>\delta>0$ such that $[\mathcal{F}_1(t)\vee\mathcal{F}2t<\varepsilon$for $0\le t\le\delta$. We can write

$$\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]=$$

$$=\hbar_{r,s}^{-1}\sum_{\substack{n\in\mathbb{I}_{r,s}\\ |\Delta u_{n,m}-t|/\varrho\le\delta,}}\sum_{\substack{m\in\mathbb{I}_{r,s}\\ |\Delta v_{n,m}-t|/\varrho\le\delta}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]+$$

$$+\hbar_{r,s}^{-1}\sum_{\substack{n\in\mathbb{I}_{r,s}\\ |\Delta u_{n,m}-t|/\varrho>\delta,}}\sum_{\substack{m\in\mathbb{I}_{r,s}\\ |\Delta v_{n,m}-t|/\varrho>\delta}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]$$

For the first summation above, we immediately write

$$\hbar_{r,s}^{-1}\sum_{\substack{n\in\mathbb{I}_{r,s}\\ |\Delta u_{n,m}-t|/\varrho\le\delta,}}\sum_{\substack{m\in\mathbb{I}_{r,s}\\ |\Delta v_{n,m}-t|/\varrho\le\delta}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]<\max(\varepsilon,\varepsilon^{\hbar})$$

by using continuity of $\mathcal{F}=(\mathcal{F}_1,\mathcal{F}_2)$. For the second summation, we will make followingprocedure. We have

$$\left[\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)\vee\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)\right]<1+\left[\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)\vee\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)\right]/\delta.$$

Since $\mathcal{F}=(\mathcal{F}_1,\mathcal{F}_2)$is non-decreasing and convex, it follows that

$$\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)\right]<$$

$$<\left[\mathcal{F}_1\left\{1+\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)/\delta\right\}\vee\mathcal{F}_2\left\{1+\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)/\delta\right\}\right]\le$$

$$\le\frac{1}{2}[\mathcal{F}_1(2)\vee\mathcal{F}_2(2)]+\frac{1}{2}\left[\mathcal{F}_1\left\{2\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)/\delta\right\}\vee\mathcal{F}_2\left\{2\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)/\delta\right\}\right]$$

Since $\mathcal{F}=(\mathcal{F}_1,\mathcal{F}_2)$ satisfiesΔ_2-condition ,we can write

$$\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)\right]$$

$$\le\frac{1}{2}L\left\{\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)/\delta\vee\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)/\delta\right\}(\mathcal{F}_1(2)\vee\mathcal{F}_2(2))$$

$$+\frac{1}{2}L\left\{\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)/\delta\vee\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)/\delta\right\}(\mathcal{F}_1(2)\vee\mathcal{F}_2(2))$$

$$=L\left\{\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)/\delta\vee\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)/\delta\right\}(\mathcal{F}_1(2)\vee\mathcal{F}_2(2))$$

In this way, we write

$$\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]\le\max(\varepsilon,\varepsilon^{\hbar})+$$

$$+\max\{1,[L(\mathcal{F}_1(2)\vee\mathcal{F}_2(2))/\delta]^{\mathcal{H}}\}\,\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}-t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]$$

Taking the limit as $\varepsilon\to 0$and $r,s\to\infty$, it follows that $(u,v)\in$ 2w2ϑ(ℱ,𝕡)Δ[15

Following similar arguments we can prove that $2w_0^{2\vartheta}(\mathbb{p})_\Delta\subset 2w_0^{2\vartheta}(\mathcal{F},\mathbb{p})_\Delta$ and $2w_\infty^{2\vartheta}(\mathbb{p})_\Delta\subset 2w_\infty^{2\vartheta}(\mathcal{F},\mathbb{p})_\Delta$

After step of this section, different inclusion relations among these double sequencespaces are going to be studied. Now we have

Theorem 2.7

Let $\mathcal{F}=(\mathcal{F}_1,\mathcal{F}_2)$ be a double Orlicz function. Then the following statementsareequivalent.

i. $2w_\infty^{2\vartheta}(\mathbb{p})_\Delta\subset 2w_\infty^{2\vartheta}(\mathcal{F},\mathbb{p})_\Delta$
ii. $2w_0^{2\vartheta}(\mathbb{p})_\Delta\subset 2w_\infty^{2\vartheta}(\mathcal{F},\mathbb{p})_\Delta$
iii. $\sup_{r,s}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}[\mathcal{F}_1(t)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2(t)^{\mathbb{p}_{n,m}}]<\infty$for all $t>0$.

Proof

i) $\Longrightarrow$ ii): Let (i) holds. To verify (ii), it is enough to prove$2w_0^{2\vartheta}(\mathbb{p})_\Delta\subset 2w_\infty^{2\vartheta}(\mathcal{F},\mathbb{p})_\Delta$.

Let $(u,v)\in 2w_0^{2\vartheta}(\mathbb{p})_\Delta$. Then, there exist $\ge r_0, s\ge s_0$, for$0<\varepsilon$,such that

$$\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]<\varepsilon$$

Hence there exists$\mathcal{K}>0$ such that

$$\sup_{r,s}\hbar_{r,s}^{-1}\sum_{n\in\mathbb{I}_{r,s}}\sum_{m\in\mathbb{I}_{r,s}}\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\vee\mathcal{F}_2\left(\frac{|\Delta v_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]<K$$

So, we get $(u,v) \in 2\mathbb{w}^{2\vartheta}_{\infty}(\mathbb{p})_{\Delta}$

ii) $\Longrightarrow$ iii): Let (ii) holds. Suppose that (iii) does not holds. Then for some $t > 0$

$$\sup_{r,s} \hbar_{r,s}^{-1} \sum_{n\in\mathbb{I}_{r,s}} \sum_{m\in\mathbb{I}_{r,s}} [\mathcal{F}_1(t)^{\mathbb{p}_{n,m}} \vee \mathcal{F}_2(t)^{\mathbb{p}_{n,m}}] = \infty$$

And therefore we can find a subinterval $\mathbb{I}_{(r,s)(m)}$of the set of interval $\mathbb{I}_{r,s}$ such that

$$\hbar_{(r,s)(m)}^{-1} \sum_{n\in\mathbb{I}_{(r,s)(m)}} \sum_{m\in\mathbb{I}_{(r,s)(m)}} \left[\mathcal{F}_1\left(\frac{1}{m}\right)^{\mathbb{p}_{n,m}} \vee \mathcal{F}_2\left(\frac{1}{m}\right)^{\mathbb{p}_{n,m}}\right] > m, m = 1,2,3,\ldots \quad (5)$$

Let us define $(u,v) = (u_{n,m}, v_{n,m})$as following

$$(\Delta u_{n,m}, \Delta v_{n,m}) = \begin{cases} (\frac{\varrho}{m}, \frac{\varrho}{m}), & n,m \in \mathbb{I}_{(r,s)(m)} \\ (0,0), & n,m \notin \mathbb{I}_{(r,s)(m)} \end{cases}$$

Then$(u,v) \in 2\mathbb{w}^{2\vartheta}_{0}(\mathbb{p})_{\Delta}$but by (5),$(u,v) \notin 2\mathbb{w}^{2\vartheta}_{\infty}(\mathcal{F},\mathbb{p})_{\Delta}$, which contradicts (ii).

Hence (iii) must holds.

iii) $\Longrightarrow$ i): Let (iii) hold and $(u,v) \in 2\mathbb{w}^{2\vartheta}_{\infty}(\mathbb{p})_{\Delta}$. Suppose that$(u,v) \notin 2\mathbb{w}^{2\vartheta}_{\infty}(\mathcal{F},\mathbb{p})_{\Delta}$. Then for$(u,v) \in 2\mathbb{w}^{2\vartheta}_{\infty}(\mathbb{p})_{\Delta}$

$$\sup_{r,s} \hbar_{r,s}^{-1} \sum_{n\in\mathbb{I}_{r,s}} \sum_{m\in\mathbb{I}_{r,s}} \left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}} \vee \mathcal{F}_2\left(\frac{|\Delta v_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right] = \infty \quad (6)$$

Let $t = \frac{|(\Delta u_{n,m}, \Delta v_{n,m})|}{\varrho}$ for each n, m, then by (6)

$$\sup_{r,s} \hbar_{r,s}^{-1} \sum_{n\in\mathbb{I}_{r,s}} \sum_{m\in\mathbb{I}_{r,s}} [\mathcal{F}_1(t)^{\mathbb{p}_{n,m}} \vee \mathcal{F}_2(t)^{\mathbb{p}_{n,m}}] = \infty$$

Which contradicts (iii). Hence (i) must holds.

Theorem 2.8

Let $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2)$ be a double Orlicz function. Then the following statementsare equivalent.

i) $2\mathbb{w}^{2\vartheta}_{0}(\mathcal{F},\mathbb{p})_{\Delta} \subset 2\mathbb{w}^{2\vartheta}_{0}(\mathbb{p})_{\Delta}$

ii) $2\mathbb{w}^{2\vartheta}_{0}(\mathcal{F},\mathbb{p})_{\Delta} \subset 2\mathbb{w}^{2\vartheta}_{\infty}(\mathbb{p})_{\Delta}$

iii) $\inf_{r,s} \hbar_{r,s}^{-1} \sum_{n\in\mathbb{I}_{r,s}} \sum_{m\in\mathbb{I}_{r,s}} [\mathcal{F}_1(t)^{\mathbb{p}_{n,m}} \vee \mathcal{F}_2(t)^{\mathbb{p}_{n,m}}] < \infty$for all $t>0$.

Proof

i) $\Longrightarrow$ ii): It is obvious.

ii) $\Longrightarrow$ iii): Let (ii) holds. Suppose that (iii) does not holds. Then

$$\inf_{r,s} \hbar_{r,s}^{-1} \sum_{n\in\mathbb{I}_{r,s}} \sum_{m\in\mathbb{I}_{r,s}} [\mathcal{F}_1(t)^{\mathbb{p}_{n,m}} \vee \mathcal{F}_2(t)^{\mathbb{p}_{n,m}}] = 0 \quad \text{for some } t > 0,$$

And we can find a subinterval $\mathbb{I}_{(r,s)(m)}$of the set of interval $\mathbb{I}_{r,s}$ such that

$$\hbar_{(r,s)(m)}^{-1} \sum_{n\in\mathbb{I}_{(r,s)(m)}} \sum_{m\in\mathbb{I}_{(r,s)(m)}} [\mathcal{F}_1(m)^{\mathbb{p}_{n,m}} \vee \mathcal{F}_2(m)^{\mathbb{p}_{n,m}}] < \frac{1}{m}, m = 1,2,3,\ldots \quad (7)$$

Let us define $(u,v) = (u_{n,m}, v_{n,m})$ as following

$$(\Delta u_{n,m}, \Delta v_{n,m}) = \begin{cases} (\varrho m, \varrho m), & n,m \in \mathbb{I}_{(r,s)(m)} \\ (0,0), & n,m \notin \mathbb{I}_{(r,s)(m)} \end{cases}$$

Thus, by (7) $(u,v) \in 2\mathbb{w}^{2\vartheta}_{0}(\mathcal{F},\mathbb{p})_{\Delta}$but $(u,v) \notin 2\mathbb{w}^{2\vartheta}_{\infty}(\mathbb{p})_{\Delta}$which contradicts (ii). Hence(iii) must holds.

iii) $\Longrightarrow$ i): Let (iii) holds. Suppose that $(u,v) \in 2\mathbb{w}^{2\vartheta}_{\infty}(\mathcal{F},\mathbb{p})_{\Delta}$. Therefore,

$$\hbar_{r,s}^{-1} \sum_{n\in\mathbb{I}_{r,s}} \sum_{m\in\mathbb{I}_{r,s}} \left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}} \vee \mathcal{F}_2\left(\frac{|\Delta v_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right] \to 0 \quad (8)$$

As$r,s \to \infty$. Again, suppose that $(u,v) \notin 2\mathbb{w}^{2\vartheta}_{0}(\mathbb{p})_{\Delta}$ for some number $\varepsilon > 0$ anda subinterval $\mathbb{I}_{(r,s)(m)}$ of the set of interval $\mathbb{I}_{r,s}$, we have $\left(\frac{|(\Delta u_{n,m} - t, \Delta v_{n,m} - t)|}{\varrho}\right) \geq \varepsilon$ for all n, m.

Then, from properties of the double Orlicz function, we can write

$$\left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}} \vee \mathcal{F}_2\left(\frac{|\Delta v_{n,m}|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right] \geq [\mathcal{F}_1(\varepsilon)^{\mathbb{p}_{n,m}} \vee \mathcal{F}_2(\varepsilon)^{\mathbb{p}_{n,m}}]$$

Consequently, by (8) we have

$$\lim_{r,s\to\infty} \hbar_{r,s}^{-1} \sum_{n\in\mathbb{I}_{r,s}} \sum_{m\in\mathbb{I}_{r,s}} [\mathcal{F}_1(\varepsilon)^{\mathbb{p}_{n,m}} \vee \mathcal{F}_2(\varepsilon)^{\mathbb{p}_{n,m}}] = 0$$

Which contradicts (iii). Hence (i) must holds.

Finally, in this section, we consider that $\mathbb{p}_{n,m}$ and $\mathbb{q}_{n,m}$are any bounded double sequences of strictly positive real numbers. We are able to prove $2\mathbb{w}^{2\vartheta}(\mathcal{F},\mathbb{q})_{\Delta} \subseteq 2\mathbb{w}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$only under additional conditions.

Theorem 2.9

i) If $0 \leq \inf \mathbb{p}_{n,m} \leq \mathbb{p}_{n,m} \leq 1$ for all k, then $2\mathbb{w}^{2\vartheta}(\mathcal{F})_{\Delta} \subseteq 2\mathbb{w}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$

ii) $0 \leq \mathbb{p}_{n,m} \leq \sup \mathbb{p}_{n,m} = \mathcal{H} < \infty$, then $2\mathbb{w}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta} \subseteq 2\mathbb{w}^{2\vartheta}(\mathcal{F})_{\Delta}$

Proof

i) Let $(u,v) \in 2\mathbb{w}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$ since $0 \leq \inf \mathbb{p}_{n,m} \leq \mathbb{p}_{n,m} \leq 1$ we get

$$\hbar_{r,s}^{-1} \sum_{n\in\mathbb{I}_{r,s}} \sum_{m\in\mathbb{I}_{r,s}} \left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m} - t|}{\varrho}\right) \vee \mathcal{F}_2\left(\frac{|\Delta v_{n,m} - t|}{\varrho}\right)\right] \leq$$

$$\leq \hbar_{r,s}^{-1} \sum_{n\in\mathbb{I}_{r,s}} \sum_{m\in\mathbb{I}_{r,s}} \left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m} - t|}{\varrho}\right)^{\mathbb{p}_{n,m}} \vee \mathcal{F}_2\left(\frac{|\Delta v_{n,m} - t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]$$

And hence $(u,v) \in 2\mathbb{w}^{2\vartheta}(\mathcal{F})_{\Delta}$.

Let $0 \leq \mathbb{p}_{n,m} \leq \sup \mathbb{p}_{n,m} = \mathcal{H} < \infty$, and $(u,v) \in 2\mathbb{w}^{2\vartheta}(\mathbb{p})_{\Delta}$. Then for each $0 < \varepsilon < 1$there exists a positive integer r_0, s_0 such that

$$\hbar_{r,s}^{-1} \sum_{n\in\mathbb{I}_{r,s}} \sum_{m\in\mathbb{I}_{r,s}} \left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m} - t|}{\varrho}\right) \vee \mathcal{F}_2\left(\frac{|\Delta v_{n,m} - t|}{\varrho}\right)\right] \leq \varepsilon < 1$$

for all $r \geq r_0, s \geq s_0$. This implies that

$$\hbar_{r,s}^{-1} \sum_{n\in\mathbb{I}_{r,s}} \sum_{m\in\mathbb{I}_{r,s}} \left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m} - t|}{\varrho}\right)^{\mathbb{p}_{n,m}} \vee \mathcal{F}_2\left(\frac{|\Delta v_{n,m} - t|}{\varrho}\right)^{\mathbb{p}_{n,m}}\right]$$

$$\leq \hbar_{r,s}^{-1} \sum_{n\in\mathbb{I}_{r,s}} \sum_{m\in\mathbb{I}_{r,s}} \left[\mathcal{F}_1\left(\frac{|\Delta u_{n,m} - t|}{\varrho}\right) \vee \mathcal{F}_2\left(\frac{|\Delta v_{n,m} - t|}{\varrho}\right)\right]$$

Therefore $(u,v) \in 2\mathbb{w}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$.

Using the same technique as in Theorem 2 in [14], it is easy to prove thefollowing theorem.

Theorem 2.10

Let $0 < \mathbb{p}_{n,m} \leq \mathbb{q}_{n,m}$ for all n, m and let $(\mathbb{q}_{n,m}/\mathbb{p}_{n,m})$ be bounded.Then

$2\mathbb{w}^{2\vartheta}(\mathcal{F},\mathbb{q})_{\Delta} \subseteq 2\mathbb{w}^{2\vartheta}(\mathcal{F},\mathbb{p})_{\Delta}$

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