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Research paper

A Study on Behaviour of Cylindrical and Hyperbolic Paraboloid Type of Shell Roofs for Fixed and hinged Boundary Conditions

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Abstract

Concrete shell roofs exhibit high structural efficiency, thus can be constructed very thin. Because of their light weight nature, the shape of shell roof is typically established so that it performs optimally under gravity loads and Carry load to foundation mainly through membrane action over shell surface. The behaviour of shell roof is studied by performing linear static analysis by varying geometrical parameters and boundary conditions. Behaviour is studied in terms of displacement, and stiffness can be predicted.

Keywords: *Concrete shell roofs, Finite element method, Linear static analysis, Displacement, Stiffness*.

1. Introduction:

The structural behaviour of shell roofs are compared to that of other type of structures characterized by higher mechanical efficiency. Concrete shell depends on their configuration but not on their mass for stability. If appropriate designs are carried out, shell roofs can support high load and allow one to cover important space using little material and/or thickness, leading some authors to refer them as the "shell roofs behaviour", Antonio Tomas and Pascual Marti conclude that, the behaviour of shell roofs depends on geometry and boundary condidtions [5],then they studied on optimization techniques for same geometry and found that, the structural behaviour may be achieved with slight geometric changes [6].

The performance of a set of RC roof shells with square plan analysed by Tim Michiels and Sigrid Adriaenssens, the results demonstrates that span must be more than 15 m [7]. Another study has been done by S. Arnout, G. Lombaert, et al, on the same topic which illustrates that geometry plays vital role in structural behaviour of shell roofs and optimization results help the designers to make a trade-off between aesthetic arguments, constructional requirements and the possibilities of material reduction [8].

The behaviour of shell roof is developed essentially due to their form. If possible, it would be interesting to find small modifications in their geometry without modifying their initial aesthetic configuration too much and still complying with design condition. Moreover, shell roofs provide an attractive lighting and maintain elegance from aesthetic point of view. The main objective on this study is to assess the behaviour of cylindrical and hyperbolic paraboloid shell roofs by using different geometrical parameters and boundary conditions.

The assumptions made for analysing thin shell roofs [1] are: (i) The stress in Z-direction 'σ_z' (Thickness direction) may be considered negligible compared with other two normal stresses. This assumption deduces the 3D problem to 2D problem. (ii) The point on lines normal to the middle surface before deformation remains on line normal to the middle surface even after deformation, i.e. plain section remains plain. (iii) All displacements are small enough so that change in the geometry will not alter the static equilibrium of the shell. The displacement at any point normal to the middle surface is constant, i.e. shear τ_{xz} and τ_{yz} are zero. (iv) The material is homogeneous, isotropic and executes linear elastic behaviour (Hooke's Law), which gives a linear differential equation which is easy to solve. For all practical purposes.

2. Methodology

The present study explores the behaviour of shell roofs by changing geometry and boundary conditions, cylindrical shell roof (Ref.Fig.1) and hyperbolic paraboloid shell roof (Ref.Fig.2) are considered with a square plan. All shells are supported on all four edges with fixed and hinge boundary conditions.

2.1. Initial Geometry, Material Properties and Plan Size:

The initial geometry of shell and material parameters are based on series of realized cylindrical RC shell roof with semi-central angle below 90° to avoid the bulging affect. (i) Ratio of bay with (b) to total width at plan (B): 1/1, 1/2, 1/3 and 1/4 with number of bays (n) 1, 2, 3 and 4 respectively (Ref. Fig.3). (ii) Height (h) to span (L) ratio (Aspect ratio): 1/8, 1/9, 1/10, 1/11 and 1/12 [9], corresponding height for 20m span as 2.5 m, 2.22 m, 2 m, 1.818 m and 1.67 m respectively.(iii) Thickness (t): Every 10 mm increment from 50 mm to 90 mm (Ref. Table.1).

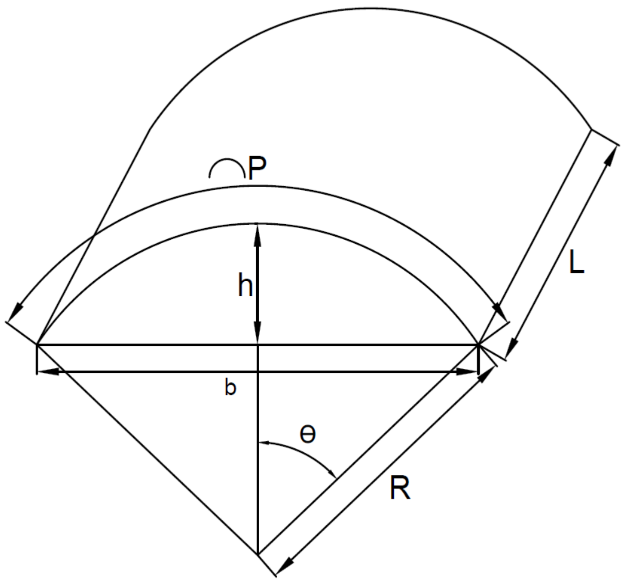

Fig.1: Isometric view of cylindrical shell roof (CS). h = Height; R = Radius; θ = Semi-central angle; L = Length; b = bay width; P = Circumference

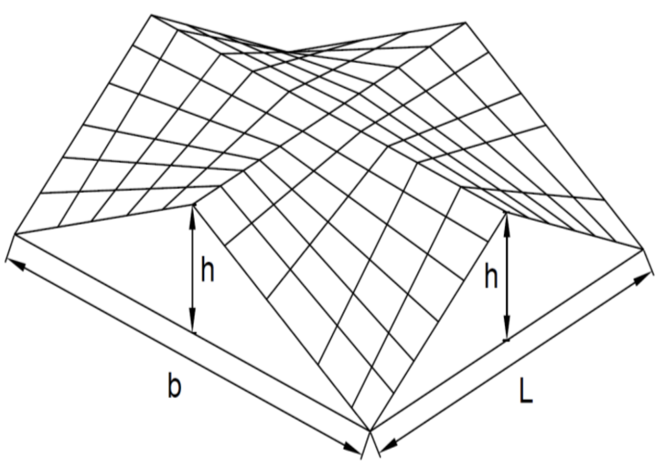

Fig.2: Isometric view of hyperbolic paraboloid shell roof (HPS). h = Height; L = Length; b = bay width

Table.1: Material properties and dimensions used for the study

Compressive strength	30	MPa
Tensile strength	3	MPa
Young's modulus	27.4	GPa
Density	25	kN/m^3
Poisson's ratio	0.2	
Plan size of initial shell	20 by 20	m^2
Height	1.67 to 2.5	m
Bay	1, 2, 3, 4	
shell thickness	10 increment from 50 to 90	mm
Boundary condition	Fixed and Hinged at ends	

2.2. Model Analysis:

Around 400 models were developed and analysed by FE analysis using available software SAP2000. A mesh convergence study shows that a mesh of 30 by 30 elements for every bay [7], further each element subdivided into 0.2 m by 0.2 m approximately, quadrilateral thin shell elements with 6 degrees of freedom per node, it is sufficiently refined to obtain consistent results. The linear static analysis performed by imposing a minimum load of 0.7 kN/m^2 [10] and combination considered from IS code [9] for both cylindrical and hyperbolic paraboloid shell roofs.

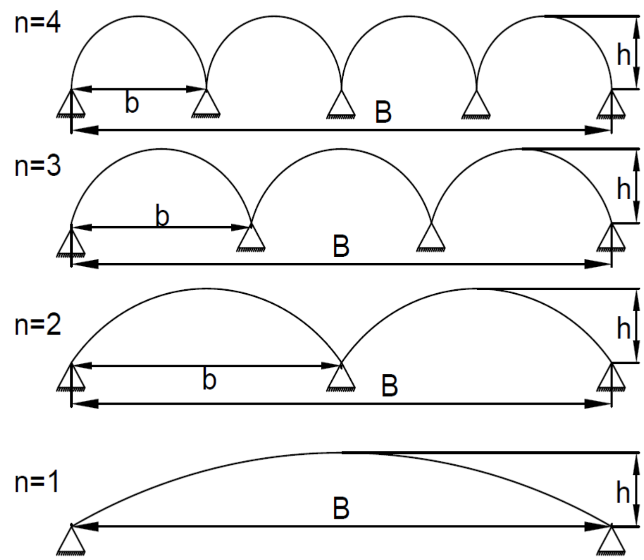

Fig.3: Cross section of cylindrical shell roof with different number of bays. n = Number of bays; h = Height; b = bay width; B = Total width of plan

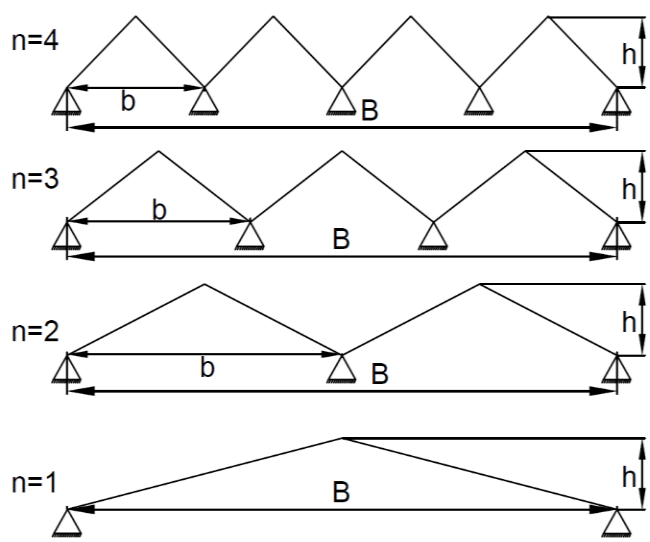

Fig.4: Cross section of Hyperbolic Paraboloid shell roof with different number of bays. n = Number of bays; h = Height; b = bay width; B = Total width of plan

2.3. Shape Variation:

Initially analysis is performed for cylindrical shell roof (CS) of span 20 m by 20 m, with varying number of bay for constant heights and thicknesses (Ref.Fig.3) to check the behaviour by changing boundary conditions. A similar analysis is carried out for hyperbolic paraboloid shell roof (HPS) (Ref. Fig.4).

Table.2: Displacements for hinged (H) support and fixed (F) support in Cylindrical shell (CS)

n	t = 50 mm; h = 2.5 m Displacement (δ) in "mm"			t = 50 mm; h = 2.22 m Displacement (δ) in "mm"			t = 50 mm; h = 2 m Displacement (δ) in "mm"			t = 50 mm; h = 1.818 m Displacement (δ) in "mm"			t = 50 mm; h = 1.67 m Displacement (δ) in "mm"		
	H	F	% Diff.	H	F	% Diff.	H	F	% Diff.	H	F	% Diff.	H	F	% Diff.
1	310	301	+2.90	353	341	+3.40	392	377	+3.82	428	411	+3.97	463	444	+4.10
2	112	108	+3.70	83	86	-3.61	98	101	-3.06	111	112	-0.90	122	121	+0.81
3	23	22	+4.30	29	30	-3.44	37	39	-5.40	43	44	-2.32	49	49	0.00
4	21	20	+4.70	18	17	+5.55	17	17	0.00	20	21	-5.00	25	25	0.00

Table.3: Displacements for hinged (H) support and fixed (F) support in Hyperbolic Paraboloid shell (HPS)

n	t = 50 mm; h = 2.5 m Displacement (δ) in "mm"			t = 50 mm; h = 2.22 m Displacement (δ) in "mm"			t = 50 mm; h = 2 m Displacement (δ) in "mm"			t = 50 mm; h = 1.818 m Displacement (δ) in "mm"			t = 50 mm; h = 1.67 m Displacement (δ) in "mm"		
	H	F	% Diff.	H	F	% Diff.	H	F	% Diff.	H	F	% Diff.	H	F	% Diff.
1	30	30	0.00	32	32	0.00	35	35	0.00	39	39	0.00	42	42	0.00
2	14	14	0.00	16	16	0.00	19	19	0.00	21	21	0.00	23	23	0.00
3	10	10	0.00	11	11	0.00	13	13	0.00	14	14	0.00	16	16	0.00
4	12	12	0.00	8	8	0.00	10	10	0.00	11	11	0.00	12	12	0.00

3. Results and Discussions

3.1. Effect of Boundary Condition:

In the present study, it is observed that the percentage difference in displacement for fixed and hinged boundary conditions are found to be negligible for cylindrical shell roofs, i.e. 0 to 5.55% (Ref.Table.2) and zero for hyperbolic paraboloid shell (Ref.Table.3) in all cases. Since there is no much percentage difference between these two boundary conditions, further results are discussed only for hinged boundary condition.

3.2. Cylindrical Shell Roof:

The peak displacement ranges from (i) 463 mm to 310 mm for heights of 1.67 m to 2.5 m, (ii) 463 mm to 17 mm for bays 1 to 4 and (iii) 463 mm to 130 mm for thickness 50 mm to 90 mm. For every increment in height, number of bay and thickness, displacement getting reduced (Ref. Fig.5-9).

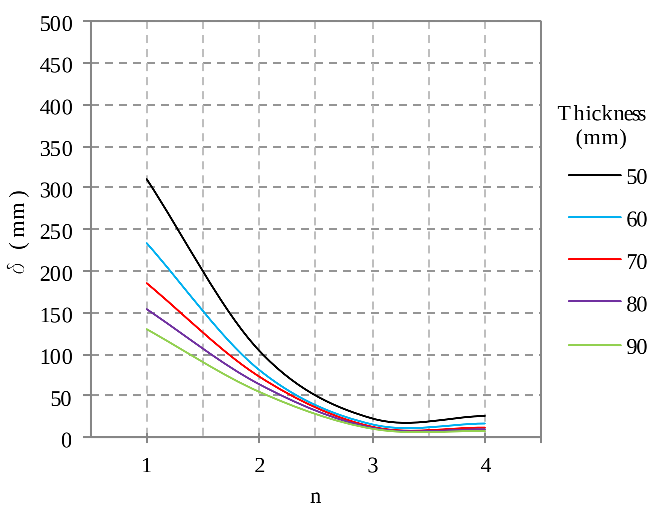

Fig.5: Displacement (δ) for cylindrical shell of height 2.5 m

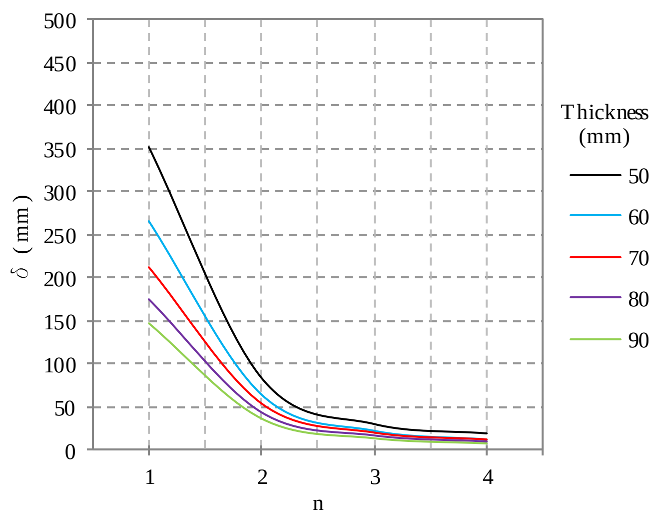

Fig.6: Displacement (δ) for cylindrical shell of height 2.22 m

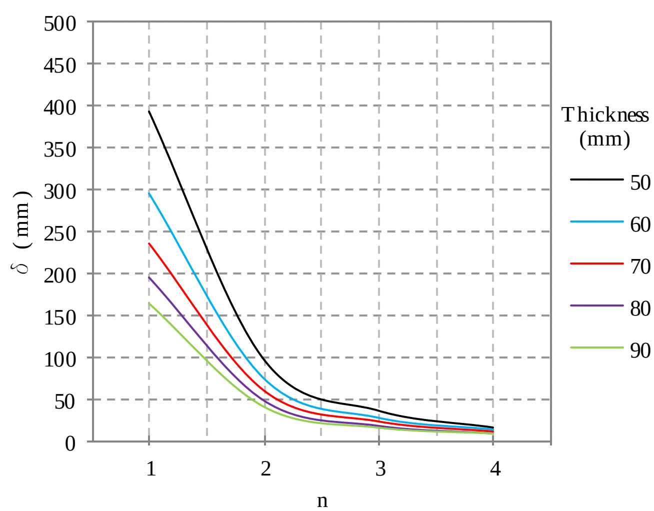

Fig.7: Displacement (δ) for cylindrical shell of height 2 m

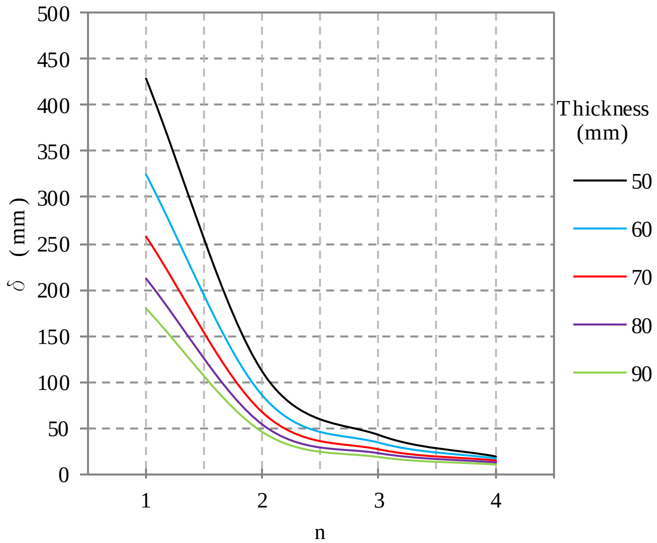

Fig.8: Displacement (δ) for cylindrical shell of height 1.818 m

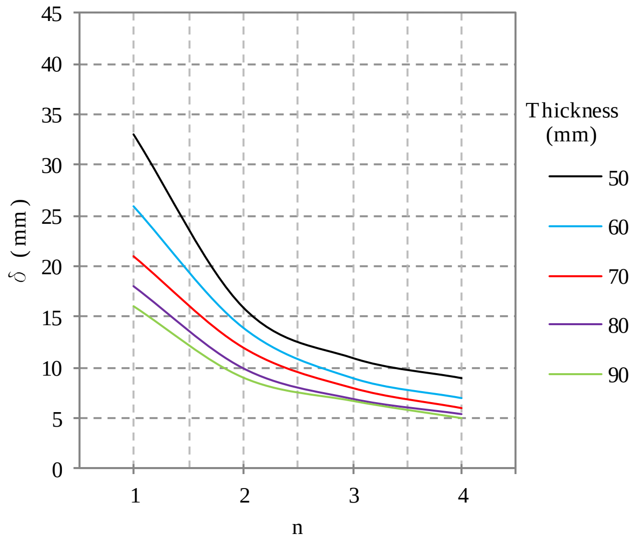

Fig.11: Displacement (δ) for hyperbolic paraboloid shell of height 2.22 m

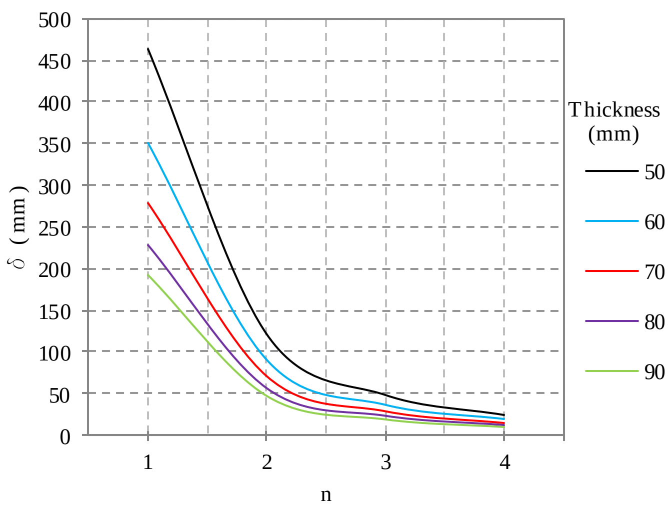

Fig.9: Displacement (δ) for cylindrical shell of height 1.67 m

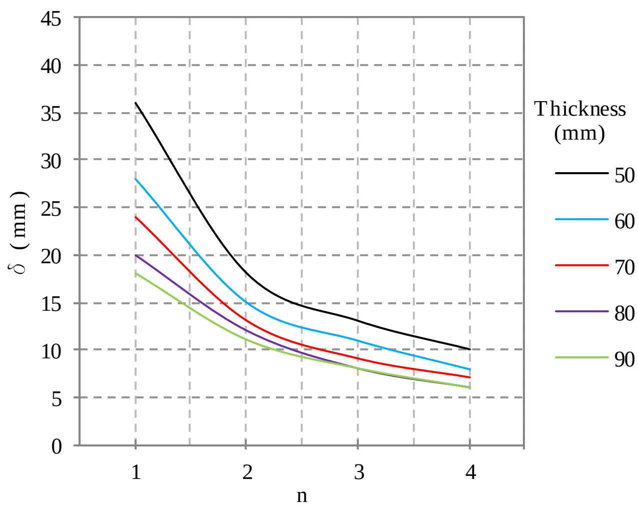

Fig.12: Displacement (δ) for hyperbolic paraboloid shell of height 2 m

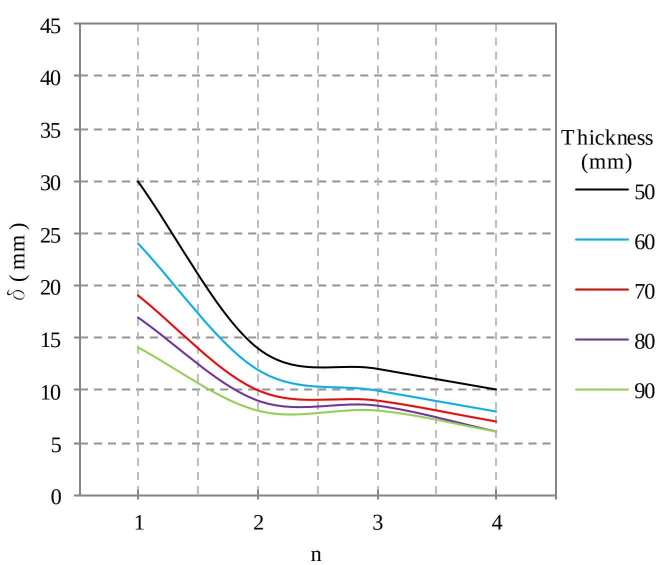

Fig.10: Displacement (δ) for hyperbolic paraboloid shell of height 2.5 m

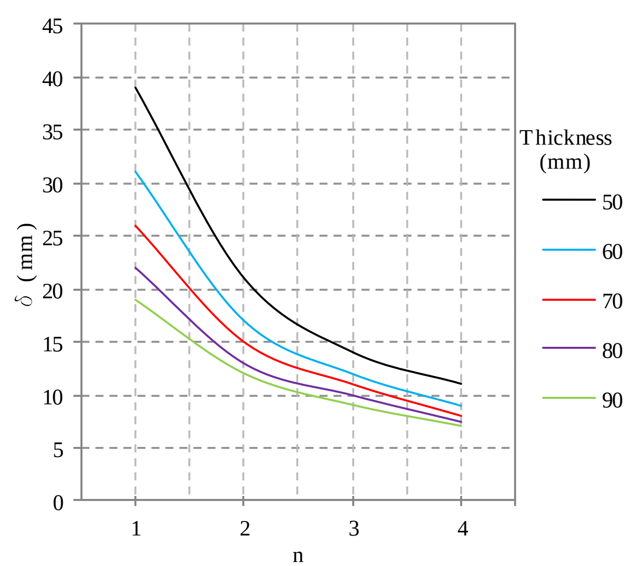

Fig.13: Displacement (δ) for hyperbolic paraboloid shell of height 1.818 m

Table.4: Displacements in cylindrical shell (CS) roof and hyperbolic paraboloid shell (HPS) roof and multiplication factor 'ξ'

n	t = 50 mm; h = 2.5 m Displacement (δ) in "mm"			t = 50 mm; h = 2.22 m Displacement (δ) in "mm"			t = 50 mm; h = 2 m Displacement (δ) in "mm"			t = 50 mm; h = 1.818 m Displacement (δ) in "mm"			t = 50 mm; h = 1.67 m Displacement (δ) in "mm"		
	CS	HPS	ξ	CS	HPS	ξ	CS	HPS	ξ	CS	HPS	ξ	CS	HPS	ξ
1	310	30	10.33	353	32	11.03	392	35	11.2	428	39	10.97	463	42	11.02
2	112	14	8.00	83	16	5.19	98	19	5.16	111	21	5.29	122	23	5.30
3	23	10	2.30	29	11	2.62	37	13	2.85	43	14	3.07	49	16	3.06
4	21	12	1.75	18	8	2.25	17	10	1.70	20	11	1.82	25	12	2.08

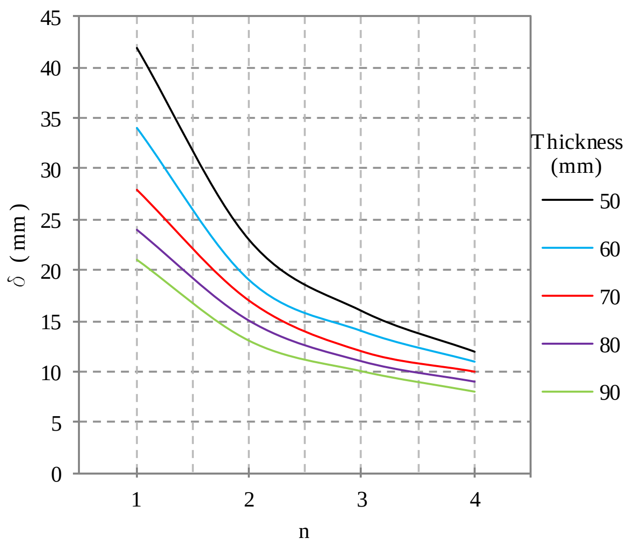

Fig.14: Displacement (δ) for hyperbolic praboloid shell of height 1.67 m

3.3. Hyperbolic Paraboloid Shell roof:

The peak displacement ranges from (i) 42 mm to 30 mm for heights of 1.67 m to 2.5 m, (ii) 42 mm to 8 mm for bays 1 to 4 and (iii) 42 mm to 8 mm for thickness 50 mm to 90 mm. For every increment in height, number of bay and thickness, displacement gets reduced (Ref. Fig.10-14).

The general discussion is made for both cylindrical shell roof and hyperbolic paraboloid roof. From figures 5-14, it is observed that, the displacement decreases sharply from one to two bay, remains constant from two to four bay for CS and reduces gradually for HPS from one to four bay.

For both CS and HPS, for every increment in thickness, there is more reduction in displacement but for heights, there is less reduction in displacement, since the large displacement variation can be found in bay and thickness compared to height so that by adjusting these three parameters material can be saved.

3.4. Effect of Shape:

From table 4, results for CS and HPS are taken considering a constant thickness of 50 mm, there is huge difference in displacement between both shapes, which is given in terms of 'ξ' (Multiplication factor). As the number of bays increases, 'ξ' reduces. The maximum and minimum multiplication factors are 11.2 and 1.7 respectively.

4. Conclusion

The behaviour of cylindrical shell and hyperbolic paraboloid shell roofs is studied in terms of displacement. As per Hooke's law, the displacement is inversely proportional to displacement for a linear elastic structures. From the present study following conclusions are drawn:

- The stiffness does not affect much by changing the boundary conditions.
- The stiffness increases with increase in height, number bays and thickness.
- The stiffness in hyperbolic paraboloid shell roof is more compared to cylindrical shell roof.
- The material can be saved by adjusting 't', 'h' and 'n'.
- The sample of design tables are specified in APPENDIX, which are expected to be useful for designers.

Acknowledgement

I owe a debt of gratitude to the second autor and institute for supporting this work.

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APPENDIX
"Design Table"

Table.5: Design table for cylindrical shell roof with hinged boundary condition

n	t = 80 mm; h = 2.5 m; Stresses in "MPa"					t = 80 mm; h = 2 m; Stresses in "Mpa"					t = 80 mm; h = 1.67 m; Stresses in "Mpa"				
	σ_x		σ_y		τ_{xy}	σ_x		σ_y		τ_{xy}	σ_x		σ_y		τ_{xy}
	T	C	T	C	--	T	C	T	C	--	T	C	T	C	--
1	19	140	154	98	57	22	153	18	120	66	24	169	20	140	72
2	17	96	31	73	46	10	64	19	44	28	11	72	19	47	31
3	7	42	15	33	24	7	52	13	30	24	7	61	11	31	25
4	7	37	14	30	24	6	45	11	25	23	5	58	8	23	23

*σ_x= Normal stress along 'X' direction; σ_y= Normal stress along 'Y' direction; τ_{xy} = Shear stress; T = Tensile stresses; C = Compressive stresses.

Table.6: Design table for hyperbolic paraboloid shell roof with hinged boundary condition

n	t = 80 mm; h = 2.5 m; Stresses in "MPa"					t = 80 mm; h = 2 m; Stresses in "MPa"					t = 80 mm; h = 1.67 m; Stresses in "MPa"				
	σ_x		σ_y		τ_{xy}	σ_x		σ_y		τ_{xy}	σ_x		σ_y		τ_{xy}
	T	C	T	C	--	T	C	T	C	--	T	C	T	C	--
1	27	51	5	149	41	33	65	6	182	50	39	77	6	22	59
2	17	42	6	50	26	20	51	8	59	31	24	59	10	69	37
3	15	42	6	30	25	17	50	8	34	30	20	58	10	39	36
4	20	70	10	36	39	13	49	7	23	26	15	56	10	28	32

*σ_x= Normal stress along 'X' direction; σ_y= Normal stress along 'Y' direction; τ_{xy} = Shear stress; T = Tensile stresses; C = Compressive stresses.