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Research paper

Quadrilateral Lateral Snake Gluing of Path and Bistar are Prime Cordial Graphs

[1]l.Tamilselvi, [2]p. Selvaraju, [3]s.Gayathri

[1,3]Vinayaka Missions Research Foundation, AVIT, Department of Mathematics, Paiyanoor, Chennai.
[2]Department of Mathematics, Vel Tech MultitechDrRangarajanDrSakunthala Engineering College
**Corresponding author Email: [1]ltamilselvi@avit.ac.in, [2]pselvar@yahoo.com*

Abstract

The graph is called a Quadrilateral Snake graph. Which is defined as series connection of non-adjacent vertices of 'N' number of cycle and these vertex set and edge set are described below

$$V(QS_N) = \{c_k\}_{k=1}^{N+1} \cup \{u_i\}_{i=1}^{N} \cup \{v_j\}_{j=1}^{N}$$

$$E(QS_N) = \{c_k u_k\}_{k=1}^{N} \cup \{c_k v_k\}_{k=1}^{N} \cup \{u_k c_{k+1}\}_{k=1}^{N} \cup \{v_k c_{k+1}\}_{k=1}^{N}$$

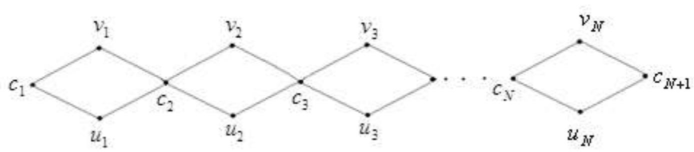

Figure 1: Quadrilateral Snake graph

A prime cordial labeling of a graph G with vertex set V(G) is a bijection such that each edge is assigned the label 1 if and 0 if then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph which admits a prime cordial labeling is called a prime cordial graph.
In this paper we prove that the graphs,

$$G_1 = P_m(QS_n), m \equiv 0(\mathrm{mod}\ 2), \forall n \geq 1 \text{ and } G_2 = B_{m,n}(QS_N), m, n \geq 2, m = n, N \geq 1 \text{ are prime cordial}.$$

Keywords: *Graph labeling, Prime cordial labeling, bistar graph, Path graph and Quadrilateral Snake graph.*

1. Introduction

The graph labeling problem that appears in graph theory has a fast development recently. This is not only due to its mathematical importance but also because of the wide range of the applications arising from this area, for instance, x-rays, crystallogrophy, coding theory, radar, astronomy, circuit design, design of good Radar type codes, Missile Guidance Codes and Convolution Codes with optimal autocorrelation properties and communication design.
The concept of prime cordial labeling was introduced by Sundaram [7] *et al.* and in the same paper they have investigated several results on prime cordial labeling. Vaidya and Vihol [8] have also discussed prime cordial labeling in the context of graph operations. For an exhaustive survey of these topics one may refer to the excellent survey paper of Gallian (2011)[2].

2. Main Results

$B_{n,n}$ is the n-bistar obtained from two disjoint copies of $K_{1,n}$ by joining the centre vertices by an edge. $B_{m,n}$ is the bistar obtained from two disjoint copies of $K_{1,m}$ and $K_{1,n}$ by joining the centre vertices by an edge. The graph $G_1 = P_m(QS_n)$ is defined as an isomorphic Quadrilateral snake one copy gluing with each m , n is the number of Block (i.e. C_4) of Quadrilateral snake QS_n in one copy.
The graph $G_2 = B_{m,n}(QS_N)$ is defined as an isomorphic Quadrilateral snake one copy gluing with each m , n .N is the

number of Blocks (i.e. C_4) of Quadrilateral snake QS_N in one copy.

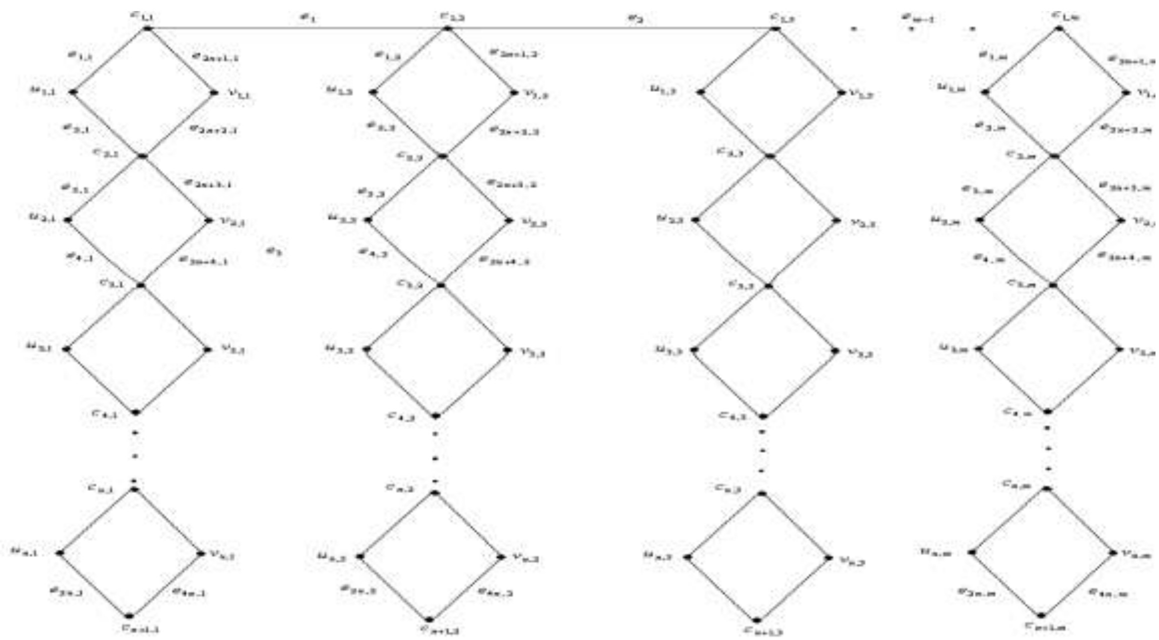

Figure 2: The graph $P_m(QS_n)$

Theorem1:
The graph $G_1 = P_m(QS_n), m \equiv 0(\mathrm{mod}\, 2), \forall n \geq 1$ is prime cordial

Proof
The graph $G_1 = P_m(QS_n)$ has $m(3n+1)$ vertices and $m(4n+1)-1$ edges. The graph of $G_1 = P_m(QS_n)$ has given in Figure 2.The prime cordial labeling for vertices of G is conveniently defined as set A,B and C as follows

$$A = \left\{ f(c_{i,j}) \,/\, \begin{array}{ll} 6i+2j+6n(j-1)-7, & i=1,2,3,\ldots,n+1, \quad 1 \leq j \leq \frac{m}{2}, \\ 2(3i-2)+2\left(j-1-\frac{m}{2}\right)(3n+1), & i=1,2,3,\ldots,n+1, \quad \frac{m}{2} < j \leq m \end{array} \right\}$$

$$B = \left\{ f(u_{i,j}) \,/\, \begin{array}{ll} 6i+2j+6n(j-1)-5, & i=1,2,3,\ldots,n, \quad 1 \leq j \leq \frac{m}{2}, \\ 2(3i-1)+2\left(j-1-\frac{m}{2}\right)(3n+1), & i=1,2,3,\ldots,n, \quad \frac{m}{2} < j \leq m \end{array} \right\}$$

$$C = \left\{ f(v_{i,j}) \,/\, \begin{array}{ll} 6i+2j+6n(j-1)-3, & i=1,2,3,\ldots,n, \quad 1 \leq j \leq \frac{m}{2}, \\ 6i+2\left(j-1-\frac{m}{2}\right)(3n+1), & i=1,2,3,\ldots,n, \quad \frac{m}{2} < j \leq m \end{array} \right\}$$

With the above labeling the corresponding edge labeling are defined by

$$A' = \left\{ f'(e_k) \,/\, \begin{array}{ll} 1, & 1 \leq k \leq \frac{m}{2} \\ 0, & \frac{m}{2} < k \leq m-1 \end{array} \right\}$$

$$B' = \left\{ f'(e_{i,j}) \,/\, \begin{array}{ll} 1, & 1 \leq i \leq 4n, 1 \leq j \leq \frac{m}{2} \\ 0, & 1 \leq i \leq 4n, \frac{m}{2} < j \leq m \end{array} \right\}$$

Table1: showing edge conditions of $G_1 = P_m(QS_n), m \equiv 0(\bmod 2), \forall n \geq 1$

Number of block (n)	Edge conditions
n is even (or) n is odd	$\lvert E_1 \rvert = \lvert E_0 \rvert + 1$

By the definition of prime cordial labeling, from the Table 1, it is clear that the number of edges assigned the label 0 and the number of edges assigned the label 1 differ by at most 1 .

Hence $G_1 = P_m(QS_n), m \equiv 0(\bmod 2), \forall n \geq 1$ is prime cordial.

The following is an illustrative example of labeling as given in the proof of theorem-1

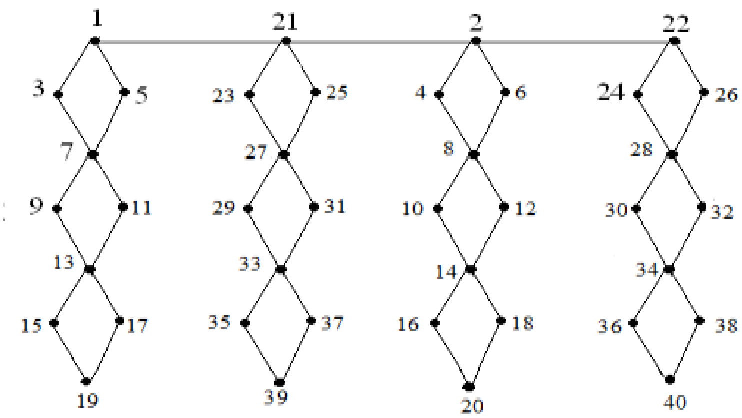

Figure 3: prime cordial labeling of $P_4(QS_3)$

Theorem-2

The graph $G_2 = B_{m,n}(QS_N), m, n \geq 2, m = n, N \geq 1$ is prime cordial.

Proof

Let the graph $G_2 = B_{m,n}(QS_N)$ has $(m+n)(1+3N)+2$ vertices, $(m+n)(1+4N)+1$ edges.

The graph of $G_2 = B_{m,n}(QS_N)$ is given in figure-4.

We define vertex labeling of $G_2 = B_{m,n}(QS_N)$ as follows.

$f(c_0) = 1$, $f(c_0') = 2$

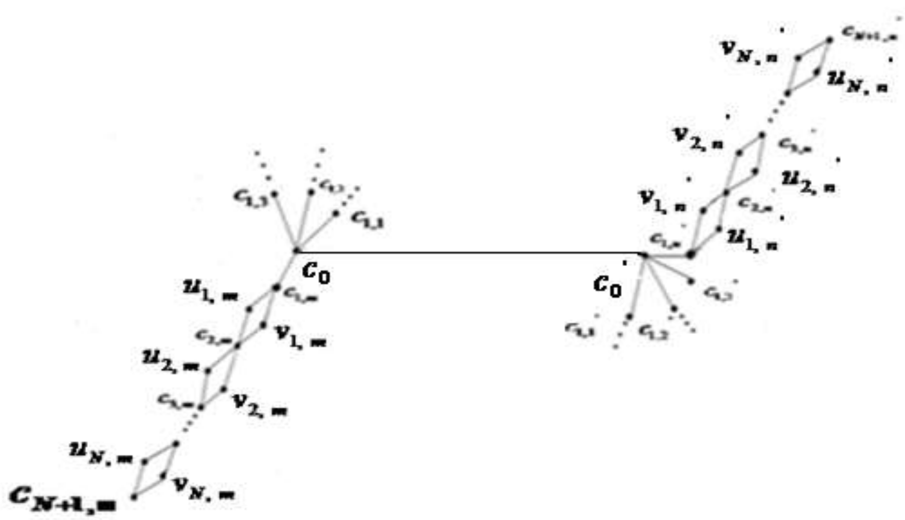

Figure 4: bistar graph $G_2 = B_{m,n}(QS_N)$

The prime cordial labeling for vertices of G is conveniently defined as set A,B,C,D,E &F as follows

$$A=\left\{ f(c_{i,j}) / \ 2(3i+j)-5+6(j-1)N, i=1,2,3,\ldots N+1, 1\le j\le m \right\}$$
$$B=\left\{ f(c_{i,j}') / \ 2(3i+j)-4+6(j-1)N, i=1,2,3,\ldots N+1, 1\le j\le n \right\}$$
$$C=\left\{ f(u_{i,j}) / \ 2(3i+j)-3+6(j-1)N, i=1,2,3,\ldots N, 1\le j\le m \right\}$$
$$D=\left\{ f(u_{i,j}') / \ 2(3i+j)-2+6(j-1)N, i=1,2,3,\ldots N, 1\le j\le n \right\}$$
$$E=\left\{ f(v_{i,j}) / \ 2(3i+j)-1+6(j-1)N, i=1,2,3,\ldots N, 1\le j\le m \right\}$$
$$F=\left\{ f(v_{i,j}') / \ 2(3i+j)+6(j-1)N, i=1,2,3,\ldots N, 1\le j\le n \right\}$$

With the above labeling the corresponding edge labeling are defined by

$$f'(c_0c_0')=1$$
$$f'(e_i)=1, \ if \ 1\le i\le m$$
$$f'(e_j)=0, \ if \ 1\le j\le n$$
$$f'(e_{i,j})=1, \ if \ 1\le i\le 4N, 1\le j\le m$$
$$f'(e_{i,j})=0, \ if \ 1\le i\le 4N, 1\le j\le n$$

Table 2: showing edge conditions of $G_2=B_{m,n}(QS_N)$

Bistar $B_{m,n}$ (m &n)	Number of block (N)	Edge conditions
m & n is even(or) odd	N is even (or) odd	$\lvert E_1\rvert=\lvert E_0\rvert+1$

By the definition of prime cordial labeling, from the Table 2 , it is clear that the number of edges assigned the label 0 and the number of edges assigned the label 1 differ by at most 1 .

Hence $G_2=B_{m,n}(QS_N), m,n\ge 2, m=n, N\ge 1$ is

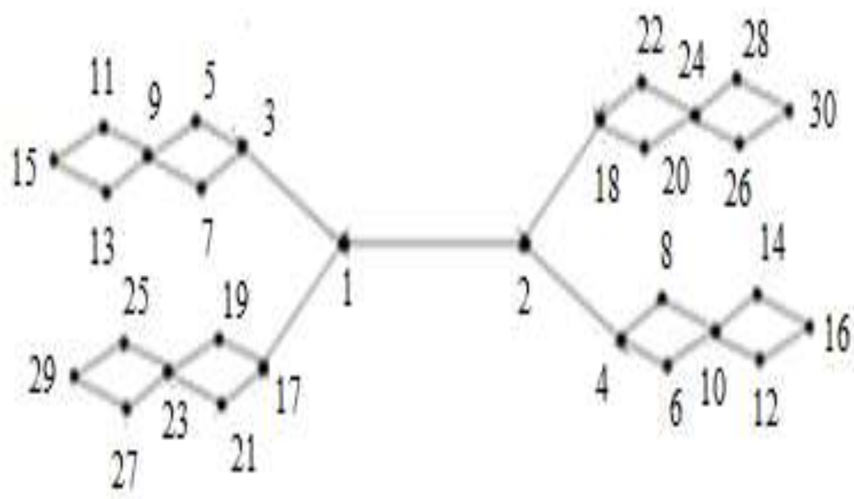

Figure 5: prime cordial labeling of bistar graph $B(2,2)(QS_2)$

3. Conclusion

According to literature survey, more work has been done in prime cordial labeling for cycle and path related graphs. In our work we determine the prime cordial labeling for new classes of Quadrilateral snake gluing of path graph and bistar graph.

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