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Research paper

Effect of semi vertex angle on stability derivatives for an oscillating cone for constant value of specific heat ratio

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Abstract

In this work we estimate stiffness derivatives for an oscillating non-slender axi-symmetric cone in pitch at Mach number greater than five. Results indicate that there is continous decrease in the stiffness derivative with the rise in the semi vertex angle for all the Mach number. The variation in the stiffness derivative is only visible for Mach numbers in the range from 5 to 7 for specific heat ratio of 1.67. For Mach number greater than 9 the stiffness derivative becomes independent of the geometrical parameter of the cone. The stiffness derivative becomes independent with the semi vertex angle for Mach 9 and above. Only at Mach 5 the variation of the stiffness derivative is distinct as at this Mach number the flow is in the transition zone from supersonic to hypersonic. The damping derivatives for h = 0, 0.2, 0.4, and 0.6 have similar trends, where the magnitude of the damping derivatives is decreasing with Mach number as well as with the semi vertex angle. However, the damping derivatives behave differently for h = 0.8 and 1.0 there is non-linear decrement in the magnitude of the damping derivatives, achieves minima, then again there sudden increase in the magnitude. Although this study is for sharp edged cone with attached shock wave case, once theory is developed it can easily extended for most practical cases with detached cases where nose bluntness will play an important role to tackle the aerodynamic heating.

Keywords: *Use about five key words or phrases in alphabetical order, Separated by Semicolon.*

1. Introduction

Aerodynamic launch vehicle encounter various flow regime during their flight, hence it is necessary to compute the pressure distribution and hence their stability derivative during the oscillation for simple shapes like cone and ogives which are very common in practice. Also, it is needed to take up the studies at fixed value of the specific heat ratio. In case we there is a need to acquire the data for a specific purpose. Keeping this in mind this study was conducted to compute the pressure distribution on the surface of the cone and hence their stability derivatives right from low hypersonic Mach numbers to higher Mach numbers.

The similarity parameter with the assumptions of attached shock wave and Mach number after the shock being > 2.5 was developed by Ghosh ([1]) at high Mach numbers. The similarity parameter was considered on the compression side of the cone and the pressure on the lee side was neglected as it will be negligible at hypersonic Mach numbers.

Crasta and Khan extended Ghosh's work to oscillating planar, non-planar wedges, and wings with straight and curved leading edges ([3-18], [20], [22-30]) to compute the stability derivatives in pitch and roll for considerable angle of incidence at supersonic as well as hypersonic Mach numbers. Ayesha et al. ([19], [32-33]), and Renita et al. ([21], [34]) derived analytical solutions to evaluate the stiffness and damping derivatives for oscillating cones, wedges, and wings in pitch and roll for quasi-steady and unsteady cases.

Results are obtained for supersonic/hypersonic flow for a perfect gas over oscillating cones for different semi vertex angles and the Mach numbers.

2. Analysis

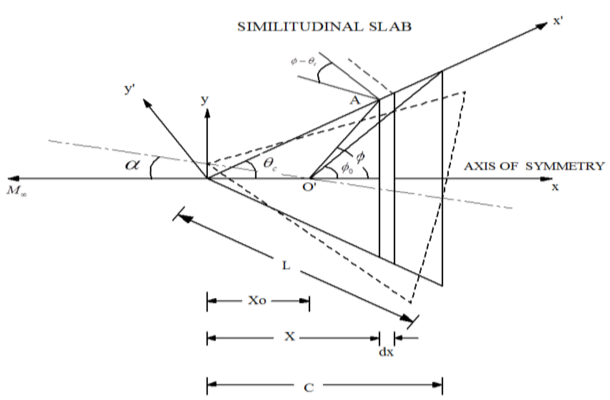

Fig. 1: Cone Geometry.

From the geometry we have

$$\tan\phi = \frac{x\tan\theta_c}{(x-x_0)}\ ; \quad \tan\phi_0 = \frac{c\tan\theta}{(c-x_0)}$$

Where ϕ is the angle subtended by A at O' with x-axis, and for different location of A, ϕ varies from π to ϕ_c, θ_c is the cone semi-angle and the chord length is c.

The Stiffness stability derivative presented by C_{m_α}, is

$$C_{m_\alpha} = \left[\frac{\partial M}{\partial\alpha}\right]_{\alpha,q\to 0} \frac{1}{\frac{1}{2}P_\infty U_\infty^2 S_b c}$$

Where

S_b = base area of the cone $= \pi(c\tan\theta_C)^2$,

c = the cone chord length.

The equation for the pressure ratio of a steady cone at zero incidence (Ghosh K., 1984), with the assumption that the bow shock is attached with the nose of the cone, is

$$\frac{P_{bo}}{P_\infty} = 1 + \gamma M^2{}_{po}\left(1 + \frac{1}{4}\varepsilon\right) \tag{1}$$

Where the density ratio is

$$\varepsilon = \frac{2 + (\gamma - 1)M^2{}_{po}}{2 + (\gamma + 1)M^2{}_{po}} \tag{2}$$

In addition, M_{po} the Mach number at the surface of the equivalent piston, operating in a conico-annular space; P_{bo} is the surface pressure on the body of the cone at zero incidences.

$$M_{po} = M_\infty \sin\theta_c$$

Where θ_c is the semi vertex angle of the cone.
Now

$$\frac{dP_{bo}}{dM_{po}} = 2\gamma P_\infty M_{po}\left[1 + \frac{1}{4}\left(1 + \frac{1}{2}M_{po}\cdot\frac{d\varepsilon}{dM_{po}}\right)\right] \tag{3}$$

Where

$$\frac{d\varepsilon}{dM_{po}} = \frac{-8M_{po}}{\left[2 + (\gamma + 1)M^2{}_{po}\right]^2} \tag{4}$$

After considerable algebra and simplification we get the following expressions for the stability derivatives.

$$C_{m_\alpha} = D\left[h^3(1-2n^2) - (1-h)\{H(2+h) + n^2h(1+2h)\}\right] \tag{5}$$

In addition, the expression for damping derivative is

$$C_{m_q} = \frac{D}{2}\left[h^4(2n^2 - 3n^4 - 1) - (1-h)\{H(3H + h(H + 2n^2) + 2h^2n^2) + n^4h^2(1+3h)\}\right] \tag{6}$$

Where

$$D = \frac{2}{3(1+n^2)}\left[1 + \frac{1}{4}\left(\varepsilon + \frac{1}{2}K\frac{d\varepsilon}{dM_{po}}\right)\right] \tag{7}$$

And

$$n = \tan\theta_c$$

Various results have been plotted and discussed.

3. Results and discussion

Based on the theory and corresponding derivation of the stiffness and damping derivatives results are being derived for a range of Mach number, semivertex angle and the pivot positions. The variations of stiffness derivative with semi vertex angle at a fixed pivot position for Mach numbers in the range from 5 to 15 is shown in Fig. 1. From the figure it is observed that there is continous decrease in the stiffness derivative with the increment in the semi vertex angle for all the Mach number of the present study. The physical reason for this decrement in the Stiffness derivative is due to the increase in the surface area of the cone with increase in the semi-vertex angle and another reason for this behaviour is due to the shift of the center of pressure towards the trailing edge with the increase in the semi vertex angle. When we calculate percentage change in the stiffness derivative for a fixed value of semi vertex angle of five degrees for various Mach number ranges they are 3 % for Mach [5 to 7], 2.6 % for Mach [7 to 9], 1 % for Mach [9 to 10], and 2.7 % for Mach [10 to 15]. It is also seen that there is no definite fixed trend as it is due to the combine deffect of the Mach number, pressusre distribution on the cone surface, and the semi vertex angle which is the geometrical parameter of the cone. This variation the stiffness derivative is only visible for Mach numbers in the range from 5 to 7. Once Mach number is greater than 9 the stiffness derivative becomes independent of the geometrical paprameter of the cone as the semi vertex angle. when we evaluate stiffness derivative at the nose of the cone. Since the cone is axi-symmetric body it will have different trend as compared with the similar results for wedge as the wedge geometry is 2-D.

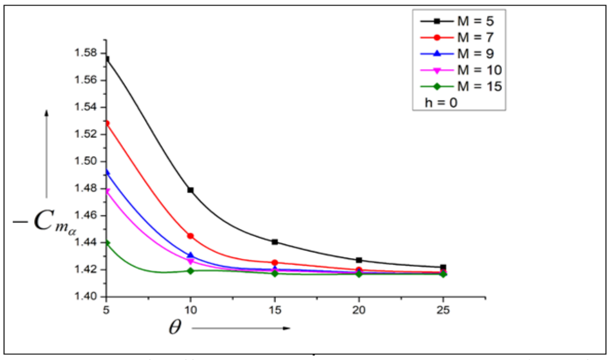

Fig. 2: Variation of Stiffness Derivative with Semi-Vertex Angle (Θ) for H = 0.0.

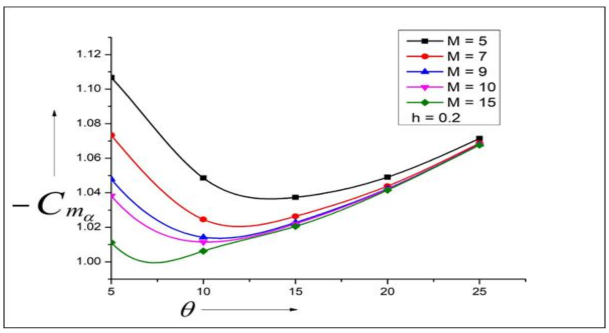

Fig. 3: Variation of Stiffness Derivative with Semi-Vertex Angle (Θ) for H = 0.2.

Results for pivot position h = 0.2 are presented in Fig. 2, keeping rest of the geometrical and kinematical parameters same, but in view of the location of the pivot position we are seeing a different trend as compared to the results for pivot position h = 0.0. The non-linear trend in the decrease of the stiffness derivative is up to semi vertex angle 15 degrees for Mach 5, for Mach 7 this value is 12 degrees, for Mach 9 and 10 it is 10 degrees, and for Mach 15 it is 7.5 degrees. After reaching the minimum value then there is sudden increase in the stiffness derivatives. It is also seen that there is no change in the stiffness derivatives for Mach 9, 10, and 15 with the semi vertex angle of the cone. The stiffness derivative becomes independent of Mach number and this trend is on the expected line. Only at Mach 5 the variation of the stiffness derivative is distinct as this Mach number the flow is in the transition from supersonic to hypersonic flow conditions.
Results for pivot position of h = 0.4 are presented in Fig. 3, keeping all other parameters the same. The trend is nearly the same with the exception that the magnitude of the stiffness derivative has reduced considerably as compared to the values for h= 0.2. Since the expression for stiffness derivatives contains non-linear terms, hence the trends are as they are expected.
For pivot position h = 0.6 the stiffness derivatives results are shown in Fig. 4. The results for this case are totally different, as the location of h = 0.6 seems to be very much close to the location of centre of the centre of pressure for the cone, hence we do not see any variations in the stiffness derivative with Mach number, however, the progressive increasing trend continues with the increase in the semi vertex angle, as the increased value of the semi vertex angle will have larger area, and the cone being the body of revolution it will have increased value of the pressure on the surface of the cone.
Results for h = 0.8, and 1.0 are shown in Figs. 5 and 6, the only difference in this figures and the previous one for h = 0.6 is that the magnitude of the stiffness derivative has decreased considerable due to the movement of the pivot position towards the trailing edge or at the trailing edge. Due to this there will change in the pitching moment values.

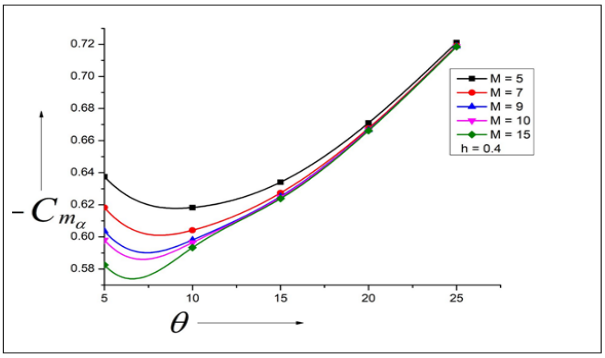

Fig. 4: Variation of Stiffness Derivative with Semi-Vertex Angle (Θ) for H = 0.4.

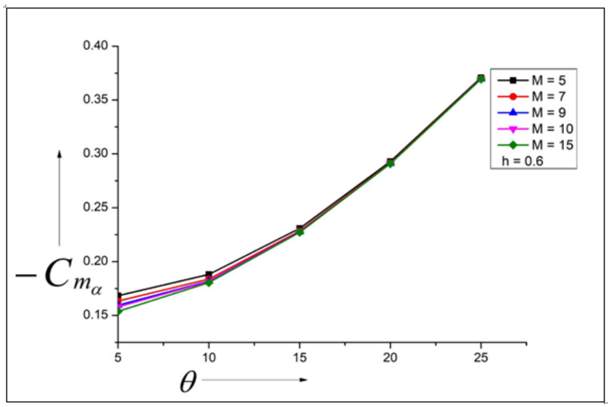

Fig. 5: Variation of Stiffness Derivative with Semi-Vertex Angle (Θ) for H = 0.6.

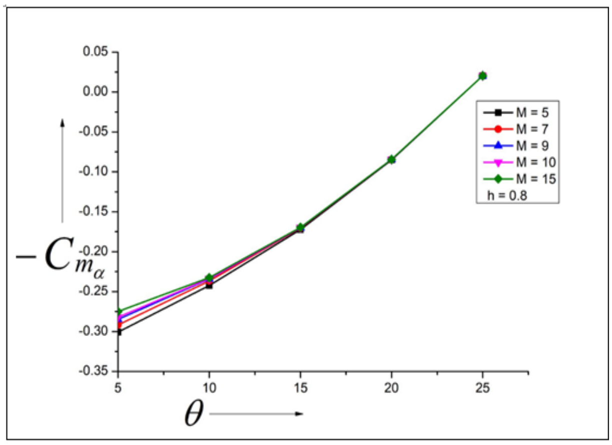

Fig. 6: Variation of Stiffness Derivative with Semi-Vertex Angle (Θ) for H = 0.8.

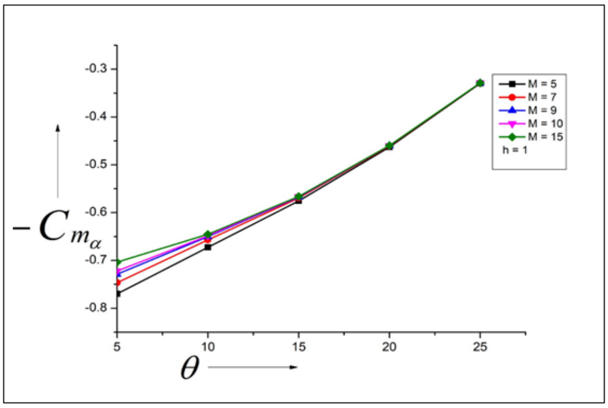

Fig. 7: Variation of Stiffness Derivative with Semi-Vertex Angle (Θ) for H = 1.0.

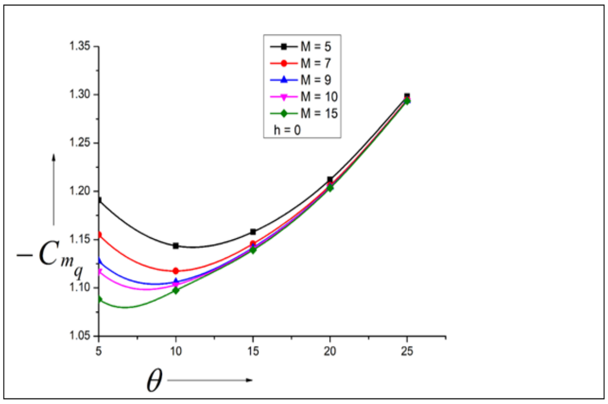

Fig. 8: Variation of Damping Derivative with Semi-Vertex Angle (Θ) for H = 0.0.

Figs. 7 to 10 present the damping derivatives for h = 0, 0.2, 0.4, and 0.6. In all these figures it is seen that the magnitude of the damping derivatives is decreasing with the Mach number as well as with the semi vertex angle. This trend of decrease in the damping derivative is different at various pivot positions it is ranging from six degrees to 7.5 degrees, and then after attaining the minima there is sudden increase in the damping derivatives. The physics behind this phenomena is the same as was discussed in the case of the stiffness derivatives.
However, the damping derivatives behave differently for h = 0.8, there is non-linear decrement in the magnitude of the damping derivatives, achieves minima, and then again there sudden increase in the magnitude. Whereas, for h = 1.0, there is continuous decline in the damping derivatives, and damping derivatives become independent Mach number for semi vertex angle 15 degrees and above. This trend is attributed to the pivot position, the inertia level, and the geometrical variables.

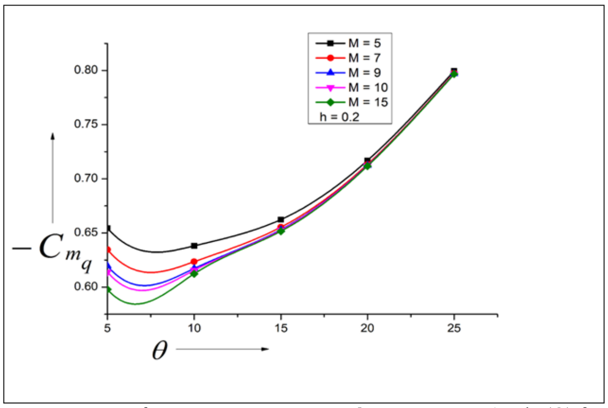

Fig. 9: Variation of Damping Derivative with Semi-Vertex Angle (Θ) for H = 0.2.

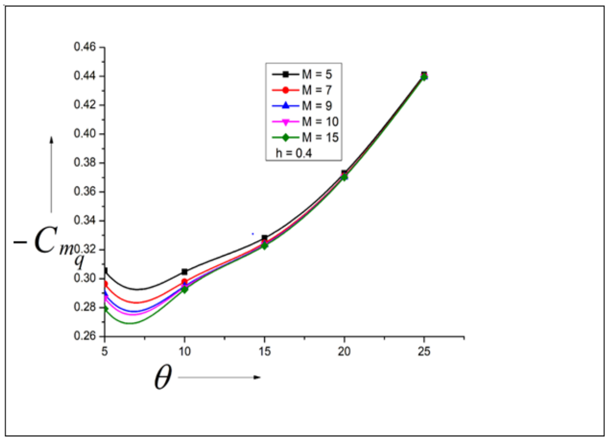

Fig. 10: Variation of Damping Derivative with Semi-Vertex Angle (Θ) for H = 0.4.

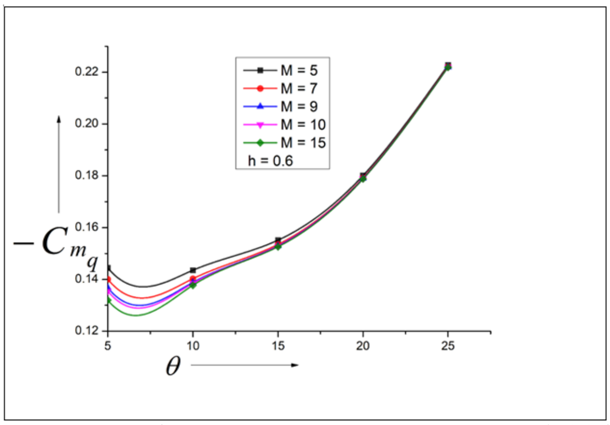

Fig. 11: Variation of Damping Derivative with Semi-Vertex Angle (Θ) for H = 0.6.

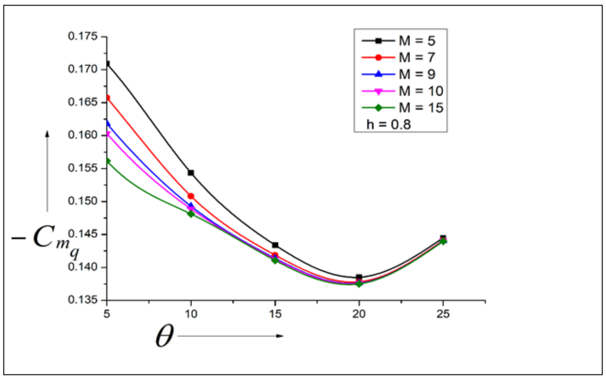

Fig. 12: Variation of Damping Derivative with Semi-Vertex Angle (Θ) for H = 0.8.

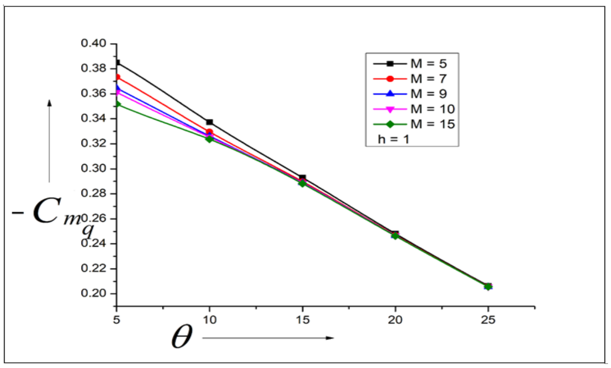

Fig. 13: Variation of Damping Derivative with Semi-Vertex Angle (Θ) for H = 1.0.

4. Conclusions

Based on the above discussion we can draw the following conclusions:

It is found that there is continous decrease in the stiffness derivative with the inhancement in the semi vertex angle of the cone for all the Mach number of the present study.

It is also seen that there is no definite and fixed trend as the results are due to the combined effect of the Mach number, pressure distribution on the cone surface, and the semi vertex angle of the cone. The variation the stiffness derivative is only visible for Mach numbers in the range from [5] to [7]. Once Mach number is greater than [9] the stiffness derivative becomes independent of the geometrical parameter of the cone as the semi vertex angle; when we evaluate stiffness derivative at the nose of the cone.

Results for pivot position h = 0.2, 0.4 and 0.6 keeping rest of the geometrical and kinematical parameters same are behaving differently in view of the location of the pivot position. After reaching the minimum value then there is sudden increase in the stiffness derivatives.

It is also seen that there is no change in the stiffness derivatives for Mach 9, 10, and 15 with the semi vertex angle. The stiffness derivative becomes independent of Mach number.

Only at Mach [5] the variation of the stiffness derivative is distinct as this Mach number the flow is in the transition zone from supersonic to hypersonic flow conditions.

Results for h= 0.8 and 1.0 behave differently the magnitude of the stiffness derivative has decreased considerable due to the movement of the pivot position towards the trailing edge.

Results for the damping derivatives for h = 0, 0.2, 0.4, and 0.6 have similar trends, it is seen that the magnitude of the damping derivatives is decreasing with the increase in the Mach number as well as with the semi vertex angle. This trend of decrease in the damping derivative is different at various pivot position it is ranging from six degrees to 7.5 degrees.

After attaining the minima there is sudden increase in the damping derivatives. However, the damping derivatives behave differently for h = 0.8, there is non-linear decrement in the magnitude of the damping derivatives, achieves minima, then again there is sudden increase in the magnitude. Whereas, for h = 1.0, there is continuous decline in the damping derivatives, and damping derivatives become independent of Mach number for semi vertex angle 15 degrees and above.

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