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Research paper

Achieving Power System Stability for Two Area Hydro Power System via LQR Techniques

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Abstract

Maintaining synchronism between different parts of power system is getting difficult over time. Frequency perturbation in power system can cause stalling in loads. If synchronism between generator and the power systems is lost at any time, voltage and current fluctuations may happen that gives catastrophic effect to the end users and power systems. In this paper, two area hydropower system was represented in state space form. The controllability and observability of the system were tested. The frequency deviation observer for both areas was formulated. Hence, the state feedback via integration of Linear Quadratic Regulator (LQR) and PI controller was designed for load frequency control (LFC) to preserve the stability of the power system due to load frequency deviation. The performance and efficacy of the proposed controller were then compared with traditional Proportional-Integral (PI) controller. The effectiveness of the proposed controller approach was demonstrated through two area hydropower system using MATLAB with SIMULINK toolbox.

Keywords: *frequency deviation; linear quadratic regulator; load frequency control; two area hydropower system.*

1. Introduction

An interconnected power system consists of control areas, which are connected to each other by tie line. In an interconnected power system, there are two main variables that are disturbed during transient power load area which are the area frequency and tie line power interchange. The key is to maintain the steady state at null position. Any sudden small load perturbation in any of the interconnected area will cause the deviation of frequency of all areas and also the tie line powers [1]. In case of a hydro power, the power system frequency regulation can be affected due to water flow fluctuation. This condition leads to imbalance between power generation and power demand, and as a result, frequency will deviate from its nominal value. Significant frequency deviations may cause under or over frequency relay operations, and finally disconnect some parts of system loads and generations [4-5].

The main objectives of load frequency control are to minimize the frequency deviation (Δf) and the tie line power interchange (P_{tie}) (ensure their steady state errors to be at zero) [3]. Any mismatch between generation and demand causes the system frequency to deviate from its nominal value. Thus, high frequency deviation may lead to system collapse. This necessitates a very fast and accurate controller to maintain the nominal system frequency.

Conventional LFC uses an integral controller [2], therefore, the Linear Quadratic Regulator (LQR) is the optimal theory of pole placement method which is that all the poles at desired location using state feedback gain matrix. Using feedback, the poles of the system can be shifted so it can shape the closed loop characteristic of system to meet the design requirement. Even though pole placement method can give the desired characteristic, but placing poles to the left-hand side (LHS) of the S-plane require massive control energy u. The LQR is an optimal solution considering the control signal u by minimizing the quadratic cost function [6-7]. In other words, LQR stabilizes the frequency changes by minimal control energy u.

In this paper, a two area hydropower system is represented in state space form. The integration of PI and LQR controller with full state observer is proposed in order to control the frequency deviation. The controllability and observability of the system is tested. The frequency deviation observer for both areas is formulated. Hence, the state-feedback (pole placement) through LQR is designed to preserve the stability of the power system due to load frequency deviation. It is expected that the proposed controller guarantees the stability of the frequency deviation and the performance terms due to the injection of the multifarious load perturbation.

2. Modeling of a Two Area Hydro Power System

Figure 1 shows the transfer function model of a two area hydro power systems with conventional PI controller. The system consists of governor turbine, transient droop compensation, hydro turbine and generator with load. In the interconnected power system, the two equal hydro power plants are interconnected through tie line. In this phase, a mathematical model of system in Fig. 1 is derived. The parameters of a power system under studies are depicted in Table 1.

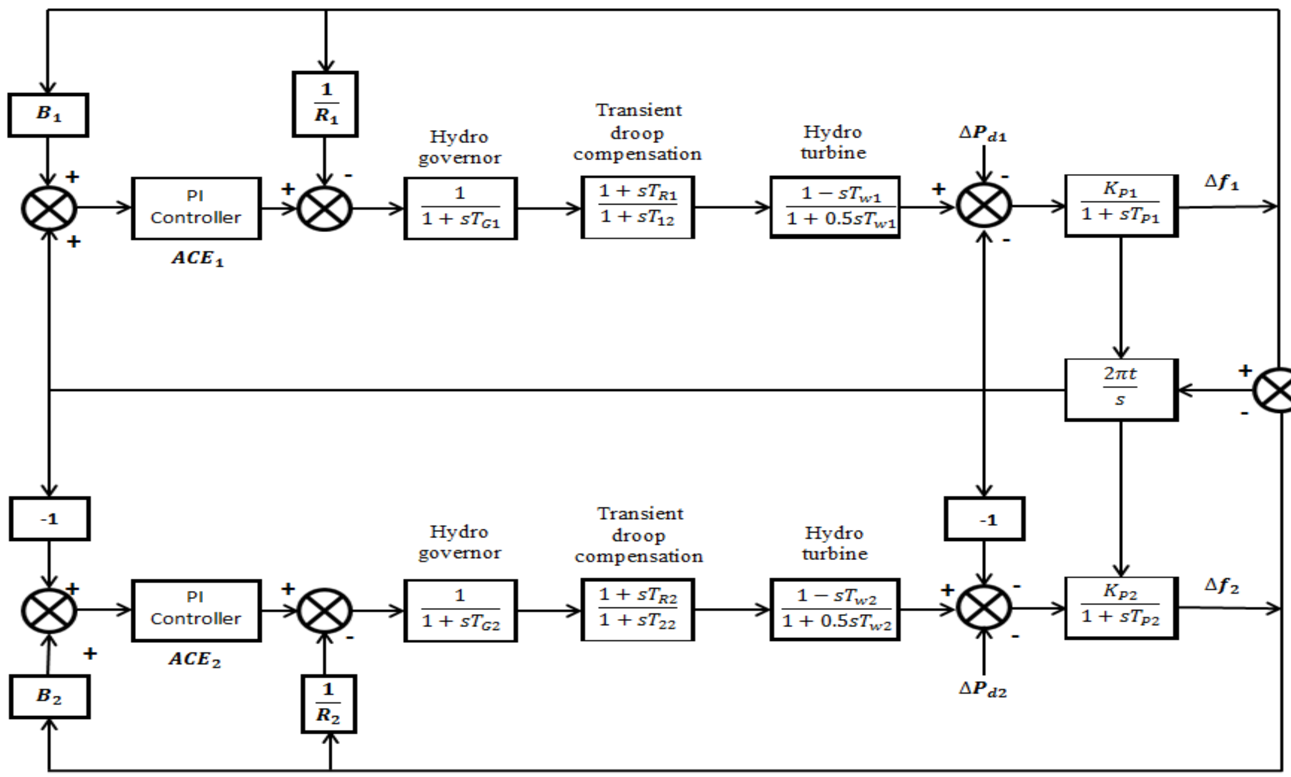

Fig. 1: Block diagram of two area hydropower system

Table 1: Nomencalture

Parameter	Description
K_P	Power system gains of area 1 and area 2
T_P	Power system time constant of area 1 and area 2
T_G	Governor time constant of area 1 and area 2
T_R	Reset time of area 1 and area 2
T_{12}	Permanent and transient droop of area 1 and area 2
T_W	Water starting time of area 1 and area 2
R	Governor speed regulation parameter of area 1 and area 2
Δf	Incremental frequency deviation
ΔP_d	Load disturbance

2.1. Power Generating Units

The hydro areas have been modeled using transfer function. Speed governor, turbine and generator constitute the various parts namely the hydro governor, transient droop compensation, hydro turbine and generation load model.

The hydro governor $G_G(s)$ is represented in (1), hydro turbine $G_T(s)$ is represented in (2), transient droop compensation $G_C(s)$ and load machine $G_P(s)$ are represented in (3) and (4) respectively.

$$G_G(s) = \frac{1}{1+T_G(s)} \tag{1}$$

$$G_T(s) = \frac{1-T_W(S)}{1+0.5T_W(s)} \tag{2}$$

$$G_C(s) = \frac{1+T_R(s)}{1+T_{12}(s)} \tag{3}$$

$$G_P(s) = \frac{K_P(s)}{1+T_P(s)} \tag{4}$$

2.2. State Space Model of a Two Area Hydropower System

The hydropower system can be expressed as in (5) and (6) [8-9]:

$$\dot{x} = Ax + Bu + Fd \tag{5}$$

$$y = C_x \tag{6}$$

where the system matrices $A \in \mathcal{R}^{nxn}, B \in \mathcal{R}^{nxm}$ and $C \in \mathcal{R}^{mxn}$. The state variable $x \in \mathcal{R}^n$, the control $u \in \mathcal{R}^m$ and the output $y \in \mathcal{R}^p$. Matrix $F \in \mathcal{R}^{nxm}$ is the load perturbation. The following assumptions are satisfied:

A1) states x is not available, but output y is available to measure.

A2) the number of input channel equals the number of output channel (i.e., $m = p$).

A3) the triple (A, B, C) is minimal realization [10].

A4) for a defined Hurwitz system matrices $A_o \in \mathcal{R}^{nxm}$, there exist two symmetric positive definite matrices $P \in \mathcal{R}^{nxn}$ and $Q \in \mathcal{R}^{nxn}$ such that P satisfies structural constraint $PH = C^T F^T$ with some selected matrix $F \in \mathcal{R}^{mxp}$.

From Figure 1, each block represents its own definition. Δf_1 and Δf_2 are defined as the frequency deviation in area 1 and area 2. ΔPt_{w1} and ΔPt_{w2} are defined as the output of the hydro turbine for both area 1 and area 2. ΔPg_2 and ΔPg_4 are defined as the output measure for the transient droop compensation at area1 and area 2. ΔPg_1 and ΔPg_3 are defined as the output measure of the hydro governor for area 1 and area 2. $\Delta P_{tie(1,2)}$ is the output measure of the tie tile power changes between both area, while $\int ACE_1\, dt$ and $\int ACE_2\, dt$ are defined as the area control area in area 1 and area 2 in the system of two area hydro power.

The state space model of the two area systems is derived based on each individual block in term of state variables based on Fig. 1. For specific system in Fig. 2, $n = 11$ and $m = 2$. Defining the following state variables:

$$x_1 = \Delta f_1 \qquad x_2 = \Delta Pt_{w1}$$

$$x_3 = \Delta Pg_2 \qquad x_4 = \Delta Pg_1$$

$$x_5 = \Delta f_2 \qquad x_6 = \Delta Pt_{w2}$$

$$x_7 = \Delta Pg_4 \qquad x_8 = \Delta Pg_3$$

$$x_9 = \Delta P_{tie(1,2)} \qquad x_{10} = \int ACE_1\, dt$$

$$x_{11} = \int ACE_2\, dt$$

$$\dot{x}_1 = -\frac{1}{T_{P1}}x_1 + \frac{K_{P1}}{T_{P1}}x_2 - \frac{K_{P1}}{T_{P1}}x_9 - \frac{K_{P1}}{T_{P1}}d_1 \quad (7)$$

$$\dot{x}_2 = \frac{2T_{R1}}{R_1T_{G1}T_{12}}x_1 - \frac{2}{T_{W1}}x_2 + \left[\frac{2}{T_{W1}} + \frac{2}{T_{12}}\right]x_3 + \left[\frac{2T_{R1}}{T_{G1}T_{12}} - \frac{2}{T_{12}}\right]x_4 - \frac{2T_{R1}}{T_{G1}T_{12}}u_1 \quad (8)$$

$$\dot{x}_3 = -\frac{T_{R1}}{R_1T_{G1}T_{12}}x_1 - \frac{1}{T_{12}}x_3 + \left[\frac{1}{T_{12}} - \frac{T_{R1}}{T_{G1}T_{12}}\right]x_4 + \frac{T_{R1}}{T_{G1}T_{12}}u_1 \quad (9)$$

$$\dot{x}_4 = -\frac{1}{R_1T_{G1}}x_1 - \frac{1}{T_{G1}}x_4 + \frac{1}{T_{G1}}u_1 \quad (10)$$

$$\dot{x}_5 = -\frac{1}{T_{P2}}x_5 + \frac{K_{P2}}{T_{P2}}x_6 + \frac{K_{P2}}{T_{P2}}x_9 - \frac{K_{P2}}{T_{P2}}d_2 \quad (11)$$

$$\dot{x}_6 = \frac{2T_{R2}}{R_2T_{G2}T_{22}}x_5 - \frac{2}{T_{W2}}x_6 + \left[\frac{2}{T_{W2}} + \frac{2}{T_{22}}\right]x_7 + \left[\frac{2T_{R2}}{T_{G2}T_{22}} - \frac{2}{T_{22}}\right]x_8 - \frac{2T_{R2}}{T_{G2}T_{22}}u_2 \quad (12)$$

$$\dot{x}_7 = -\frac{T_{R2}}{R_2T_{G2}T_{22}}x_5 - \frac{1}{T_{22}}x_7 + \left[\frac{1}{T_{22}} - \frac{T_{R1}}{T_{G1}T_{12}}\right]x_8 + \frac{T_{R1}}{T_{G2}T_{22}}u_2 \quad (13)$$

$$\dot{x}_8 = -\frac{1}{R_2T_{G2}}x_5 - \frac{1}{T_{G2}}x_8 + \frac{1}{T_{G2}}u_2 \quad (14)$$

$$\dot{x}_9 = 2\pi T x_1 - 2\pi T x_5 \quad (15)$$

$$\dot{x}_{10} = B_1x_1 + x_9 \quad (16)$$

$$\dot{x}_{11} = B_2x_5 - x_9 \quad (17)$$

The following matrices $A \in R^{11X11}$, $B \in R^{11X2}$ and $F \in R^{11X2}$ are shown in Appendix A. The conceptual block diagram of hydro power system is shown in Fig. 2.

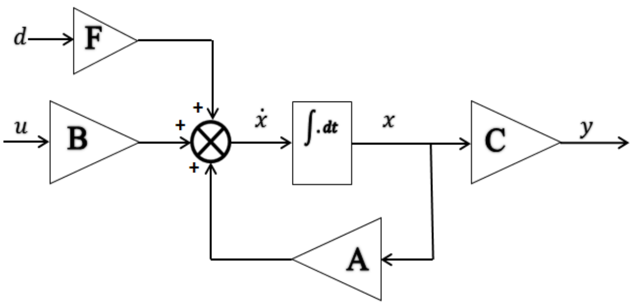

Fig. 2: State space model of a two area hydropower system

3. LQR Controller with Full State Observer

The design of optimal control linear system with quadratic performance index is called linear quadratic regulator (LQR) [11]. The LQR method gives algorithm to design optimal controller and generally known as LQR method. A recursive algorithm for selecting the state weighting matrix of a LQR problem to result in a set of desired closed-loop eigenvalues is presented. By dealing two or more dimensional matrix equations, an aggregation is used so that each process step is being placed either a real pole or a complex conjugate pair regardless of the systems [12]. The designed controller is used in feedback of system and thus it is also known as optimal feedback controller. The proposed control structure is shown in Fig. 3.

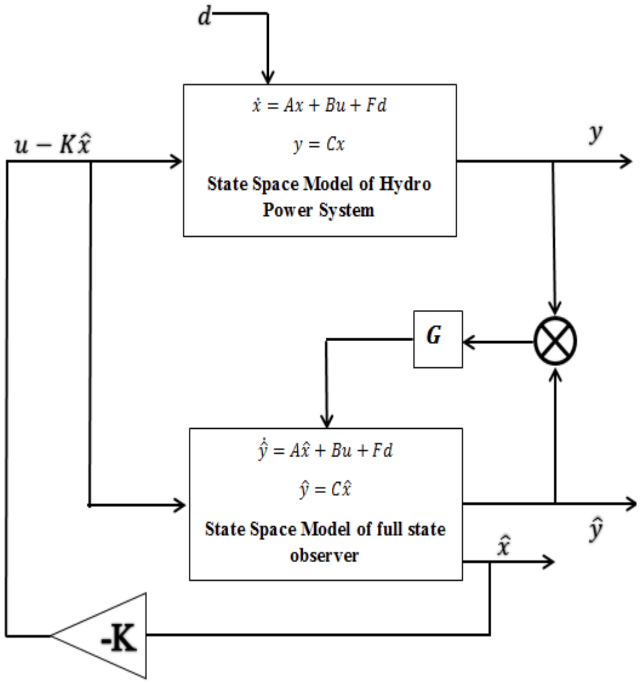

Fig. 3: Proposed algorithm and control structure

The objective is to develop a robust linear observer from the measured input and output signal to estimate the system state. Based on Figure 3, the state space model of an observer for system (5) and (6) is described in the form as (18) and (19) [13] where y is the actual output and $\hat{y}$ is the estimated output:

$$\dot{\hat{x}} = A\hat{x} + Bu + G(y - \hat{y}) + Fd \quad (18)$$

$$y = C\hat{x} \quad (19)$$

The optimal control law is given as (20), where K is the feedback gain:

$$u = -Kx \quad (20)$$

where $G \in \mathcal{R}^{nxp}$ is the gain for estimation correction term, $d \in \mathcal{R}^{m}$ is an external discontinuous control law and $F \in \mathcal{R}^{nxm}$ is the control injection map such that CF is non-singular. Having only estimated states in our possession, the first intention is to design the observer such that the linear quadratic performance index (J) in (21) is minimized.

$$J = \frac{1}{2}\int_0^{\infty}(\hat{x}^TQ\hat{x} + u^TRu)dt \quad (21)$$

$$J = \frac{1}{2}\int_0^{\infty}[(B_1x_1 + x_9)^2 + (B_2x_5 - x_9)^2 + (x_{10})^2 + (x_{11})^2 + (u_1)^2 + (u_2)^2]dt$$

$$J = \frac{1}{2}\int_0^{\infty}[{B_1}^2{x_1}^2 + 2B_1x_1x_9 + 2{x_9}^2 \; {B_2}^2{x_5}^2 - 2B_2x_5x_9 + {x_{10}}^2 \; {x_{11}}^2 + {u_1}^2 + {u_2}^2]dt$$

Note that Q and R are square positive definite matrices. Using above relation, the 'Q' and 'R' matrices as shown in Appendix B can be directly obtained by solving the performance index using (21).

For this part, the LQR+PI controller has been designed in simulation works by tuning the value of matrix Q and weight factor R. The values of matrix Q and weight factor R have been used in designing the LQR controller as shown in Appendix B. By substituting the values of parameters given in Table 2 [3] in the matrices A, B, Q and R, the value of feedback gain K can be obtained using simulation in MATLAB.

Table 2: Parameters value

Parameters	Value
$K_{P1} = K_{P2}$	120 Hz/pu
$T_{P1} = T_{P2}$	20s
$T_{G1} = T_{G2}$	48.75s
$T_{R1} = T_{R2}$	5s
$T_{12} = T_{22}$	0.513s
$T_{W1} = T_{W2}$	1s
$R_1 = R_2$	2.4 Hz/pu Mw
$B_1 = B_2$	0.425 pu Mw/Hz
$2\pi T$	0.4442

The matrix 'S' as attached in Appendix C has been real, positive definite and symmetric. For the eigenvalues of matrix 'S' are real and positive values as shown in (22), while the feedback gain matrix 'K' is shown in (23). Hence, the control inputs of the two area hydropower system are shown in (24) and (25).

$$eig(S) = \begin{bmatrix} 0.0001 \\ 0.0106 \\ 0.0895 \\ 0.2966 \\ 0.5516 \\ 1.5196 \\ 1.7832 \\ 2.6609 \\ 4.6833 \\ 44.4480 \\ 178.3151 \end{bmatrix} \tag{22}$$

$$K = \begin{bmatrix} 0.1384 & 0.1620 \\ 0.7286 & 0.8055 \\ 2.6927 & 2.9837 \\ 5.6961 & -0.2433 \\ 0.1345 & 1.2247 \\ 0.1901 & 2.2200 \\ 0.1738 & 3.0235 \\ -0.0977 & -1.4016 \\ -0.7435 & 0.2781 \\ 0.7956 & 0.6058 \\ -0.6058 & 0.7956 \end{bmatrix}^T \tag{23}$$

$$u_1 = \begin{bmatrix} -0.1384x_1 \\ -0.7286x_2 \\ -2.6927x_3 \\ -5.6961x_4 \\ -0.1345x_5 \\ -0.1901x_6 \\ -0.1738x_7 \\ 0.0977x_8 \\ 0.7435x_9 \\ -0.7956x_{10} \\ 0.6058x_{11} \end{bmatrix}^T \tag{24}$$

$$u_2 = \begin{bmatrix} -0.1620x_1 \\ -0.8055x_2 \\ -2.9837x_3 \\ 0.2433x_4 \\ -1.2247x_5 \\ -2.2200x_6 \\ -3.0235x_7 \\ 1.4016_8 \\ -0.2781x_9 \\ -0.6058x_{10} \\ -0.7956x_{11} \end{bmatrix}^T \tag{25}$$

The closed loop system matric 'A_C' has been obtained as shown in Appendix C. The eigenvalues of open loop system matrix 'A' are shown in (26). Two eigenvalues are zero and the remaining has negative real parts indicating that the system is marginally stable before applying the optimal control strategy.

While the eigenvalues of closed loop system matrix 'A_C' are shown in (27). The real parts of all the eigenvalues of 'A_C' are negative indicating that after applying the optimal control strategy, the system is asymptotically stable.

$$eig(A) = \begin{bmatrix} 0.0000 + 0.0000i \\ 0.0000 + 0.0000i \\ 0.1265 + 2.3278i \\ 0.1265 - 2.3278i \\ -3.0913 + 0.0000i \\ -2.8486 + 0.0000i \\ -1.3538 + 0.0000i \\ -0.6217 + 0.0000i \\ -0.1284 + 0.2959i \\ -0.1284 - 0.2959i \\ -0.0203 + 0.0000i \end{bmatrix} \tag{26}$$

$$eig(A_C) = \begin{bmatrix} -1.1069 + 2.7163i \\ -1.1069 - 2.7163i \\ -3.5225 + 0.0000i \\ -2.8859 + 0.0000i \\ -1.0486 + 1.5310i \\ -1.0486 - 1.5310i \\ -1.4103 + 0.0000i \\ -09931 + 0.0000i \\ -0.1402 - 0.0924i \\ -0.1402 - 0.0924i \\ -0.0272 + 0.0000i \end{bmatrix} \tag{27}$$

4. Results and Discussion

The performance of LQR with full state observer was compared with the PI controller as shown in Fig. 4, 5, 6 and 7. The responses shown here are in the form of dynamic responses of each area frequencies and the power of tie line for the two area power system model. The stability for closed loop system for the model has been determined by their eigenvalues using two different controllers; PI controller and LQR+PI controller, as shown in Table 3.

Fig. 4 and 5 show the dynamic responses of deviation in frequency for both the areas $(\Delta f_1, \Delta f_2)$, Fig. 6 shows the power deviation in tie line $(\Delta P_{tie(1,2)})$ and Fig. 7 shows the different change in frequency (Hz) for both areas for the power system has two control areas with the load disturbance of load powers which are the input disturbance taken as, $d_1 = 1\%, d_2 = 1\%$ at $t = 20$ seconds.

Fig. 4 and 5 compare the frequency deviation (in %) at area 1 and area 2 when the system was disturbed by persistent perturbation. At 1% load perturbation, the system with PI controller oscillated with a high settling deviation at $\Delta f_{(1,2)PI} = 0.0435\,pu$ and reached its steady state at $t = 29$ seconds. Compared with the system of LQR+PI controller, both in area 1 and area 2 gave a steady state as $t = 23$ seconds with a low settling deviation $\Delta f_{(1)LQR+PI} = 0.0237\,pu$ for area 1 and $\Delta f_{(2)LQR+PI} = 0.0186\,pu$ for area 2.

Fig. 6 shows the changes of power tie line in the system between the PI controller and the LQR+PI controller. Before the presence of LQR, the system gave a high changes of power at $\Delta tie\ line_{(1,2)PI} = 0.0793\,pu$ at $t = 36$ seconds. By adding the full state observer, the power tie line system was reduced until the oscillation of the system was stable to $\Delta tie\ line_{(1,2)LQR+PI} = 0.0059\,pu$ at $t = 21$ seconds.

The frequency demand in any system was at 50Hz, as shown in Fig. 7 the frequency (Hz) with PI controller was very high which was at $F(Hz)_{PI} = 52.9372$ Hz, while the system that combined with LQR controller led the system to return the frequency back to its desired value at $F(Hz)_{LQR+PI} = 50.0237$ Hz.

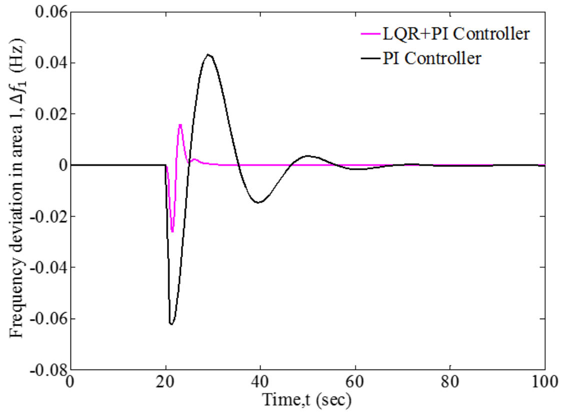

Fig. 4: Change in frequency deviation in area 1

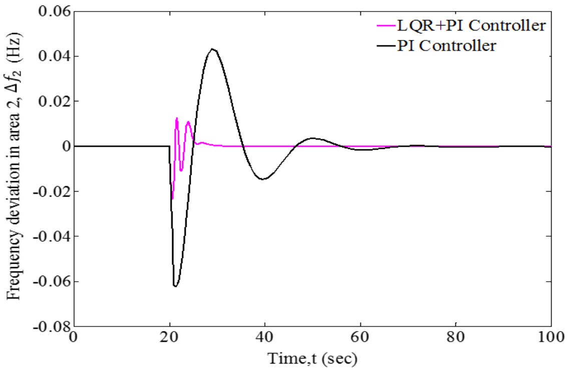

Fig. 5: Change in frequency deviation in area 2

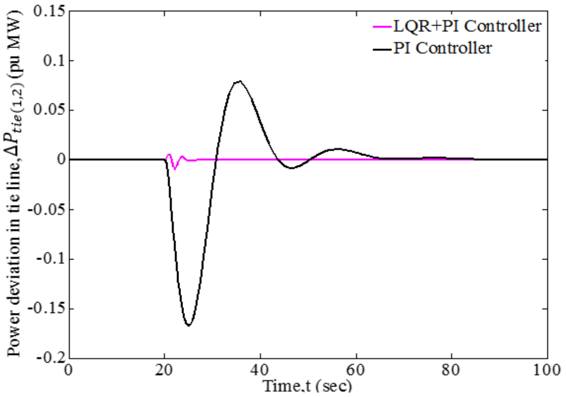

Fig. 6: Change in tie line power between area 1 and area 2

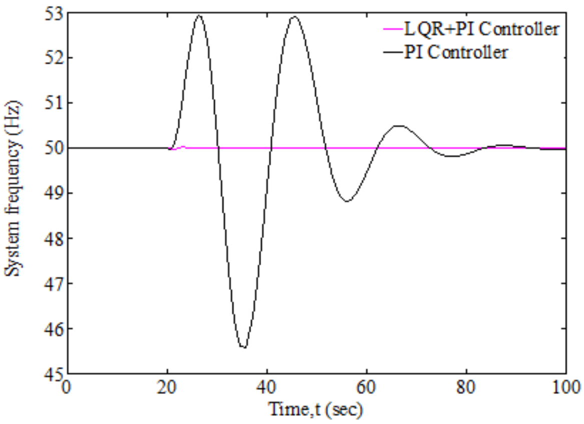

Fig. 7: The frequency changes for the power system

Table 3: Comparison of the deviation between LQR+PI controller with PI controller

		LQR+PI	PI
Δf_1	=	0.0237 pu	0.0435 pu
Δf_2	=	0.0186 pu	0.0435 pu
$\Delta P_{tie(1,2)}$	=	0.0059 pu	0.0793 pu
$f(Hz)$	=	50.0237 Hz	52.09372 Hz

Table 3 reveals that mixed LQR+PI algorithm provides better settling performance as compared with conventional PI controller.

5. Conclusion

The main aim of this paper is the design of the LQR with the combination of PI controller method for the two area hydro power system. With this developed system, the integration of the LQR with the PI controller in the system shows it robustness compared with the conventional PI controller system.

The proposed control algorithm is simulated and the performance of LQR+PI controller and PI controller results are literally compared and analysed.

By synthesizing the LQR+PI control method in the system, the power system shows low overshoot and shorter settling time due to frequency deviation that results from load changes. The stability of the proposed control scheme is achieved in response to the load perturbation in the initial state and the proposed LQR+PI control scheme offers the controllability and the observability set of solutions.

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Appendix A

System matrix A

$$A = \begin{bmatrix}
\frac{-1}{T_{P1}} & \frac{K_{P1}}{T_{P1}} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-K_{P1}}{T_{P1}} & 0 & 0 \\
\frac{2T_{R1}}{R_1T_{G1}T_{12}} & \frac{-2}{T_{W1}} & \frac{2}{T_{W1}}+\frac{2}{T_{12}} & \frac{2T_{R1}}{T_{G1}T_{12}}-\frac{2}{T_{12}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-T_{R1}}{R_1T_{G1}T_{12}} & 0 & \frac{-1}{T_{12}} & \frac{1}{T_{12}}-\frac{T_{R1}}{T_{G1}T_{12}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-1}{R_1T_{G1}} & 0 & 0 & \frac{-1}{T_{G1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{-1}{T_{P2}} & \frac{K_{P2}}{T_{P2}} & 0 & 0 & \frac{K_{P2}}{T_{P2}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{2T_{R2}}{R_2T_{G2}T_{22}} & \frac{-2}{T_{W2}} & \frac{2}{T_{W2}}+\frac{2}{T_{22}} & \frac{2T_{R2}}{T_{G2}T_{22}}-\frac{2}{T_{22}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{-T_{R2}}{R_2T_{G2}T_{22}} & 0 & \frac{-1}{T_{22}} & \frac{1}{T_{22}}-\frac{T_{R2}}{T_{G2}T_{22}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{-1}{R_2T_{G2}} & 0 & 0 & \frac{-1}{T_{G2}} & 0 & 0 & 0 \\
2\pi T & 0 & 0 & 0 & -2\pi T & 0 & 0 & 0 & 0 & 0 & 0 \\
B_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & B_2 & 0 & 0 & 0 & -1 & 0 & 0
\end{bmatrix}$$

System matrix B

$$B = \begin{bmatrix}
0 & 0 \\
\frac{2T_{R1}}{T_{G1}T_{12}} & 0 \\
\frac{T_{R1}}{T_{G1}T_{12}} & 0 \\
\frac{1}{T_{G1}} & 0 \\
0 & 0 \\
0 & \frac{2T_{R2}}{T_{G2}T_{22}} \\
0 & \frac{T_{R2}}{T_{G2}T_{22}} \\
0 & \frac{1}{T_{G2}} \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}$$

System matrix F

$$F = \begin{bmatrix}
\frac{-K_{P1}}{T_{P1}} & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & \frac{-K_{P2}}{T_{P2}} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}$$

Appendix B

System matrix Q

$$Q = \begin{bmatrix} B_1{}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & B_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & B_2{}^2 & 0 & 0 & 0 & -B_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ B_1 & 0 & 0 & 0 & -B_2 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

System matrix R

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Appendix C

System matrix S

$$S = \begin{bmatrix} 0.8471 & 2.0104 & 4.3975 & 3.0871 & 0.1200 & 0.1030 & 0.1022 & -0.0562 & -1.5530 & 1.6991 & -0.5514 \\ 2.0104 & 5.7363 & 14.1432 & 9.5226 & 0.6817 & 0.6954 & 0.5452 & -0.3050 & -5.3268 & 4.5652 & -1.7540 \\ 4.3975 & 14.1432 & 38.6541 & 30.3052 & 2.0663 & 2.3711 & 1.9783 & -1.1111 & -13.7275 & 11.7879 & -5.6884 \\ 3.0871 & 9.5226 & 30.3052 & 168.0223 & -0.2896 & -0.2800 & -0.1790 & 0.1222 & -6.3153 & 12.9107 & -8.2951 \\ 0.1200 & 0.6817 & 2.0663 & -0.2896 & 0.6318 & 0.8350 & 0.7841 & -0.4309 & -0.8183 & 0.4046 & 0.4616 \\ 0.1030 & 0.6954 & 2.3711 & -0.2800 & 0.8350 & 1.2638 & 1.3709 & -0.7319 & -0.5437 & 0.4658 & 0.5592 \\ 0.1022 & 0.5452 & 1.9783 & -0.1790 & 0.7841 & 1.3709 & 1.7957 & -0.8718 & 0.0303 & 0.3988 & 0.5122 \\ -0.0562 & -0.3050 & -1.1111 & 0.1222 & -0.4309 & -0.7319 & -0.8718 & 2.3645 & 0.0051 & -0.2491 & -0.2250 \\ -1.5530 & -5.3268 & -13.7275 & -6.3153 & -0.8183 & -0.5437 & 0.0303 & 0.0051 & 6.3256 & -3.6918 & 1.4016 \\ 1.6997 & 4.5652 & 11.7879 & 12.9107 & 0.4046 & 0.4658 & 0.3988 & -0.2491 & -3.6918 & 5.3292 & -2.0180 \\ -0.5514 & -1.7540 & -5.6884 & -8.2951 & 0.4616 & 0.5592 & 0.5122 & -0.2250 & 1.4016 & -2.0180 & 3.3882 \end{bmatrix}$$

System matrix A_C

$$A_C = \begin{bmatrix} -0.0500 & 6.0000 & 0 & 0 & 0 & 0 & 0 & 0 & -6.0000 & 0 & 0 \\ 0.2220 & -1.7086 & 6.9754 & -1.2209 & 0.0538 & 0.0760 & 0.0695 & -0.0391 & -0.2973 & 0.3182 & -0.2423 \\ -0.1110 & -0.1456 & -2.4876 & 0.6108 & -0.0269 & -0.0380 & -0.0347 & 0.0195 & 0.1486 & -0.1590 & 0.1211 \\ -0.0114 & -0.0149 & -0.0552 & -0.1373 & -0.0028 & -0.0039 & -0.0036 & 0.0020 & 0.0152 & -0.0163 & 0.0124 \\ 0 & 0 & 0 & 0 & 6.0000 & 6.0000 & 0 & 0 & 6.0000 & 0 & 0 \\ 0.0648 & 0.3221 & 1.1932 & -0.0973 & 0.6564 & -1.1122 & 7.1077 & -4.0593 & 0.1112 & 0.2423 & 0.3182 \\ -0.3239 & -1.6103 & -5.9644 & 0.4863 & -2.5315 & -4.4378 & -7.9933 & 4.5513 & -0.5559 & -1.2111 & -1.5904 \\ -0.0033 & -0.0165 & -0.0612 & 0.0050 & -0.0337 & -0.0455 & -0.0620 & 0.0082 & -0.0057 & -0.0124 & -0.0163 \\ 0.4442 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.4250 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.0000 & 0 & 0 \end{bmatrix}$$