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Research paper

Complete synchronization of a novel 6-D hyperchaotic Lorenz system with known parameters

Ahmed S. Al-Obeidi [1], Saad Fawzi AL-Azzawi [1] *

[1] Department of Mathematics, College of Computer Sciences and Mathematics, University of Mosul, Mosul, Iraq
**Corresponding author E-mail: saad_fawzi78@yahoo.com*

Abstract

There has been an increasing interest in field of high-dimensional systems and their synchronization phenomena. This paper deals with complete synchronization between two identical 6-D hyperchaotic Lorenz systems based on nonlinear control strategy. The designed control functions for the synchronization between the drive and response systems are succeed to achieve complete synchronization with the states of both systems are measurable and the parameters are known. Numerical simulations have verified the analytical synchronization technique.

Keywords: *Chaos Synchronization; 6-D Hyperchaotic Lorenz Systems; Lyapunov Stability Theory.*

1. Introduction

Chaos control is one of the chaos treatment, which contains two aspects, namely, chaos control and chaos synchronization. Synchronization means to control a chaotic system to follow another chaotic system[1]. Chaos control and chaos synchronization play very important topic in nonlinear dynamical systems and have great significance in the application of chaos [2]. Especially, the subject of chaos synchronization has received considerable attention due to its potential applications in engineering, physics, secure communication, networks, control theory, artificial neural network. etc. [3], [4]. Historically, chaos control is discover by Ott et al. while synchronization is introduced by Pecora and Carroll in 1990s which opens the way for chaotic systems synchronization [4 - 6].

Different kinds of synchronization phenomena have been presented such as complete synchronization (CS), generalized synchronization (GS), lag synchronization, anti-synchronization (AS), projective synchronization (PS), generalized projective synchronization (GPS) [7]. The complete synchronization (full or synchronization) play an important role in engineering applications such as secure communication [8]

Initially, the synchronization phenomena are applied on low-dimensional systems (3-D chaotic system). But, for these systems, there is just one positive Lyapunov exponent. In secure communication, messages masked by such simple chaotic systems are not always safe [7], [8]. It is suggested that this problem can be overcome by using higher-dimensional hyperchaotic systems, which have increased randomness and higher unpredictability. Due to its higher unpredictability than chaotic systems, the hyperchaos may be more useful in some fields such as secure communication [9]. So, it's needed to discover hyperchaotic systems, these system are characterized as a chaotic system with more than one positive Lypunov exponent, and have more complex and richer dynamical behaviors than chaotic system. Historical, Rössler system is the first hyperchaotic systems which discover in 1979, Since then, many hyperchaotic systems have been discover [10], such as hyperchaotic Lorenz system [11], hyperchaotic Liu system[12], hyperchaotic Chen system, Modified hyperchaotic Pan system [12] as well as to propose a 5-D hyperchaotic system such as A novel 5-D hyperchaotic Lorenz system [9]., a novel hyperjerk system with two nonlinearities. Currently, a novel 6-D hyperchaotic Lorenz is discover by Yang which contains four positive Lyapunov Exponents [13], [14].

Results of previous works indicated that the chaos synchronization with unknown parameters can be easily found compare with other hyperchaotic systems with known parameters [6,11].

The main contribution of this paper is the achieving synchronization between two identical 6-D hyperchaotic Lorenz system via nonlinear control strategy when the parameters are known.

2. System description

The Lorenz system was the first 3-D chaotic system to be modeled and one of the most widely studied. The original system was modified into a 4-D and 5-D hyperchaotic systems by introducing a linear feedback controller. In 2015, Yang constructed a 6-D hyperchaotic system which contains four positive Lyapunov Exponents $LE_1 = 1.0034$, $LE_2 = 0.57515$, $LE_3 = 0.32785$, $LE_4 = 0.020937$, and two negative Lyapunov Exponents $LE_5 = -0.12087$, $LE_6 = -12.4713$. The 6-D system which is described by the following mathematical form [13], [14]:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_4 \\ \dot{x}_2 = cx_1 - x_2 - x_1x_3 + x_5 \\ \dot{x}_3 = -bx_3 + x_1x_2 \\ \dot{x}_4 = dx_4 - x_1x_3 \\ \dot{x}_5 = -kx_2 \\ \dot{x}_6 = lx_2 + hx_6 \end{cases} \quad (1)$$

Where a, b, c, d, k, l and h are constant. The Lorenz system has a hyperchaotic attractor when $a = 10, b = \frac{8}{3}$, $c = 28, d = 2$, $k = 8.4$, $l = 1$ and $h = 1$. Figures 1-3 show the 3-D attractor of the system (1), while figures 4-6 show the 2-D attractor of the system (1).

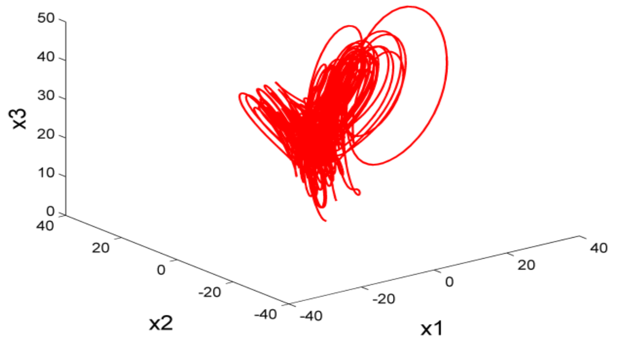

Fig. 1: 3-D attractor of the system (1) in the (x_1, x_2, x_3) space.

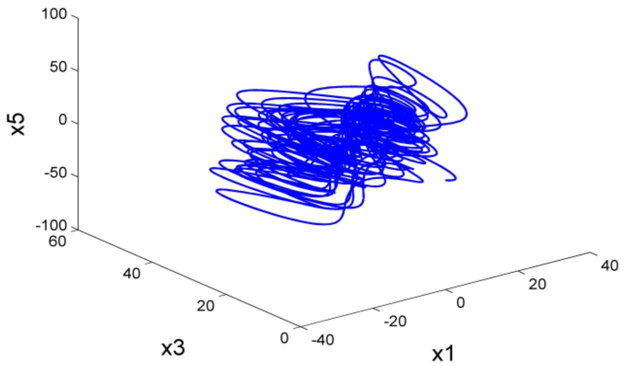

Fig. 2: 3-D attractor of the system (1) in the (x_1, x_3, x_5) space.

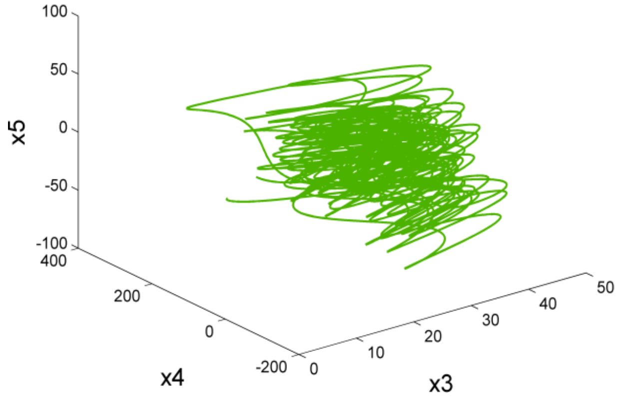

Fig. 3: 3-D attractor of the system (1) in the (x_3, x_4, x_5) space.

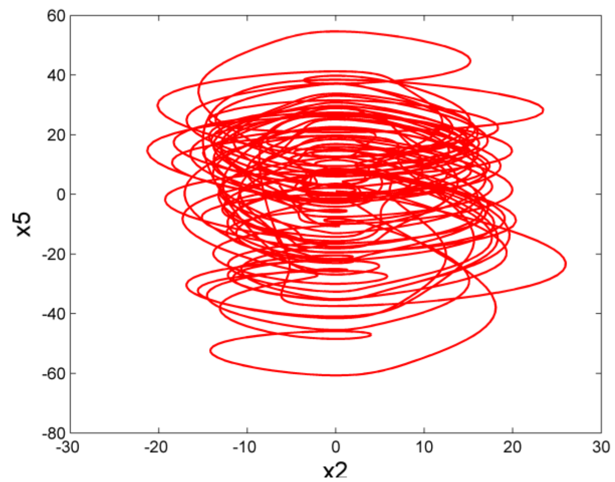

Fig. 4: 2-D attractor of the system (1) in the (x_2, x_5) plane.

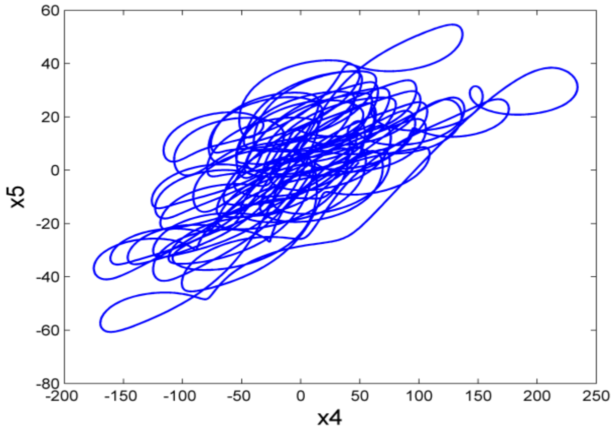

Fig. 5: 2-D attractor of the system (1) in the (x_4, x_5) plane.

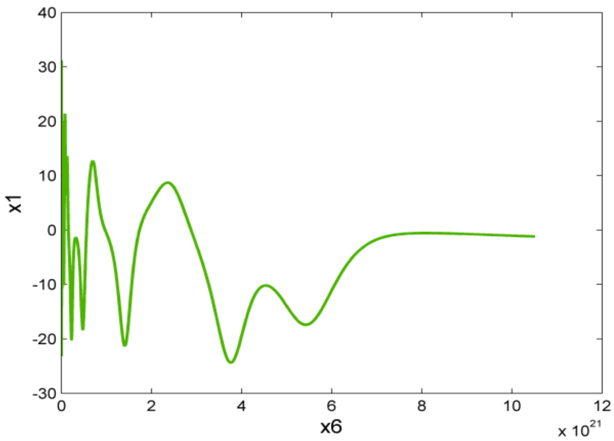

Fig. 6: 2-D attractor of the system (1) in the (x_1, x_6) plane.

3. Chaos synchronization of the 6-D Lorenz hyperchaotic systems

In this section, we study an engineering application of the 6-D Lorenz hyperchaotic system via. nonlinear chaos synchronization of two identical of Lorenz hyperchaotic system with known parameters. To begin with, the definition of synchronization used in this paper is given as

3.1. Definition

For two nonlinear dynamical systems:

$$\dot{X}_1 = F_1(X_i) \quad (2)$$

$$\dot{Y}_i = F_2(Y_i) + U(X_i, Y_i) \quad (3)$$

Where$X_i, Y_i \in R^n$, $F_1, F_2 : R^n \to R^n$, $i = 1,2,\dots,n$, $U(X_i, Y_i)$ is the nonlinear control vector, suppose that Eq. (2) is the drive system, Eq.(3) is the response system. The response system and drive system are said to be chaos synchronized or (complete /full) synchronized if for

$$\forall\, X_i, Y_i \in R^n \,,\ \lim_{t\to} \infty \, \|Y_i - \alpha_i X_i\| = 0,$$

Where α_i is the scaling factor taken the value 1 for complete synchronization[15].

3.2. Design of nonlinear controllers with known parameters

According to the above definition, assume that the system (1) be the drive system and response system is given as the following form:

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + y_4 + u_1 \\ \dot{y}_2 = cy_1 - y_2 - y_1y_3 + y_5 + u_2 \\ \dot{y}_3 = -by_3 + y_1y_2 + u_3 \\ \dot{y}_4 = dy_4 - y_1y_3 + u_4 \\ \dot{y}_5 = -ky_2 + u_5 \\ \dot{y}_6 = ly_2 + hy_6 + u_6 \end{cases} \quad (4)$$

Where $U = [u_1, u_2, u_3, u_4, u_5, u_6]^T$ is the nonlinear controller to be designed.
The synchronization error dynamics between the 6-D hyperchaotic system (1) and system (4) is defined as

$$\begin{cases} e_1 = y_1 - \alpha_1 x_1 \\ e_2 = y_2 - \alpha_2 x_2 \\ e_3 = y_3 - \alpha_3 x_3 \\ e_4 = y_4 - \alpha_4 x_4 \\ e_5 = y_5 - \alpha_5 x_5 \\ e_6 = y_6 - \alpha_6 x_6 \end{cases} \quad (5)$$

The error dynamics is calculated as the following:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + e_4 + u_1 \\ \dot{e}_2 = ce_1 - e_2 + e_5 - e_1e_3 - x_3e_1 - x_1e_3 + u_2 \\ \dot{e}_3 = -be_3 + e_1e_2 + x_1e_2 + x_2e_1 + u_3 \\ \dot{e}_4 = de_4 - e_1e_3 - x_3e_1 - x_1e_3 + u_4 \\ \dot{e}_5 = -ke_2 + u_5 \\ \dot{e}_6 = le_2 + he_6 + u_6 \end{cases} \quad (6)$$

The above system is unstable. Based on the following law $|\lambda I_6 - J_{E_1}| = 0$, the characteristic equation and eigenvalues is yield as

$$\lambda^6 + \frac{32}{3}\lambda^5 - \frac{4069}{15}\lambda^4 + \frac{1658}{15}\lambda^3 + \frac{24004}{15}\lambda^2 - \frac{9496}{5}\lambda - 448 = 0$$

$$\begin{cases} \lambda_1 = 2 \\ \lambda_2 = 1 \\ \lambda_3 = -8/3 \\ \lambda_4 = 11.36592689 - 8.10^{-9}i \\ \lambda_5 = -22.69162026 - 3.92820323010^{-9}i \\ \lambda_6 = 0.32569338 + 9.92820323010^{-9}i \end{cases}$$

So, some eigenvalues are positive real parts. Therefore, The error dynamics system(6) is unstable. Now, we try to control this system through the design the following control

Theorem 1: *For the error dynamics system (system 6) with nonlinear control $U = [u_1, u_2, u_3, u_4, u_5, u_6]^T$ such that*

$$\begin{cases} u_1 = 0 \\ u_2 = -2ce_1 - e_6 + x_3e_1 \\ u_3 = -x_2e_1 \\ u_4 = -de_1 - 2de_4 + e_1e_3 + x_3e_1 + x_1e_3 \\ u_5 = -e_5 \\ u_6 = -2he_6 \end{cases} \quad (7)$$

Then the system (6) can be controlled i.e., system (4) followed to system (1).
Proof. According to the previous discussion, the error dynamics system (6) with controller (7) become

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + e_4 \\ \dot{e}_2 = -ce_1 - e_2 + e_5 - e_6 - e_1e_3 - x_1e_3 \\ \dot{e}_3 = -be_3 + e_1e_2 + x_1e_2 \\ \dot{e}_4 = -d(e_1 + e_4) \\ \dot{e}_5 = -ke_2 - e_5 \\ \dot{e}_6 = le_2 - he_6 \end{cases} \quad (8)$$

Now, based on the Lyapunov stability theory, we construct a positive definite on R^6 Lyapunov candidate function as

$$V(e) = e^T P e = \frac{1}{2}[e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_5^2 + e_6^2] \quad (9)$$

Where

$$P = daig[1/2, 1/2, 1/2, 1/2, 1/2, 1/2] \quad (10)$$

Differentiating $V(e)$ along the error dynamics (6), we obtain of the Lyapunov function $V(e)$ with respect to time is

$$\dot{V}(e) = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + e_5\dot{e}_5 + e_6\dot{e}_6 \quad (11)$$

$$\dot{V}(e) = e_1[-ae_1 + ae_2 + e_4] + e_2[-ce_1 - e_2 + e_5 - e_6 - e_1e_3 - x_1e_3] + e_3[-be_3 + e_1e_2 + x_1e_2] + e_4[-de_1 - de_4] + e_5[-ke_2 - e_5] + e_6[le_2 - he_6]$$

$$\dot{V}(e) = -ae_1^2 - e_2^2 - be_3^2 - de_4^2 - e_5^2 - he_6^2 + (a-c)e_1e_2 + (1-d)e_1e_4 + (1-k)e_2e_5 + (l-1)e_2e_6 = -e^T Q e \quad (12)$$

Where

$$Q = \begin{pmatrix} a & \frac{c-a}{2} & 0 & \frac{d-1}{2} & 0 & 0 \\ \frac{c-a}{2} & 1 & 0 & 0 & \frac{k-1}{2} & \frac{1-l}{2} \\ 0 & 0 & b & 0 & 0 & 0 \\ \frac{d-1}{2} & 0 & 0 & d & 0 & 0 \\ 0 & \frac{k-1}{2} & 0 & 0 & 1 & 0 \\ 0 & \frac{1-l}{2} & 0 & 0 & 0 & h \end{pmatrix} \quad (13)$$

So, Q is not diagonal matrix. In order to transform above symmetric matrix to diagonal (Every diagonal matrix with positive diagonal elements are positive definite, while the symmetric (not diagonal) matrix it is possible to a positive definite.
For treatment this problem, we modify the matrix P to make the matrix Q is a positive definite. So, we modify the matrix P by the following form

$$P_1 = daig[7/5, 1/2, 1/2, 7/10, 5/84, 1/2] \quad (14)$$

Then the derivative of the Lyapunov function as

$$\dot{V}(e) = -2.8ae_1^2 - e_2^2 - be_3^2 - 1.4de_4^2 - 0.119e_5^2 - he_6^2 = -e^T Q_1 e \quad (15)$$

$$Q_1 = \begin{pmatrix} 2.8a & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.4d & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.119 & 0 \\ 0 & 0 & 0 & 0 & 0 & h \end{pmatrix} \quad (16)$$

Clearly, Q_1 is diagonal matrix with positive diagonal elements $Q_1 > 0$. Therefore, $\dot{V}(e)$ is negative definite. According to the Lyapunov asymptotical stability theory, the nonlinear controller is achieved.
Based on the second method (Linearization method), the characteristic equation and eigenvalues of system (8) is yield as

$$\lambda^6 + \frac{53}{3}\lambda^5 + \frac{1952}{5}\lambda^4 + \frac{11398}{5}\lambda^3 + \frac{26924}{5}\lambda^2 + \frac{27912}{5}\lambda + \frac{31552}{15} = 0$$

$$\begin{cases} \lambda_1 = -1 \\ \lambda_2 = -8/3 \\ \lambda_3 = -1.4092 \\ \lambda_4 = -1.8748 \\ \lambda_5 = -5.3580 + 16.4274\,i \\ \lambda_6 = -5.3580 - 16.4274\,i \end{cases}$$

It is clear that all real parts of eigenvalues are a negative, therefore the propose controller (7) is succeeded to perform complete synchronization between systems (1) and (4) .

3.3. Numerical simulation

For simulation, the MATLAB is used to solve the differential equation of controlled error dynamical system (6), based on fourth-order Runge-Kutta scheme with time step 0.001 and the and the initial values of the drive system and the response system are following $(15,-12,17,-10,-9,4)$ and $(-10,8,-2,4,2,-6)$ respectively. We choose the parameters $= 10, b = \frac{8}{3}, c = 28, d = 2, k = 8.4, l = 1$ and $h = 1$.
Figures 7-12 show the complete/full synchronization of the hyper-chaotic Lorenz system (1) and system (4). Figure 13 shows the convergent for system (6) with controller (7).

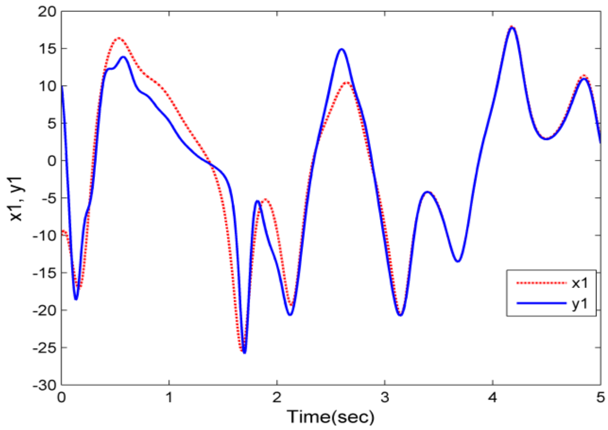

Fig. 7: Complete synchronization of the states x_1 and y_1 for the Lorenz systems (1) and (4).

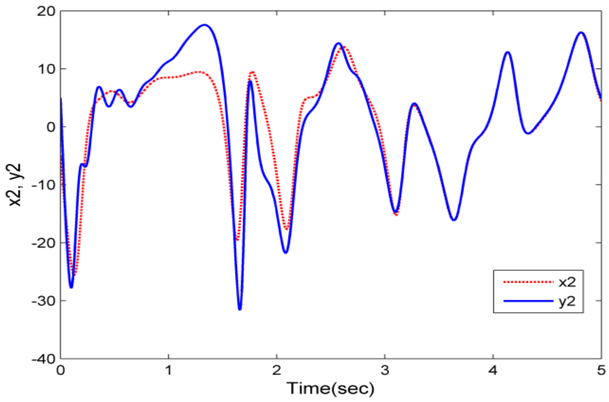

Fig. 8: Complete synchronization of the states x_2 and y_2 for the Lorenz systems (1) and (4).

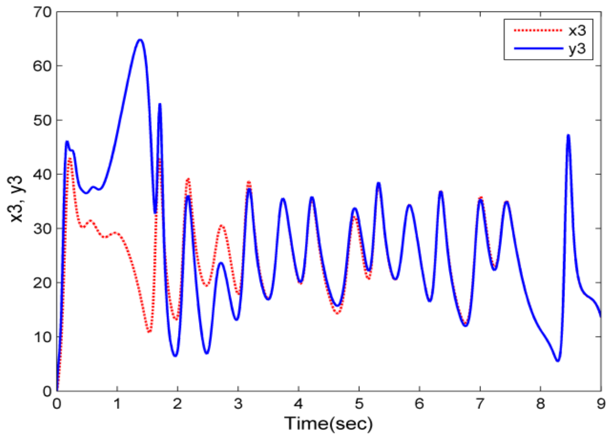

Fig. 9: Complete synchronization of the states x_3 and y_3 for the Lorenz systems (1) and (4).

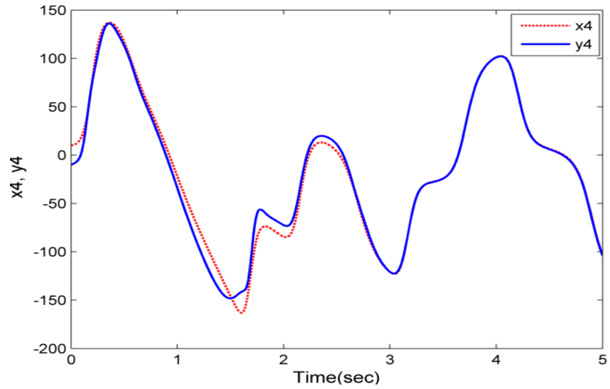

Fig. 10: Complete synchronization of the states x_4 and y_4 for the Lorenz systems (1) and (4).

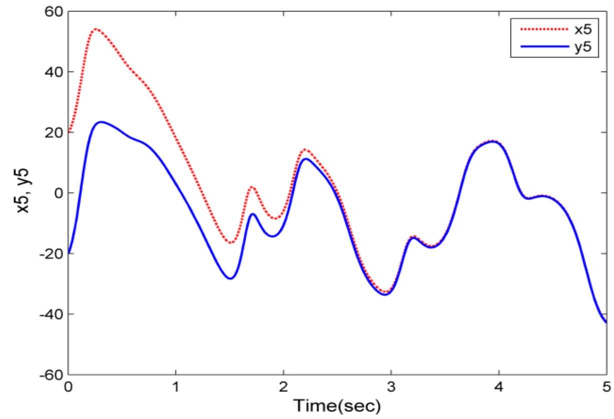

Fig. 11: Complete synchronization of the states x_5 and y_5 for the Lorenz systems (1) and (4).

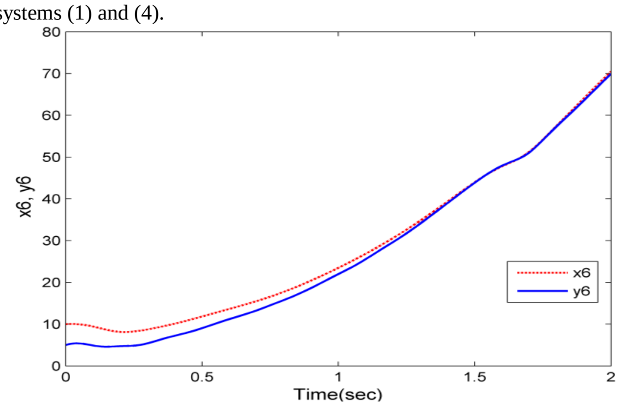

Fig. 12: Complete synchronization of the states x_6 and y_6 for the Lorenz systems (1) and (4).

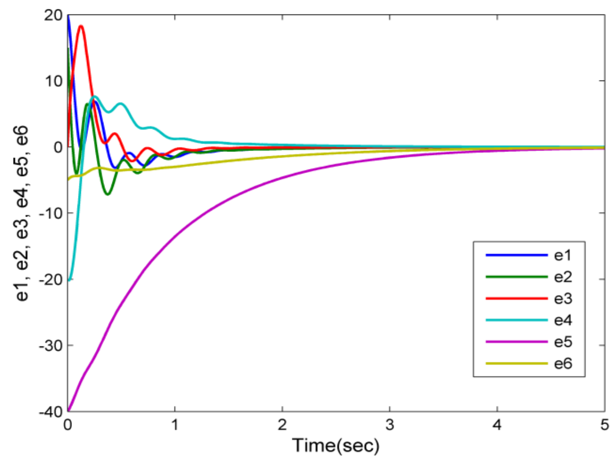

Fig. 13: The Convergent of the error dynamics (6) with controller (7).

4. Conclusion

In this article, chaos synchronization (complete synchronization) for 6-D hyperchaotic system is dealt with. When the parameters of this system are known, the controller was designed via the nonlinear control strategy for controlling this high-dimensional system based on the Lyapunov stability criteria. Obviously from this controller, we achieved complete synchronization although this system is higher-dimensional and more complex from low-dimensional chaotic system. The effectiveness of these proposed control strategies was validated by numerical simulation results.

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