

Fundus image enhancement using white top hat operation and perona-malik diffusion filter

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Abstract

Diabetes mellitus is a chronic disease characterized by increasing the blood sugar levels that are far above normal. The diabetes mellitus can damage small blood vessels in the retina called diabetic retinopathy. The diabetic retinopathy could result in blurred vision and can progress to blindness when it is left untreated. The diabetic retinopathy is one of the main causes of blindness in the western world in the working period population. One way to find out the diabetic retinopathy is by examining the eye fundus image with using a fundus camera. However, the fundus image often has noise and uneven illumination which cause the evaluation diabetic retinopathy is hard. Therefore, a method for enhancing the fundus image is necessary. This paper proposes the use of white top operation and the Perona - Malik diffusion filters for enhancing the quality of the fundus image. The white top-hat transform is used to extracts small elements and details from given images. The Perona-Malik diffusion filter is used for reducing the noise in images without removing significant parts of the image contents. From the results of experiment, it can be concluded that the proposed method is able to significantly improve fundus image.

Keywords: Diabetic Retinopathy, Fundus Image; Image Enhancement; Perona-Malik Diffusion Filter; White Top Hat Operation.

1. Introduction

Diabetes mellitus is a chronic disease characterized by increasing the blood sugar levels that are far above normal. Over time, high sugar level in the blood can cause blockages of the retinal vessels that give retina nutrients, and also cut blood intake. As a result, the eyes try to form the new blood vessels. However, the new blood vessels cannot develop properly, and it can leak easily. The diabetes mellitus can damage small blood vessels in the retina called diabetic retinopathy. Diabetic retinopathy is a microvascular diabetes mellitus complications which damages the small blood vessels. Diabetic retinopathy causes the blindness in the western world in the working period population [1].

This disease begins with a narrowing of blood vessels in the eye. The narrowing of blood vessels can lead to leakage or bleeding, and accumulation of fluids and fatty material in the retina resulting in blurred vision. If this condition is left then it can cause severe vision damage as well as blindness. The risk of diabetic retinopathy can be reduced by early diagnosing, and controlling blood sugar, blood pressure and lipids appropriately [2]

One way to find out the diabetic retinopathy is by examining the retinal image with using a fundus camera. This image is called by fundus image. From the fundus images, ophthalmologists could find the sign of the diabetic retinopathy. Ophthalmologists use the fundus image to diagnose diabetic retinopathy. The retinal image resulting from fundus cameras often has noise and uneven illumination. In addition, details of retinal fundus images such as small blood vessels, microaneurism, and exudate may be in low contrast. Enhancing the quality of fundus image will help ophthalmologists in diagnosing diseases related to retinal fundus image [3]. Therefore, a method for enhancing the fundus image is necessary.

Many methods for image enhancement has been developed recently and commonly applied, one of them is the histogram equalization method [4], [5]. However, the histogram equalization method often leads to loss of image detail and tends to produce over-enhanced image output if the probability of gray level density increases unexpectedly [6]. Morphological operation is one of methods to enhance image quality. The white top hat method is the morphological operation for improving uneven illumination. The white top-hat transform is used to extracts small elements and details from given images. It is used to enhance bright objects of interest in a dark background. Meanwhile, one of the common methods used to reduce noise is the Gaussian filter, but this method often makes the edges of the image blurred. The Perona-Malik diffusion filter is not only reducing noise but also it is effectively maintaining the edges of the image [8]. The Perona-Malik diffusion filter has been used for enhancing the IVUS (Intravascular Ultrasound) image [9] and the X-ray images of the hand bone [10].

This paper proposes the use of the white top hat operation and the Perona-Malik diffusion filters to enhance the quality of fundus image. The white top hat operation is used to enhance the interesting objects, i.e. the retinal vessel and the other symptoms of diabetic retinopathy diseases. While, the Perona-Malik diffusion filter is used to filter the noise and enhance the interesting objects in fundus images. The good quality of the fundus images will help the ophthalmologists in diagnosing the vessels and some symptoms of the diabetic retinopathy in the fundus image. The results of this research is also very important for automatically detecting symptoms of the diabetic retinopathy by the image segmentation method.

2. Related works

This section will discuss several theories and previous research which are related with this research, such as diabetic retinopathy and fundus image, contrast enhancement, morphological operation and white top hat operation, image filtering and Perona-Malik diffusion filter.

2.1. Diabetic retinopathy and fundus image

Diabetes mellitus is disease which is characterized with interferes in the body's ability to use and store sugar. The disease causes too much sugar in the blood, which can cause damage throughout the body, including the eyes. The eye disease caused by diabetes mellitus is called diabetic retinopathy. The diabetic retinopathy causes serious damage to the retina. When the small blood vessels leak blood and other fluids, diabetic retinopathy occurs. The diabetic retinopathy causes the retinal tissue to swell and results in blurred vision. The condition usually affects both eyes. If it left untreated, it can lead to blindness.

The first stage of diabetic retinopathy occurs abnormality on retina called by microaneurysms as shown in Figure 1. They are the earliest clinically visible changes in diabetic retinopathy. The microaneurysms are localised capillary dilatations which seem as small red dots. The microaneurysms are the tiny aneurysms, or swelling, in the side of a blood vessel in the retina of the eye. They causes enlargement of retinal capillary, can rupture and leak blood. Several researches show that the microaneurysms can predict the progress of diabetic retinopathy. The microaneurysm that rupture can cause hemorrhage which can be seen in Figure 2. It is a disorder of the eye where bleeding occurs into the light-sensitive tissue on the back wall of the eye.

After the microaneurysm appeared, hard exudate may appear as shown in Figure 3. They look small white or yellowish white deposits with sharp margins. The hard exudate appears shiny, waxy, or glistening. The hard exudate is located in the outer layers of the retina. The hard exudate is a leaky lipid formation of weakened blood vessels.



Fig. 1: Microaneurim.



Fig. 2: Hemorrhage.

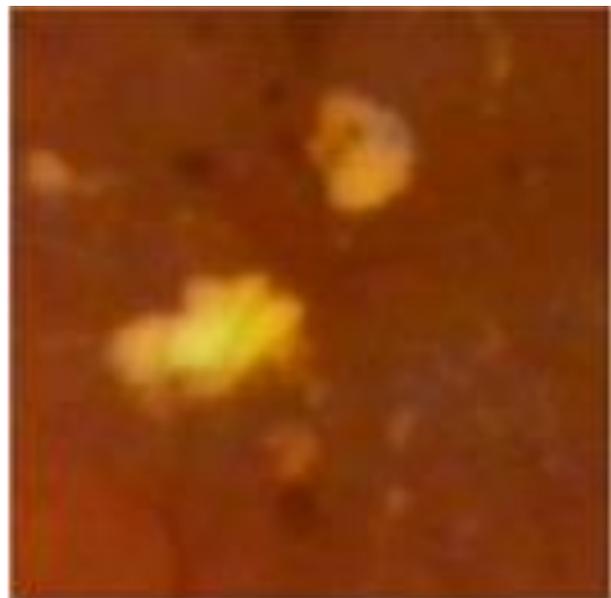


Fig. 3: Hard exudate.



Fig. 4: Soft Exudate.

Along with the severity of retinopathy disease, the blood vessels may become inhibited causing microinfarct in the retina called

soft exudate as shown in Figure 4. The lack of oxygen caused by microinfarct causes the development of new vessels (neovascularization), as shown in Figure 5. These phenomenon can cause a sudden loss of vision. The diagnosis of diabetic retinopathy using fundus image is indispensable because the disease is progressive [9], an example of fundus image can be seen in Figure 6.

2.2. Contrast enhancement

Image enhancement is a technique usually used to improve the quality of the degraded images. Contrast of images is the difference between the highest intensity values and the lowest intensity values of the image. Contrast is one of the important parts of an image for enhancement. There are many reasons due to which some images have poor contrast. There are some reasons why the images have poor contrast. These are caused by poor illumination, lack of expertise of the operator or adverse external conditions such as foggy weather when the image is acquired. These poor contrast images cannot be successfully used for further analysis or processing by many image processing applications.

Contrast enhancement becomes a necessary pre-processing step in many applications of the image processing algorithms. The contrast enhancement improves the image contrast in order to make various features more easily seen. Contrast enhancement will



Fig. 5: Neovascularization.



Fig. 6: Example of Fundus Image.

Increase the total contrast of the image. The contrast enhancement is done by making light colours lighter and dark colours darker at the same time. For this reason, the image contrast enhancement becomes important task in the image processing.

There are two types of the image contrast which are local and global. Many image contrast enhancement algorithms have been created over the years. The conventional image enhancement techniques are broadly divided into two categories, including the spatial and frequency domain techniques. The techniques for enhancing the contrast of images can be categories into two. The first categories is used for enhancing the local contrast and the second categories are used for enhancing the global contrast of the images. The performance of these techniques depends on the condition of the image to be enhanced.

Histogram Equalization (HE) is one of the most known methods for image contrast enhancement. This approach is generally useful for enhancing images with poor intensity. HE is commonly applied for enhancing an image because it is very simple and comparative better performance for almost all images types. The operation of HE is developed by remapping the gray levels of the image based on the gray levels probability distribution. HE flattens and stretches the dynamic range of the images histogram and it results in overall the enhancement of images contrast.

However, when HE is implemented to process digital images, it suffers from major drawbacks especially. Firstly, HE transforms the histogram of the original image into a flat uniform histogram with a mean value that is in the middle of gray level range. HE may reduce the local details within these objects, especially smooth and small ones. HE also produces over enhancement and saturation artifacts [11].

2.3. Morphological operation and white top hat operation

Mathematical morphology provides an excellent tool for image processing. It is a powerful tool for image analysis, which was developed about forty years ago. It provides a unified and powerful approach to solve image processing problems. Mathematical morphology is based on shape concept from the set theory. The identification of objects and object features through their shape makes mathematical morphology become an obvious approach for various machine vision and recognition processes. Morphological operations provide a powerful approach to overcome the image processing problem. Unlike other tools, morphological operators relate directly to shape.

The approach of operation morphology has been proposed for image contrast enhancement. It can simplify images by preserving their essential shapes and eliminating noise when it is used appropriately. In morphology, objects in the image are represented as two sets. The morphological algorithm is based on the operation between two sets, ie image and structural elements (SE). The shape and size of SE is determined according to the purpose of the related application. Operation welded and erosion are two basic operations in successive morphological operations, defined by (1) and (2), where $f(x, y)$ is a gray scale image and g is SE.

$$(f \oplus g) = \max \{f(x-k, y-l) | (k, l) \in g\}, \quad (1)$$

$$(f \otimes g) = \max \{f(x+k, y+l) | (k, l) \in g\}, \quad (2)$$

The opening and closing operations are defined successively defined by (3) and (4), respectively.

$$f \circ g = (f \otimes g) \oplus g, \quad (3)$$

$$f \bullet g = (f \oplus g) \otimes g, \quad (4)$$

The shape of the structuring element g plays a crucial role in extracting features or objects of given shape from the image. However, for a categorical extraction of features or objects from the image based on shape and size, we must incorporate a second attribute to the structuring element which is its scale. A morphological operation with a scalable structuring element can extract

features based not only on shape but also on size. Also features of identical shape but of different size are now treated separately.

The top hat transform is one of the important operations of mathematical morphology used to extract image features related to the used structuring element. The top hat transform has two versions: white top hat and black top hat transformation. The white top hat transform (WTH) is used to extract bright or white features of image related to the used structuring element. It is given as the difference of the original image and the opened image. It is defined as: Top hat transformation is one of the most important operations in morphological operations used to extract the image features associated with the structural elements used. One type of top hat transformation is the White Top Hat (WTH) used to extract the bright or white image features associated with the elements of the structure used. The WTH transform is defined by (5) [7].

$$WTH(f) = f - (f \circ g) \quad (5)$$

Where f is the image and g is the structuring element. Similarly, the black top hat transformation is used to extract the darker or black features of image related to the used structuring element [12].

2.4. Image filtering

Image smoothing is a method of improving the quality of images. The image quality becomes an important factor for the human vision point of view. The image usually contains noise which is not easily reduced in image processing. The image quality is affected by the presence of noise. The image filtering method is used to remove or to reduce noise in the images. It has been widely applied in many fields, such as image display, image transmission and image analysis, etc.

Many conventional filtering methods have been proposed for removing noise in images. However; the capability of conventional filters based on pure numerical computation is broken down rapidly when they are put in heavily noisy environment. Median filter is the most used method [13], but it cannot work efficiently when the noise rate is above 0.5. Yang and Toh [14] proposed a heuristic rules for improving the performance of traditional multilevel median filter. Russo and Ramponi [15] used a heuristic knowledge to develop the fuzzy rule based operators for smoothing, sharpening and edge detection. However, they can perform smoothing efficiently but not in brightness. In practice, an image usually contains some different types of noise. So good image smoothing algorithm should be able to deal with different types of noise. However, image smoothing [16] often causes blur and offsets of the edges. While the edge information is much important for image analysis and interpretation. So, it should be considered to keep the precision of edge's position in image smoothing.

2.5. Perona-Malik diffusion filter

Perona-Malik diffusion filter has been applied for enhancing image and filtering out noise. The Perona-Malik model is a non-linear anisotropic diffusion model proposed by Perona and Malik. It is a image processing technique based on a partial differential equation. The speed of diffusion in the Perona-Malik model is controlled by an edge stopping function. The sharp edges and fine details can be preserved well by using appropriate the edge stopping function. An edge stopping function which stops diffusion from low image gradient onwards well preserves the sharp edges and fine details. Perona-Malik model can successfully remove noise while preserving edges and small structures as long as the diffusion coefficient or edge stopping function and gradient threshold parameter (K) are estimated correctly. The gradient magnitude threshold parameter K that differentiates between image gradients due to noise and those gradients that are more likely to represent image edges. Since these are the important parameters that decides rate of diffusion, if not properly estimated will result

in either over smoothed image or will leave the image with most of the noise unfiltered. Since there are several edge stopping functions, their ability to filter the noise and ability to preserve the edges when used in Perona-Malik mod 1 need to be studied and evaluated to make an appropriate choice.

This model may fail when the gradient due to noise is close to gradient of image edges. So a better version of PM model was suggested by Catte in 1992. In this method the image should be smoothed first by convolving with Gaussian filter then the gradient to be computed. Gabor suggested an inverse heat equation.

The Perona-Malik diffusion filter is able to smooth the image once but still preserve the edge of the image. It is used to filter noise and preserve the edges of an image. The basic idea of the PMD process is to get an increasingly smoothed image $u(x, y, t)$ from an original image $u_0(x, y)$, indexed by diffusion parameter t . This process can be interpreted as an image convolution by a Gaussian kernel $G(x, y, t)$ with an increasing width as in (6).

$$I(x, y, t) = I_0(x, y) * G(x, y, t) \quad (6)$$

The PMD filter equation is defined by (7), where $c(x, y, t) = g(\|\nabla I(x, y, t)\|)$ is a diffusion coefficient. ∇I denotes a gradient of an image.

$$I_t = \frac{\partial I}{\partial t} = \text{div}(c(x, y, t)\nabla I) = c(x, y, t)\Delta I + \nabla c(x, y, t)\nabla I \quad (7)$$

$g(\cdot)$ Refers to an edge stopping function, which is a decreasing function of the gradient of image, which is defined by (8),

$$g(\nabla I) = \frac{1}{1 + \left(\frac{\|\nabla I\|}{K}\right)^2} \quad (8)$$

Where K is a parameter that controls the strength of diffusion.

The edge stopping function $g(z)$ should satisfy theoretically two conditions. The first condition is $\lim_{z \rightarrow 0} g(z) = 1$, so that the diffusion rate is high within uniform or inner regions. The second condition is $\lim_{z \rightarrow \infty} g(z) = 0$, so that the diffusion is close to zero across the boundaries of objects. The important property of edge stopping function is that it should has a zero value or insignificant value for this gradient that corresponds to edges.

The discrete form of the Perona-Malik diffusion filter is given by equation (9).

$$I_s^{(n+1)} = I_s^{(n)} + \frac{\lambda}{|\phi_s|} \sum g(\nabla I_{s,p}^{(n)}) \nabla I_{s,p}^{(n)}, \quad (9)$$

Where $s = (x, y)$ is the coordinate of the pixel in question and the neighboring p . is an intensity on s with n is iteration. ϕ_s represents 4 neighboring pixels in the diffusion direction of North, West, South and East. $|\phi_s|$ is the number of the neighbor pixels. λ is a parameter. $g(\cdot)$ called an edge stopping function which is a monoton-dropping function of the image gradient. The image gradient is calculated using (10)

$$\nabla I_{s,p}^{(n)} = I_p^{(n)} - I_s^{(n)}. \quad (10)$$

3. Proposed method

In this paper we will discuss how to improve fundus image by using the white top hat operation and the Perona-Malik diffusion filter. The flow diagram of the proposed method is represented in Figure 8. The input of the proposed method is fundus image. For

example, the input is fundus image in Figure 6. The input image is in RGB (Red, Green, Blue) format.

The next step, the fundus image is simplified by taking only the green component of the color image. The green component is used because it can defend the feature / feature found in the fundus image. Figure 9 shows the red, green and blue components of the fundus image. For the figures, it can be shown that the green component of the fundus image can represent the objects in fundus image (optical disk, retina's blood vessels, exudates and microaneurim) better than the red and blue components. For this reason, only the green component is taken for the next process.

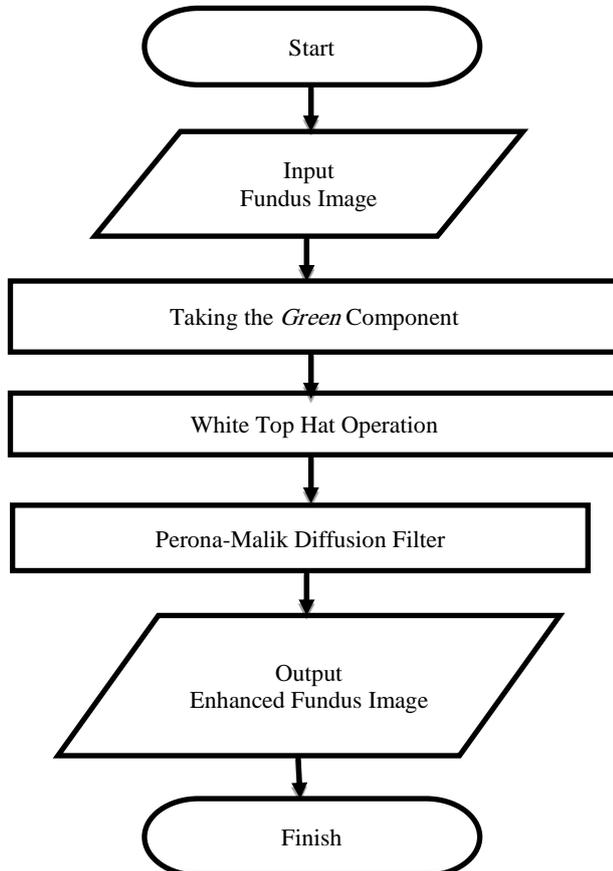


Fig. 8: Flowchart of the Proposed Method.

The next step is that enhance and extract the interesting objects, i.e. the retinal vessel and the other symptoms of diabetic retinopathy diseases. In this step, the white top hat is used for extracting the interesting objects, ie. optical disk, retina's blood vessels, exudates and microaneurim. From Figure 10, it can be shown that the optical disk, retina's blood vessels, exudates and microaneurim are represented better than fundus image in Figure 9 (b). The white top hat operation is succesfully extracted the interesting components. However, the white top hat operation gives negative effect, after applying the white top hat operation the noise seen clearer. To reduce the negative effect of this method, the filtering method is needed to be applied.

This proposed method uses the Perona-Malik diffusion filter to reduce the noise caused the white top hat operation. Figure 11 show the image after applying the Perona-Malik diffusion filter on Figure 10. It can be show that the image is more smooth compared the image before applying the Perona-Malik diffusion filter.

4. Results and discussion

The proposed method is evaluated by using data of fundus images shown in Figure 12, the number of data used is 4. For evaluation.

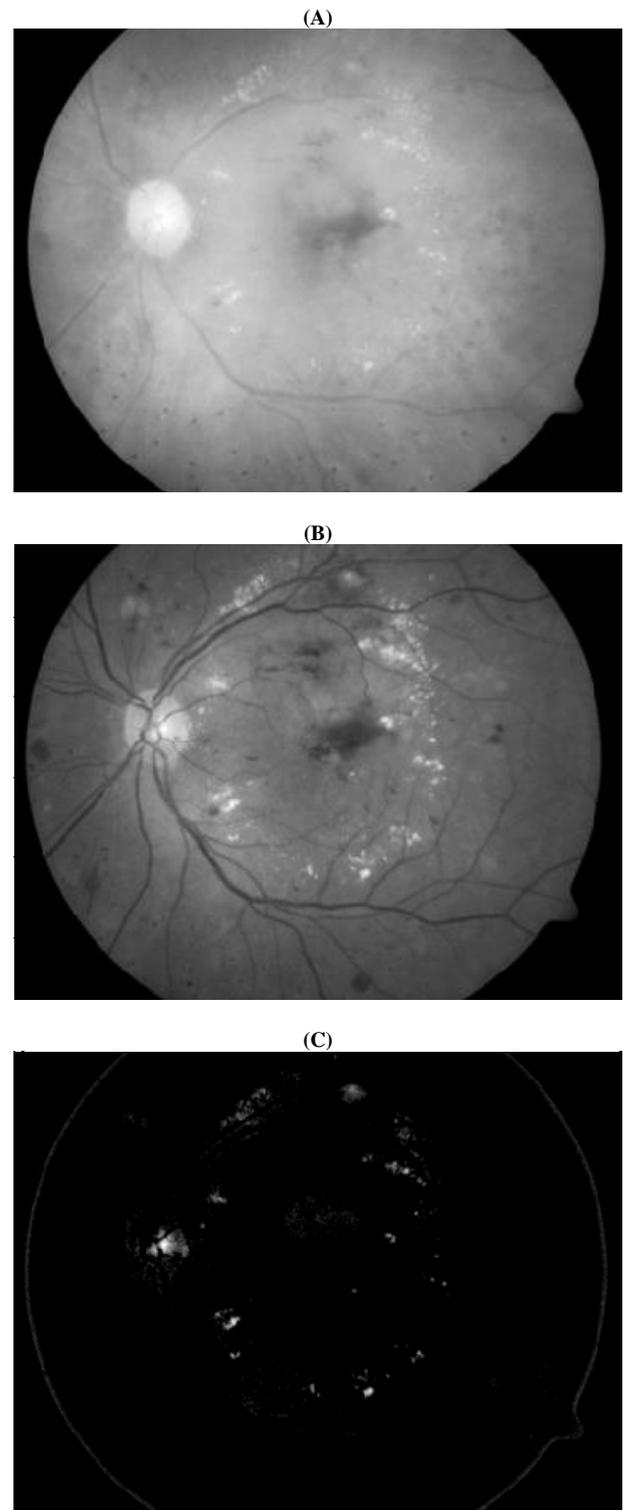


Fig. 9: Components of RGB Image Format. (A) Red. (B) Green. (B) Blue.

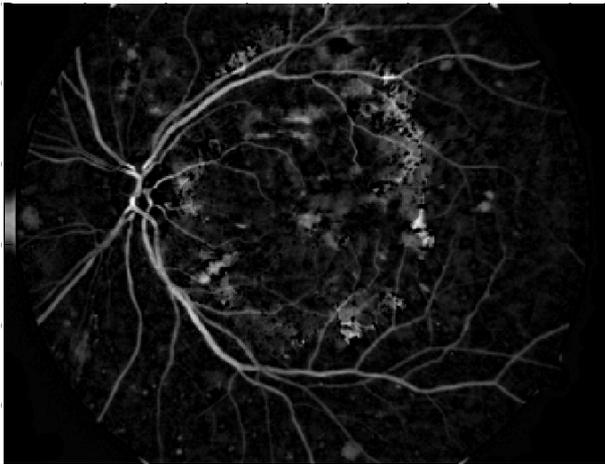


Fig. 10: White Top Hat Operation Result of the Green Component in Figure 9.

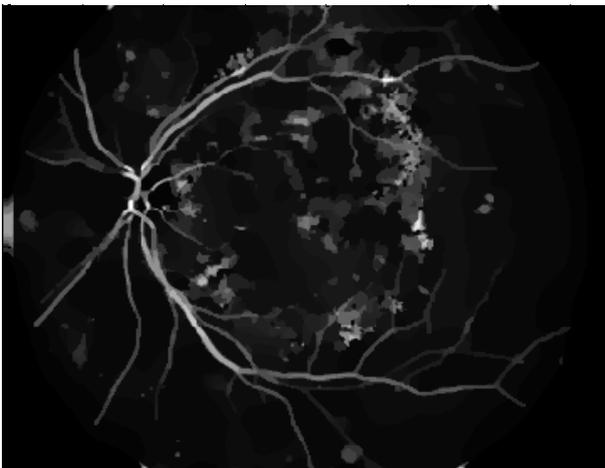


Fig. 11: Image After Applying the Perona-Malik Diffusion Filter of Figure 10.

the performance, the various data of fundus images is used. Some images are normal fundus images and the others are the fundus images have sign diseases, i.e. retinopathy diabetic. It can be shown that the test images has variety of color, brightness level and noise. The various fundus images are used to see the robustness of the proposed method. The test images are taken from the site <http://www.it.lut.fi/project/imageret/diaretdb1/>. Figure 13 show the green component of images in Figure 12. It can be seen that the green component of images is able to show the interesting objects in the fundus images. However, several interesting objects cannot be seen clearly because some parts of images has uneven illumination, which causes the blood vessels and other interesting objects appear less obvious. For this reason, the images need to be enhanced.

It will then attempt to use the white top hat of morphology operation to improve uneven lighting conditions. This method is influenced by the type of element structure and size. In this experiment used the structure of disk elements with size eight. Figure 14 shows the white top hat operation successfully used to correct uneven illumination in the fundus image. However, the noise in the fundus image is still visible.

The next step is to filter the fundus image by using the Perona-Malik diffusion filter to reduce the noise. The value used in this experiment is 0.5 and K equals 0.4 is a measure of time change

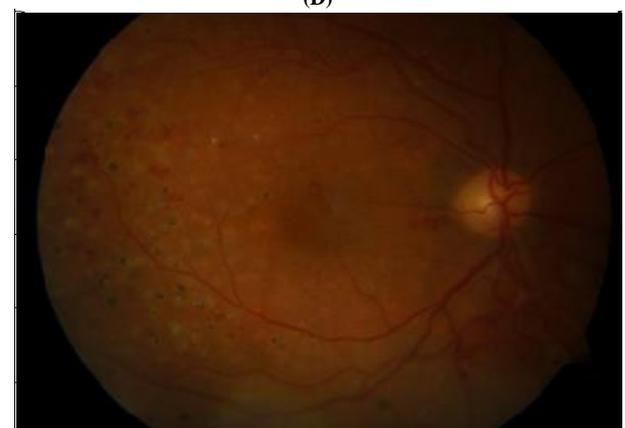
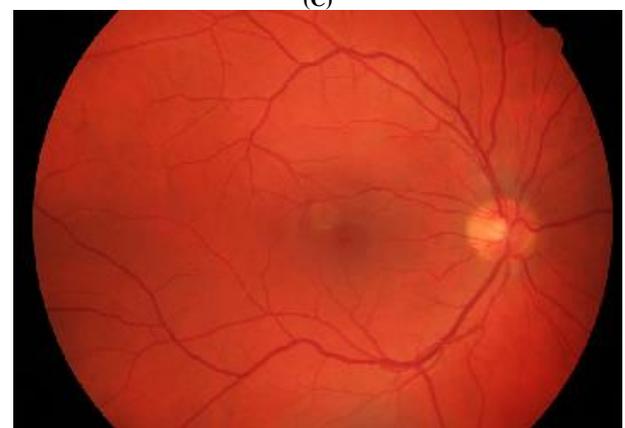
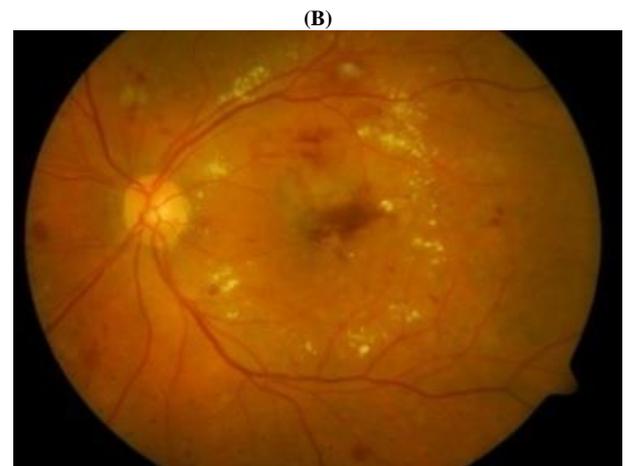
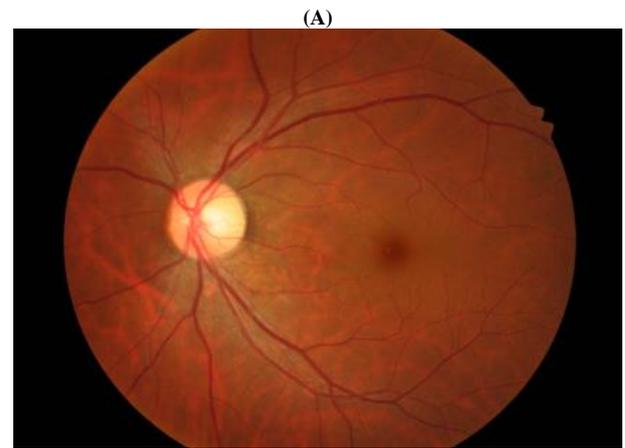


Fig. 12: Test Image. A) Image Test 1st. (B). Image Test 2nd. (C). Image Test 3th. D). Image Test 4th.

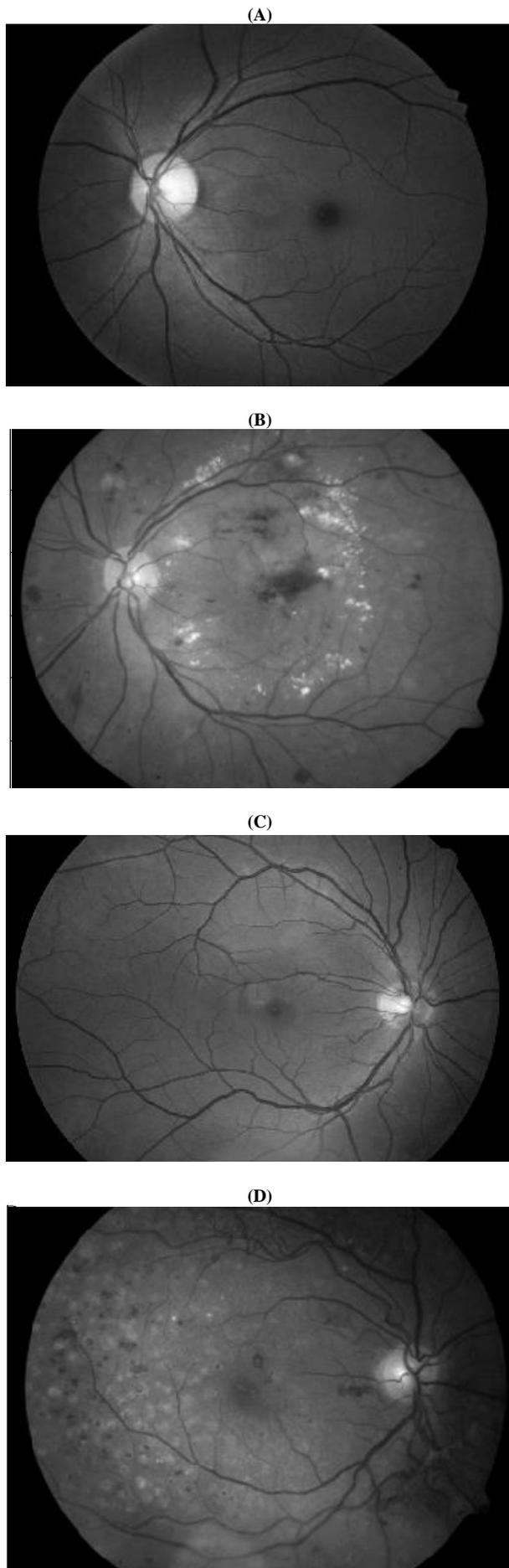


Fig. 13: Gray Scale Image of the Test Images. (A) Image Test 1st. (B). Image Test 2nd. (C). Image Test 3th. (D). Image Test 4th.

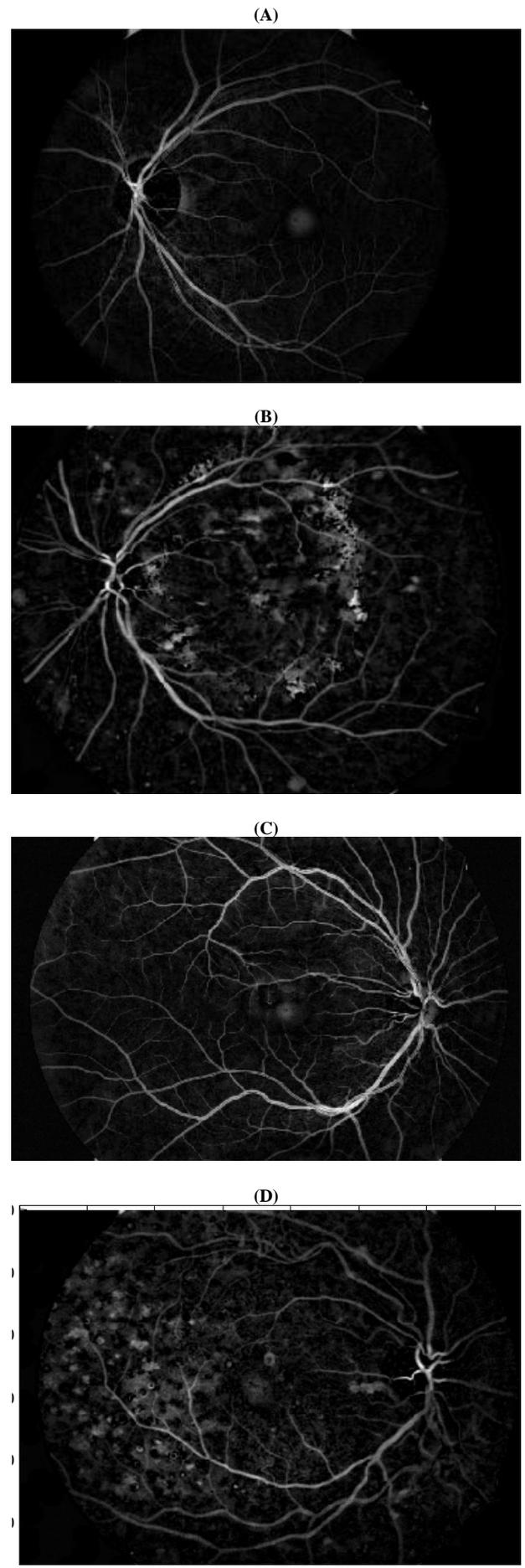


Fig. 14: Results of Morphological Operations from Gray Scale Images. (A). Test Image 1st. (B). Test Image 2nd. (C). Test Image 3th. (C). Test Image 4th.

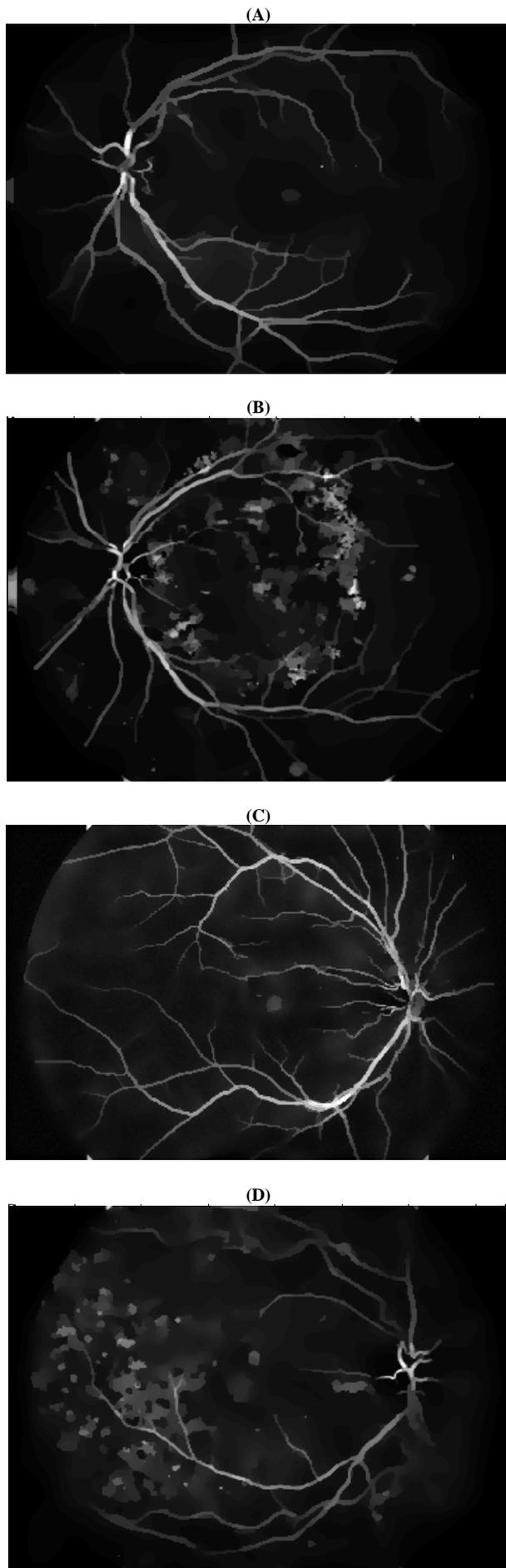


Fig. 15: The Images Results Of Morphological Operations after Applying the Perona-Malik Diffusion. (A). Test Image 1st. (B). Test Image 2nd. (C). Test Image 3rd. (D). Test Image 3rd.

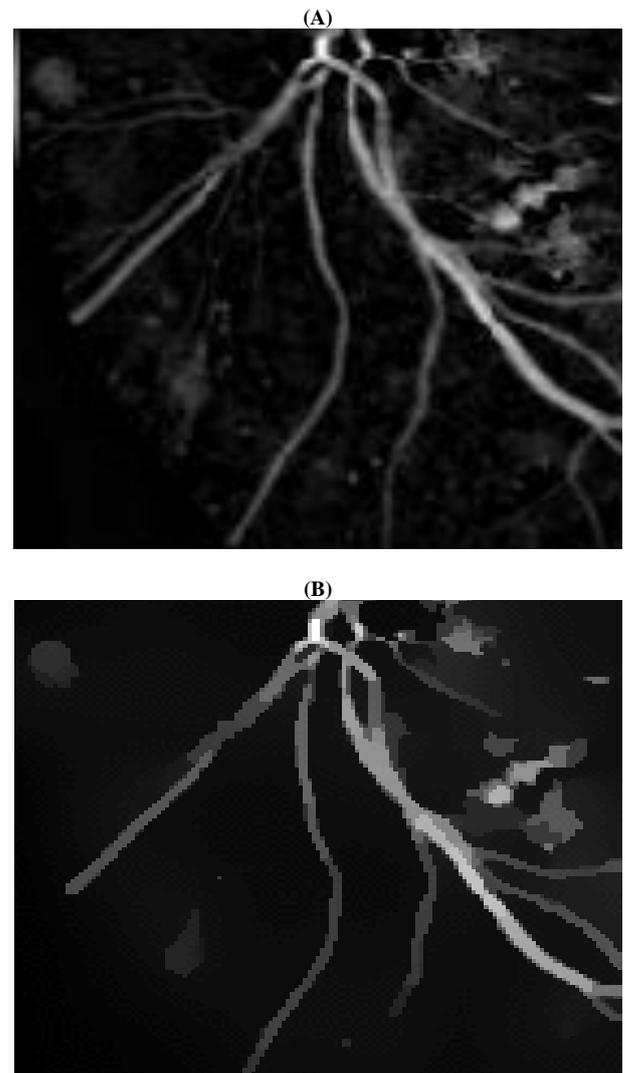


Fig. 16: Comparison of Fundus Image before and after Applying the Perona-Malik Diffusion Filter Process. (A) Before Applying the Perona-Malik Diffusion Filter Process. (B) After Applying the Perona-Malik Diffusion Filter Process.

(Δt) of diffusion. The smaller the size of the time change the longer the diffusion process is required, and the longer the time-varying size changes. But if the size of the time size is too large to cause unfavorable results. The parameter K will control the speed of the diffusion process. Figure 15 shows that the Perona-Malik diffusion filter significantly reduce the noise and enhance the edge of interesting objects in fundus image.

Figure 16 also shows that the Perona-Malik diffusion significantly reduces the noise in the fundus image. The experimental results show that the proposed method is able to improve all the fundus images.

5. Conclusion

From the results of experiments and discussions that have been done then it can be concluded that the proposed method is able to enhance the interesting objects in the image fundus and reduce the noise in fundus image. White top hat morphological operations successfully perform uneven illumination on the fundus image. The Perona-Malik diffusion filter is now to reduce the noise present and enhance the edge of interesting objects in the fundus image.

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