International Journal of Engineering &Technology, 7 (4) (2018) 6402-6407

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET
doi: 10.14419/ijet.v7i4.18480
Research paper

Evaluation of different observation methods of the unknown points using three points resection

Hayder A. Alkanaani *

Surveying Dept., institute of technology-Baghdad, Middle Technical University Baghdad, Iraq
**Corresponding author E-mail: haider_abdalhadei@yahoo.com*

Abstract

In the current times, landscape works in general, and locating points in particular, depend on modern software and survey techniques, since they adopt AI (artificial intelligence) in calculations, thus the results are better if compared to manual calculations which have errors and mistakes. Here, the need to invent methods to check the accuracy of results has emerged, in order to reduces mistakes and errors. Additionally, these methods are adopted to find the unknown coor-dinates or check the calculated coordinates for those points using different methods, whether they were traditional or use the modern survey devices.
The process of calculating the coordinates with the use of engineering drawing featured by the dependence on pixels when they locate any point on the drawing screen, so, it is a very accurate method compared to visual devices and manual calculations. And depending on the above concept, the research depended three known points, and the unknown location of a point was found through these points using (Resection) method. The point was located in three different locations and its position was calculated using four different methods (manual cadastral computations, modern devices, engineering drawing software and websites). The results obtained from the three methods were compared in the end of the research, the comparison showed that the use of (manual cadastral computations), which depends the accuracy of 3 ranks after comma (millimeters) in calculations and for distance of approximately half kilometer between the known points and the unknown point, caused various errors among the four methods, while the methods that depend on software and websites gave a great match in their coordinates. The reason behind that was excluding the distance calculation as in manual and instruments methods. It was shown that distance and distribution of points play an important role in locating the unknown point accurately. Also it's shown that the use of devices and manual calculations are not proportional for finding precise results, where one of them was better than the other if right steps are followed.

Keywords: *Resection; Kaestner; Tienstra; Unknown Coordinates; Cadastral Computations.*

1. Introduction

Engineering surveys have witnessed many developments, which made the process of locating points on Earth's surface a very easy, and the importance of this field increased due to the development of techniques represented by survey devices and software used for location detection. Yet, the common attribute of all techniques is that they all depend on the concept of finding the coordination of a certain point with the dependence on two or more known points even in those new modern devices that relay on satellites technique represented by (GNSS), where the static observation concept that uses Single receiver, in fact, is related to three Reference Stations or three (or more) satellite locations are depended, these satellites represent the lines links them and the required point (Base Lines) (NISTOR and BUDA 2015) . Note that these techniques are featured by providing more accurate coordinates than others.
This paper studies the possibility of finding the coordinates of the unknown point through the three known points by depending on different methods, and the two angles (α, β) represent the concept of work to determine the direction from the unknown point towards the three points with resection method. The two directions which are represented by two straight lines will have an intersection point, which represents the required one. The difference among the four methods depends on cancel the measures of distance (CP), (BP) and direction (θ & ω) between the known and the unknown points which will have some errors. On the other side, the unknown coordinates were calculated through using cadastral survey equations first, total stations second and websites at last to compare them and show if they can be adopted as reference points that can be used in other surveys.
This paper aims to evaluate the accuracy of smart calculation methods and visual survey devices in finding the unknown coordinates of points.

2. Resection

Resection refers to the calculation of a certain point location through three points of known coordinates and (α , β) angles which are resulted from the observation of the three points. Typically, resection is used to link the coordinates of newly constructed region to an old established city, or to start a certain project through point of know coordinates that surround the region… etc.
There are three cases of resection depending on the point that its coordinates are required as the following figures (1- A,B,C) show (Milburn and Allaby 1987) :

1) The required point (P1) out of the triangle towards the base.
2) The required point (P2) out of the triangle towards the vertex.

3) The required point (P3) inside the triangle.

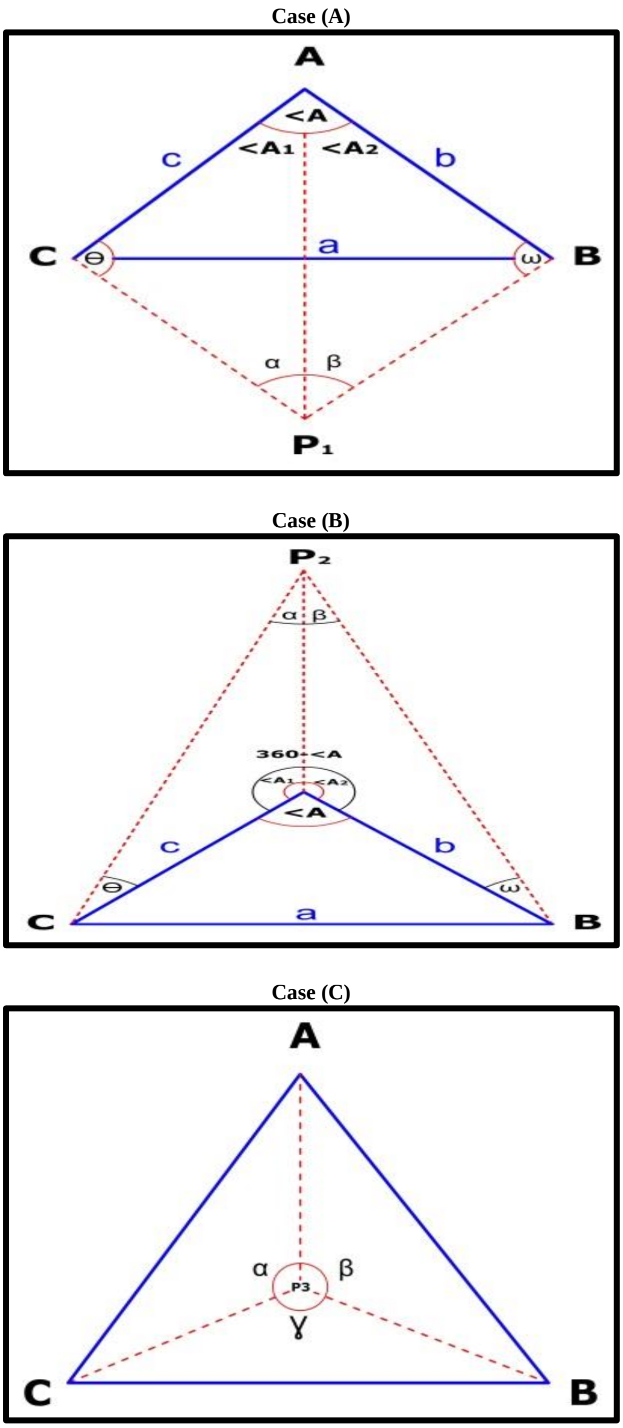


Fig. 2: Illustrates the Cases of Three Points Resection.

The concept of resection in survey works depends on determining, at lease, three points in the study area, which are depended to find the coordinates of the unknown point. And since the three points form a triangle where its sides, their directions and its internal angles can be calculated using inverse calculations, the location of the required point may occupy one of three possible locations, case (A) the point is opposite to one of the triangle sides, case (B) opposite to one of the triangle vertex or case (C) inside the triangle. It is required to find the coordinates of point (P) through the vertex of the triangle (A, B, C) using different methods to compare the results and determine the optimum method. Where, the coordinates of vertexes are known, and the unknown point (P) can be occupied, hence, from which the angle (α) between directions (PC & PA) and (β) between the direction (PA & PB) can be calculated, the main requirement is locating (P) from these parameters.

3. The procedure

Due to figure (1), the locations of points (P1, P2, P3) in the three cases (A, B and C) will be detected using four different methods. The first method uses equations for survey engineering after observing the three direction angles (α, β, γ) using Total Station Leica 06 plus device of accuracy (1") as a theodolite device, and the second method uses smart survey visual devices through the pervious device and its resection function, the reason of using the same device is to avoid differences result from the repeated installation of device on the point, its accuracy and the technical errors of devices. The third method depends on engineering drawing software represented by Autodesk programs of survey works such as (Civil 3D) and through creation button with resection method. While the last method depends on a website specialized with calculating the coordinates of unknown points based on Tree points resection (Tienstra's Method).

3.1. Text font of entire document

The mechanism of this method involves processing the location of the unknown point depending on parameters like observation supplied by coordinates. There are lots of mathematical methods with which the coordinates of an unknown point can be calculated (Joseph and Joaquim 2009) . But the most common methods that include all cases are KAESTNER-BURKHARDT METHOD and Tienstra's Method. But Tienstra's Method has a problem cleared by Professor K. Snelgrove in Memorial University (St. John's, Newfoundland & Labrador). He explained the limitation of Tienstra's Method that has caused some unexpected Results for his students when using this method. The problem is that unless point (P) located within the triangle (A, B and C), there are two Possible locations of (P) that will satisfy the given data. Tienstra's Method will find that only one of is to be the wrong one. The method specifies the wrong location when the angle covered by points A, B and C is more than 180 degrees, as well as if all four points are located on (or sufficiently close to) a circle, or the 3 known reference points located on a straight line. All these will causing errors in the results (Mike 2017).

1) Kaestner-Burkhardt Method.

Trigonometric functions method: this method will be adopted in cases (1 and 2) because two angles are required (α & β). This method is as following (eBook):

a) Length (a, b and c), directions of triangle sides and then the internal angels of triangle are calculated from the coordinates of the its vertexes.

$$a, b, c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (1)$$

$$Az = \tan^{-1}[\frac{(x_2 - x_1)}{(y_2 - y_1)}] \quad (2)$$

b) To calculate the angles (ω, θ), (tan φ, < S) are calculated through:

$$< S = \theta + \omega = 360 - (\alpha + \beta + < A) \quad (3)$$

$$\tan \varphi = \frac{\sin \omega}{\sin \alpha} = \frac{c . \sin \beta}{b . \sin \alpha} \quad (4)$$

$$\tan \theta = \frac{\sin < S}{\tan \varphi + \cos < S} \quad (5)$$

$$\tan \omega = \frac{\sin < S}{\cot \varphi + \cos < S} \quad (6)$$

c) AP_1 side length is calculated using sine rule

$$\left.\begin{array}{l} \text{Right triangle } \frac{AP_1}{\text{Sin } \omega} = \frac{b}{\text{Sin } \beta} \\ \text{Or} \end{array}\right\} \quad (7)$$

Left triangle $\frac{AP_1}{\text{Sin }\theta} = \frac{c}{\text{Sin }\alpha}$

d) calculating the direction of side AP1 from (AZ_{AC} and angle A_1) or (AZ_{AB}, angle A_2).

$$< A1 = 180 - (\theta + \alpha) \quad (8)$$

$$< A2 = 180 - (\omega + \beta) \quad (9)$$

$$\left.\begin{array}{l} AZ_{AP1} = AZ_{AC} \pm < A2 \\ \text{Or} \\ AZ_{AP1} = AZ_{AB} \pm < A1 \end{array}\right\} \quad (10)$$

e) P_1 coordinates are calculated from the direction AZ_{AP1} , AP_1 length and A Coordinate .

$$EP1 = EA + (AP1) \text{ Sin } AZ_{AP1} \quad (11)$$

$$NP1 = NA + (AP1) \text{ Cos } AZ_{AP1} \quad (12)$$

The required mathematical relations to calculate the unknown coordinates in the second case match the five mentioned steps except for one relation represented with angle value (S) which is shown in the next point as follows:

$< S = \theta + \omega = 360 - (\alpha + \beta + 360 + < A)$

$$\rightarrow < S = < A - \alpha - \beta \quad (13)$$

2) TIENSTRA'S METHOD (Harvey 2017)

Quantum equations method, it is more accurate since it requires the measurements of three angles towards the known points (γ, α, β), where the required point is inside the known triangle and as the following equations:

$$a = tan^{-1}[\frac{E_C - E_A}{N_C - N_A}] - tan^{-1}[\frac{E_B - E_A}{N_B - N_A}] \quad (14)$$

$$b = tan^{-1}[\frac{E_A - E_B}{N_A - N_B}] - tan^{-1}[\frac{E_C - E_B}{N_C - N_B}] \quad (15)$$

$$C = tan^{-1}[\frac{E_B - E_C}{N_B - N_C}] - tan^{-1}[\frac{E_A - E_C}{N_A - N_C}] \quad (16)$$

$$\left.\begin{array}{l} K_1 = \frac{1}{Cot\ a - Cot\ \gamma} \\ K_2 = \frac{1}{Cot\ b - Cot\ \alpha} \\ K_3 = \frac{1}{Cot\ c - Cot\ \beta} \end{array}\right\} \quad (17)$$

$$E_{P3} = \frac{K_1 * E_A + K_2 * E_B + K_3 * E_C}{K_1 + K_2 + K_3} \quad (18)$$

$$N_{P3} = \frac{K_1 * N_A + K_2 * N_B + K_3 * N_C}{K_1 + K_2 + K_3} \quad (19)$$

3.1.1. The point (p) is opposite to the side

This case is represented by placing a visual survey device in point (P1), then angles (α , β) between the three known points and the occupied point are observed, and using the lengths and directions of triangle sides, the location of the unknown point can be determined. Note that in order to reach the optimum possible accuracy to compare the results to those of other methods, the work will be through setting the lengths data to closest (Mm), and angles to closest (second).

Depending on pervious relations, and field survey illustrated in figure (2) which shows (case 1: the location is below the triangle base), the calculations were performed and they were as following:

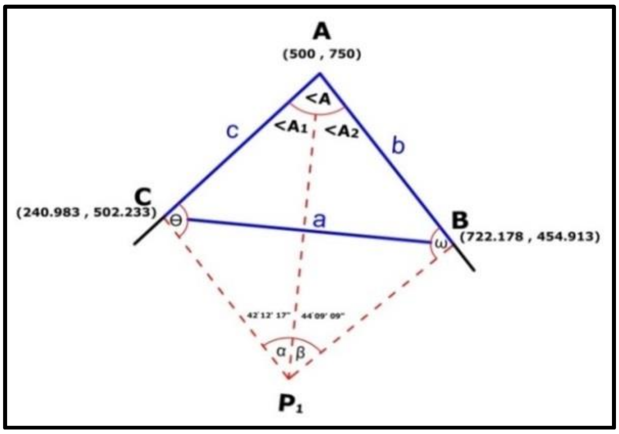


Fig. 2: Illustrates the Location of the Required Point is Below the Triangle Base Opposite to CB Side.

- Length, Azimuth of a triangle Sides:

By formula (1), a = 483.516 m, b= 369.377 m, c= 358.439 m

By formula (2), Az_{CA}= 46º 16' 18", Az $_{AB}$= 143º 01' 22", and from both directions determine <A >

< A = (46º 16' 18" + 180º) – 143º 01'22" = 83º 14' 56"

- The angles (θ & ω):
- By formulas (3, 4, 5, 6)

< S = 190º 23' 38", $\varphi = 45^{o}10' 37"$, θ = 97º 04' 50", ω = 93º 18' 08"

- The length (AP_1) by Sin Law.
- By formula (7), $AP_1 = 529.496\ m$ (or) $AP_1 = 529.399\ m$

Avg. of AP1 = 529.448 m

- The direction (AP1).

By formula (8), < A1 = 40o 42' 53"

By formula (10), AzAP1 = 185o 33' 25"

- Coordinate of point (P1).

By formula (11), EP1 = 448.731 m

- By formula (12), NP1 = 223.04 m

3.1.2. The point is opposite to the vertex

The case is represented by placing one of the visual survey devices which are not programmed at (P2), then the angles (α, β) between the three known points and the occupied point are observed. Using the lengths and directions of Sides of triangle, the location of the unknown point can be obtained. Note here that we will work on setting the data of lengths and angles (Mm and seconds) as in case (1) to reach the optimum possible accuracy.

Depending on the previous mathematical relation and the field work shown in figure (3), that illustrates the second case (the location of the point is above the known triangle), the calculations were as following:

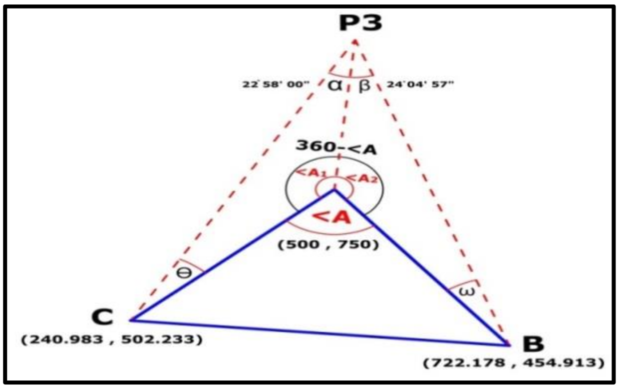


Fig. 3: Shows the Location of the Required Point above the Triangle (Opposite to the Vertex).

- Compute the length, Azimuth of the triangle Sides, (by the same formulas) :

a =483.516 m, b = 369.377 m, c =358.420 m

Az_{CA} = 46º 16' 18", Az_{AB} =143º 01' 23" , < A = 83º 14' 55"

- The angles (θ & ω):

< S = 36º 11' 58", $\varphi = 45\,^{o}25'\,09"$

$\theta = 17^{o}57'37" , \omega = 18^{o}14'21"$

- The length (AP2) by Sin Law:

$AP_2 = 283.247\ m$ (or) $AP_2 = 283.320\ m$

Avg. of AP1 = 283.284 m

- The direction (AP2)

< A_1 = 139º 04' 23", Az_{AP2} = 365º 20' 41"

Az_{AP2} = 5º 20' 41"

- The Coordinate of point (P2)

EP_2 = 526.387 m, NP_2 = 1032.052 m

3.1.3. The point inside the triangle

This case is represented by placing the thiolate device at (P3), then angles (α, β) between the three known points and the occupied point are observed. Using the lengths and directions of sides of triangle, the location of the unknown point can be obtained. Note here that we will work on setting the data of lengths and angles (Mm and seconds) as in previous cases (1 & 2) to reach the optimum possible accuracy.

Depending on the previous mathematical relation and the field survey shown in figure (4), that illustrates the second case (the location of the point is above the known triangle), the calculations were as following:

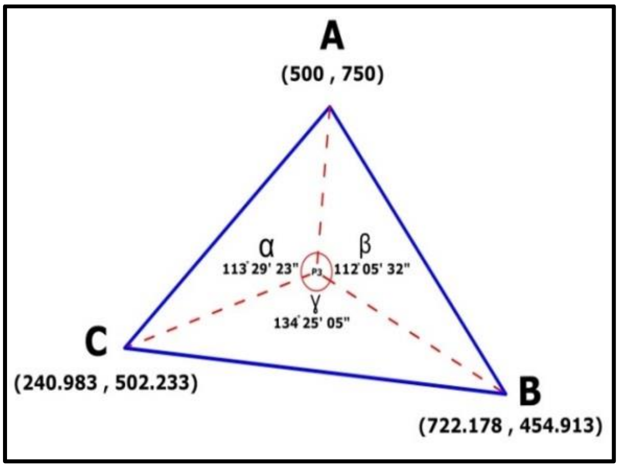


Fig. 4: Shows the Location of the Required Point Inside the Triangle.

By formulas (14, 15, 16) we find:

$a = 83.249 \, , b = 47.407 \, , c = -130.655$

By formula (17):

$K_1 = 0.912 \, , K_2 = 0.739 \, , K_3 = 0.791$

By formulas (18, 19):

$E_{P3} = 483.336\ m \, , N_{P3} = 580.445\ m$

The results were as shown in table (1).

Table 1: The Coordinates of (P1, P2, P3) Using Mathematical Calculation Method.

Resection Method	Case	X (m)	Y (m)
Cadastral Formulas	(P1) opposite the side	448.731	223.04
	(P2) opposite the vertex	526.387	1032.052
	(P3) inside triangle	483.336	580.445

3.2. Instruments (total station)

The device Total Station type (Leica 06 Plus) of accuracy (5”) was used. It is the same device that was used to measure the angles in the previous method (as a thiolate device). And through resection function, the coordinates were calculated and they were as follows:

Table 2: The Coordinates of (P1, P2, P3) Using Smart Survey Devices Method.

Resection Method	Case	X (m)	Y (m)
Instruments T.S (Leica 06 plus)	(P1) opposite the side	448.057	223.142
	(P2) opposite the vertex	526.294	1032.065
	(P3) inside triangle	483.372	580.414

3.3. Programs (civil 3D)

Civil 3D was adopted to calculate the coordinates of (P_1, P_2, P_3) through the angles (α, β, γ) that where observed using the mentioned device. The process of finding the data of the unknown points requires inserting the field coordinates of the three points and observing the two angles between these points, this is done through installing a thiolate device on the unknown points and find the angles (α, β) along with the coordinates of points (A, B, C) in three cases. But, to work on this program it is required determining the coordinates of the three known points, the angle between the first and the second point and the angle between the first and the third points. After these requirements are provided, the program will determine the unknown point on a drawing board through moving the sides right and left with fixed (B, C) until the angles (α = 42º 12' 17") and (β = 44º 09' 09") match as it is shown in figure (5) that illustrates case (1) and table (3).

The idea of the program (Civil 3D) is inspired by the concept of determining the unknown point depending on the intersection of straight lines through direction angles. Josep M. Font-Llagunes and Joaquim A. Batlle worked on this methodology in their study (New Method That Solves the Three-Point Resection Problem Using Straight Lines Intersection) (Joseph and Joaquim 2009).

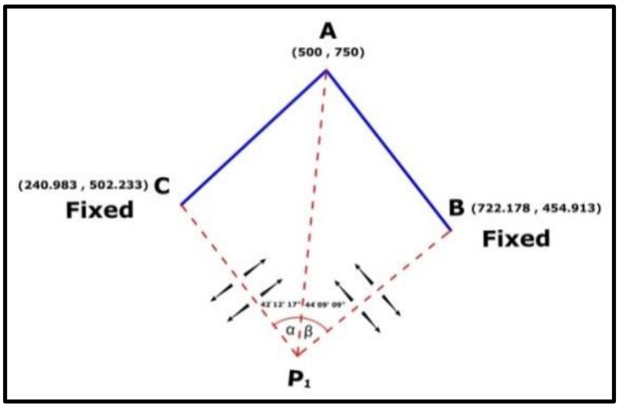


Fig. 5: An Example on How the Program Works on Finding Point (P) Case (1).

Table 3: The Coordinates of (P_1, P_2, P_3) Using Civil 3D

Resection Method	Case	X (m)	Y (m)
Programms (Civil 3D)	(P1) opposite the side	448.173	223.118
	(P2) opposite the vertex	526.378	1032.060
	(P3) inside triangle	438.315	580.373

3.4. The websites

The website is made by (Mesa Mike), it depends on the mathematical relations of (Tienstra's Method). The coordinates of an unknown point (P) can be calculated after providing the website with the data of the three points and angles (α, γ) as the following figure shows (6).

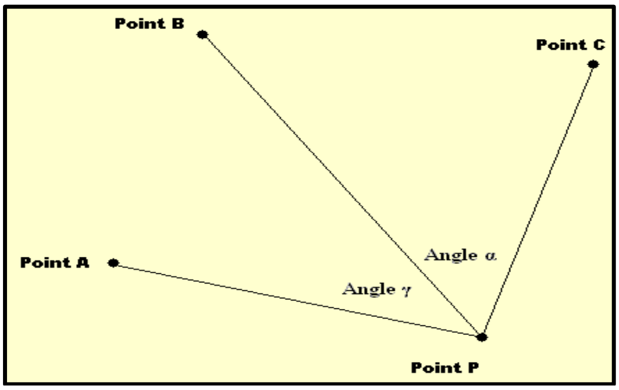
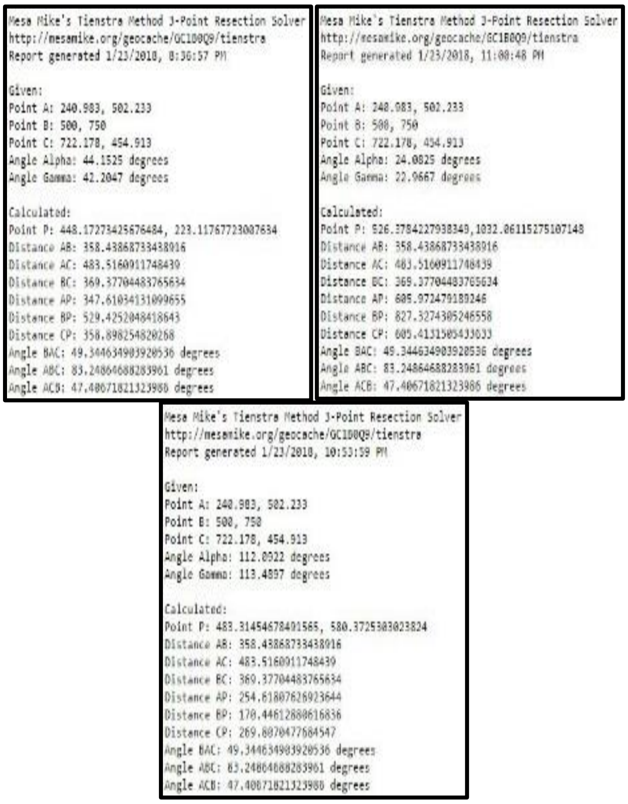


Fig. 6: A Model to Sort the Data by the Mentioned Website.

After the website is provided with required data, the following reports were obtained, see figure (7-A) point :

a) Point Opposite the side
b) Point Opposite the vertex
c) Point Inside the triangle



Mesa Mike's Tienstra Method 3-Point Resection Solver
http://mesamike.org/geocache/GC1D0Q9/tienstra
Report generated 1/23/2018, 8:36:57 PM

Given:
Point A: 240.983, 502.233
Point B: 500, 750
Point C: 722.178, 454.913
Angle Alpha: 44.1525 degrees
Angle Gamma: 42.2047 degrees

Calculated:
Point P: 448.17273425676484, 223.11767723007634
Distance AB: 358.43868733438916
Distance AC: 483.5160911748439
Distance BC: 369.37704483765634
Distance AP: 347.61034131099655
Distance BP: 529.4252048418643
Distance CP: 358.898254820268
Angle BAC: 49.344634903920536 degrees
Angle ABC: 83.24864688283961 degrees
Angle ACB: 47.40671821323986 degrees

Mesa Mike's Tienstra Method 3-Point Resection Solver
http://mesamike.org/geocache/GC1B0Q9/tienstra
Report generated 1/23/2018, 11:00:48 PM

Given:
Point A: 240.983, 502.233
Point B: 500, 750
Point C: 722.178, 454.913
Angle Alpha: 24.0825 degrees
Angle Gamma: 22.9667 degrees

Calculated:
Point P: 526.3784227938349,1032.06115275107148
Distance AB: 358.43868733438916
Distance AC: 483.5160911748439
Distance BC: 369.37704483765634
Distance AP: 605.972479189246
Distance BP: 827.3274305246558
Distance CP: 605.413150543303
Angle BAC: 49.344634903920536 degrees
Angle ABC: 83.24864688283961 degrees
Angle ACB: 47.40671821323986 degrees

Mesa Mike's Tienstra Method 3-Point Resection Solver
http://mesamike.org/geocache/GC1D0Q9/tienstra
Report generated 1/23/2018, 10:53:59 PM

Given:
Point A: 240.983, 502.233
Point B: 500, 750
Point C: 722.178, 454.913
Angle Alpha: 112.0922 degrees
Angle Gamma: 113.4897 degrees

Calculated:
Point P: 483.31454678491565, 580.3725303023824
Distance AB: 358.43868733438916
Distance AC: 483.5160911748439
Distance BC: 369.37704483765634
Distance AP: 254.61807626923644
Distance BP: 170.44612880616836
Distance CP: 269.8870477684547
Angle BAC: 49.344634903920536 degrees
Angle ABC: 83.24864688283961 degrees
Angle ACB: 47.40671821323986 degrees

Fig. 7: The Reports from Website.

And according to the above reports, the results of the unknown point coordinates (P1, P2, P3) as table (4) shows.

Table 4: The Coordinates of (P1, P2, P3) Using the Website

Resection Method	Case	X (m)	Y (m)
web sites	(P1) opposite the side	448.173	223.118
	(P2) opposite the vertex	526.378	1032.06
	(P3) inside triangle	483.315	580.373

According to what mentioned above, the detected coordinates of the unknown point (P), which represents the point location below the triangle base, (P2) is opposite to the vertex and (P3) is inside the triangle using the four methods (mathematical, devices, programs and websites) as illustrated in table (5).

Table 5: The Coordinates Detected Using Four Methods

Resection Method	Case	X (m)	Y (m)
Cadastral Formulas	(P1) opposite the side	448.731	223.04
	(P2) opposite the vertex	526.387	1032.052
	(P3) inside triangle	483.336	580.445
Instruments T.S (Leica 06 plus)	(P1) opposite the side	448.057	223.142
	(P2) opposite the vertex	526.294	1032.065
	(P3) inside triangle	483.372	580.414
Programms (Civil 3D)	(P1) opposite the side	448.173	223.118
	(P2) opposite the vertex	526.378	1032.060
	(P3) inside triangle	483.315	580.373
web sites	(P1) opposite the side	448.173	223.118
	(P2) opposite the vertex	526.378	1032.060
	(P3) inside triangle	483.315	580.373

The similar coordinate values in both websites and program indicates the accuracy of the unknown coordinates data, considering that the error values are minimized because the work is not affected by closing the values, the distances are not affected by the striped measures or by the slop of device reflector and the effect of the weather condition in observation (when the devices are used). Depending on the assumption above, the values calculated using the program and websites represent reference values, while other values are subsequent that should be checked and reasons behind errors should be identified. The coordinates can be graphically represented as in figures (8, 9 and 10):

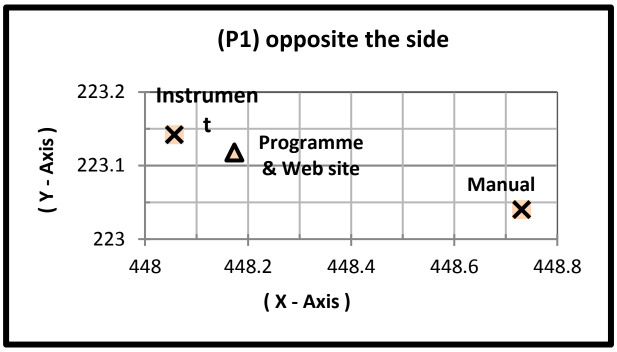


Fig. 8: Illustrating the Data of Point (P1) with the Four Methods.

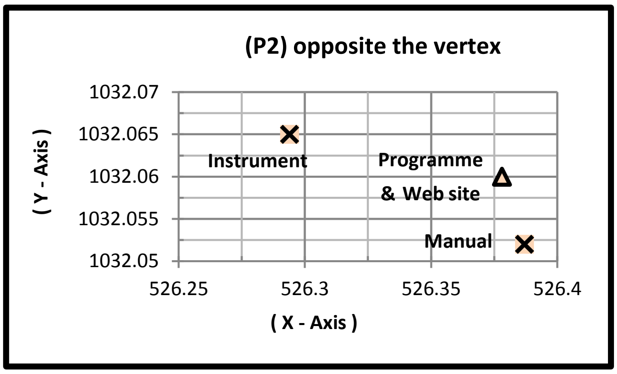


Fig. 9: Illustrating the Data of Point (P2) With the Four Methods.

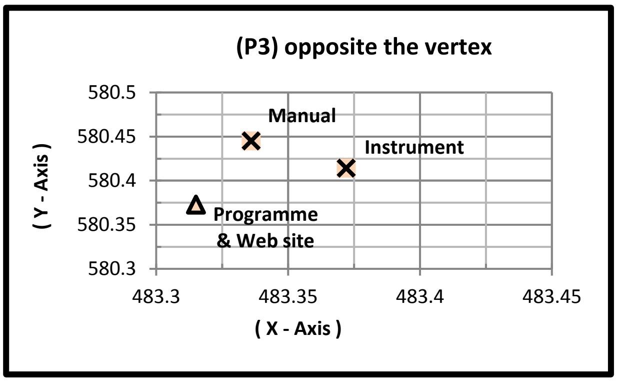


Fig. 10: Illustrating the Data of Point (P3) with the Four Methods.

In the three figures above, the symbol (Δ) refers to the coordinates of observation of unknown points (P1, P2, P3) using (Civil 3D) program and website, while the other symbol (X) of the point refers to observation which uses mathematical manual calculations method and Instrument method which is based on (Total Station Leica 06 Plus) device. It is conducted by the previous results that these errors are resulting from neglecting values after the comma which are the most common reasons with the accurate survey. In addition to technical mistakes resulted from the use of survey devices. It is noticed that the error values decreased in the third method (inside triangle) due to the distribution of points around the unknown point and detecting points through angles, which features this method than the other two.

4. Conclusion

It was conducted by this study that despite the small size of observation area, the observation process may have many errors of large values relatively. It was shown that the values of these errors are decreased if the points around the required point were distributed in a geometric homogenous pattern, in other words, as the angles (α, β, γ) surrounding the point are smaller; the error value are increased, similar to the distribution of satellites around observation point (DOP).
The results indicated that calculations depended on software, in fact, were relatively more accurate compared to those of other methods used to find the coordinates of an unknown point; the reason behind that is the process of locating these points depends on direction only while the measures between the unknown point and the known ones are excluded compared to the use of device and mathematical relations represented by KAESTNER-BURKHARDT and Tienstra's Methods, where many errors may be occurred when the process of calculation is performed using devices, errors like invertor inclination, the effect of weather conditions on laser distances measurements and errors of manual calculations.
This means that if high accuracy is required, the resection method to calculate coordinates using software is the best method. In addition, this method is utilized to check the work when the first two methods used. Also, a periodic check can be performed on all known points in work field using resection method, and for a more accurate field work, the points may be increased in this method through forming triangles with (inside triangle and opposite to side) methods since better results were obtained compared to (point opposite to vertex) method.

Acknowledgements

The author would like to thank the (survey department / institute of technology-Baghdad) for moral support and in providing experimental instruments and program concerning to prepare this paper.

References

[1] eBook "THREE POINT RESECTION PROBLEM " Surveying Engineering Department, Ferris State University: 175-201
[2] Harvey, R. (2017). "Survey Computation, Surveying & Geospatial Engineering " School of Civil and Environmental Engineering, University of New South Wales, SYDNEY N.S.W 2052 1-287
[3] Joseph, M. and A. Joaquim (2009). "New Method That Solves the Three-Point Resection Problem Using Straight Lines Intersection." JOURNAL OF SURVEYING ENGINEERING 135(2): 39-45. https://doi.org/10.1061/(ASCE)0733-9453(2009)135:2(39).
[4] Mike, M. (2017). "Point Resection Solver Using Tienstra's Method." web site: http://mesamike.org/geocache/GC1B0Q9/tienstra/.
[5] Milburn, P. and R. Allaby (1987). "A Field-Ready Solution to the resection problem given two coordinated points" Research Station, Canadian agricultural engineering 29(1): 93-96
[6] NISTOR, S. and A. S. BUDA (2015). "ANALYSIS OF GNSS DATA USING PRECISE POINT POSITIONING." Mathematical Modelling in Civil Engineering 11(3): 31-37. https://doi.org/10.1515/mmce-2015-0013.