International Journal of Engineering & Technology, 7 (3.31) (2018) 157-159

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

On Trio Ternary Γ-Semigroups

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Abstract

In this paper the terms trio L-trio TΓ-ideal, La-trio TΓ-ideal, R-trio TΓ- ideal and trio TΓ- ideal of a TΓ-semi group are introduced and some examples are given. It is proved that (1) a TΓ-semi group T is a trio TΓ-semi group if and only if $x\Gamma T\Gamma T = T\Gamma T\Gamma x = T\Gamma x\Gamma T$ for all $x \in T$, (2) Every trio TΓ-semi group is a duo TΓ-semi group, (3) Every commutative TΓ-semi group is a trio TΓ-semi group (4) Every quasi commutative TΓ-semi group is a trio TΓ-semi group.

Keywords *duo TΓ-semi group, idempotent, prime trio TΓ-ideal, semi primary TΓ-semi group trio TΓ-semi group.*

1. Introduction

In the year 2012, A. G. Rao, and D. M. Rao[1, 2, 3] investigated on duo – semi groups. In the year 2013, K. Padmavathi, M. Ramesh and D.M. Rao[4] developed on Regular Duo Ternary Semi groups. G. Srinivasa Rao and D. M. Rao [5, 6] were introduced and developed the notions of T-semirings in 2014 and 2015. M. Sajani Lavanya and D. M. Rao[7, 8] introduced the concept of TΓ-semirings in the year 2015.

2. Preliminaries

Note 2.1 : for preliminaries refer to reference [10].

3. Trio Ternary Γ-Semi group

Def 3.1: A TΓ-semi group Q is called a L-duo ternary Γ-semi group if every left TΓ-ideal of Q is a two sided TΓ-ideal of Q.

Def 3.2: A TΓ-semi group Q is called a R-duo ternary Γ-semi group if every right TΓ-ideal of Q is a two sided TΓ-ideal of Q.

Def 3.3: A TΓ-semi group Q is called a duo TΓ-semi group if it is both a left duo TΓ-semi group and a right duo TΓ-semi group.

Th 3.4: A TΓ-semi group M is a duo TΓ-semi group if and only if $p\Gamma M\Gamma M = M\Gamma M\Gamma p$ for all $p \in M$.

Def 3.5: A TΓ-semi group M is said to be a ***L-trio TΓ-semi group*** if every left TΓ-ideal of M is a lateral TΓ-ideal and right TΓ-ideal of M.

Def 3.6: A TΓ-semi group M is said to be a ***R-trio TΓ-semi group*** if every right TΓ-ideal of M is a lateral TΓ-ideal and left TΓ-ideal of M.

Def 3.7 : A TΓ-semi group Q is said to be a ***La-trio TΓ-semi group*** if every lateral TΓ-ideal of Q is a left TΓ-ideal and a right TΓ-ideal of Q.

Def 3.8: A TΓ-semi group M is said to be a ***trio TΓ-semi group*** if it is a L-trio TΓ-semi group, a La-trio TΓ-semi group and a R-trio TΓ-semi group.

Th 3.9: A TΓ-semi group M with identity is a trio T*Γ-semi group* iff $x\Gamma M\Gamma M = M\Gamma M\Gamma x = M\Gamma x\Gamma M$ for all $x \in M$.

Proof: Suppose that M is a trio TΓ-semi group and $x \in M$. Let $t \in x\Gamma M\Gamma M$. Then $t = x\alpha u_i\beta v_i$ for some $u_i, v_i \in M$, $\alpha, \beta \in \Gamma$. Since $M\Gamma M\Gamma x$ is a left TΓ-ideal of M, $M\Gamma M\Gamma x$ is a TΓ-ideal of M. So $x \in M\Gamma M\Gamma x$, $u_i, v_i \in M$, $M\Gamma M\Gamma x$ is a TΓ-ideal of M $\Rightarrow x\alpha u_i\beta v_i \in M\Gamma M\Gamma x \Rightarrow t \in M\Gamma M\Gamma x$. Therefore $x\Gamma M\Gamma M \subseteq M\Gamma M\Gamma x$. Similarly we can prove that $M\Gamma M\Gamma x \subseteq x\Gamma M\Gamma M$.
Therefore, $x\Gamma M\Gamma M = M\Gamma M\Gamma x$ for all $x \in M$. ⟶ (1)
Let $t \in x\Gamma M\Gamma M$. Then $t = x\alpha u_i\beta v_i$ for some $u_i, v_i \in M$.
Since $M\Gamma x\Gamma M$ is a La-TΓ-ideal of M, $M\Gamma x\Gamma M$ is a TΓ-ideal of M. So $x \in M\Gamma x\Gamma M$, $u_i, v_i \in M$, $M\Gamma x\Gamma M$ is a TΓ-ideal of M $\Rightarrow x\alpha u_i\beta v_i \in M\Gamma x\Gamma M \Rightarrow t \in M\Gamma x\Gamma M$. So $x\Gamma T\Gamma T \subseteq T\Gamma x\Gamma T$.
Similarly, we can prove that $M\Gamma x\Gamma M \subseteq x\Gamma M\Gamma M$.
Therefore, $x\Gamma M\Gamma M = M\Gamma x\Gamma M$ for all $x \in M$. ⟶ (2).
Hence form (1) and (2) $x\Gamma M\Gamma M = M\Gamma M\Gamma x = M\Gamma x\Gamma M$ for all $x \in M$
Conversely Let $x\Gamma M\Gamma M = M\Gamma M\Gamma x = M\Gamma x\Gamma M$ $\forall x \in M$. Let Q be a L-TΓ-ideal of M.
Let $x \in Q$, $u_i, v_i \in M$. Then $x\alpha u_i\beta v_i \in x\Gamma M\Gamma M = M\Gamma M\Gamma x = M\Gamma x\Gamma M \Rightarrow x\alpha u_i\beta v_i = s_i\gamma t_i\delta x = p_i\chi x\varphi q_i$ for some $s_i, t_i, p_i, q_i \in M$ and $\alpha, \beta, \gamma, \delta, \chi, \varphi \in \Gamma$. Let $x \in Q$, $s_i, t_i \in M$, Q is a L-TΓ-ideal of M $\Rightarrow s_i\gamma t_i\delta x \in Q \Rightarrow x\alpha u_i\beta v_i \in Q$. So Q is a R-TΓ-ideal of M and let $x \in Q$, $p_i, q_i \in M$, A is a La-TΓ-ideal of M $\Rightarrow p_i\chi x\varphi q_i \in Q \Rightarrow x\alpha u_i\beta v_i \in Q$. Hence, Q is La-TΓ-ideal of M. So Q is a R-TΓ-ideal of M and La-TΓ-ideal of M and hence Q is a TΓ-ideal of M. Hence, M is L-trio TΓ-semi group. Similarly, we can prove that M is a R-trio TΓ-semi group as well as La-trio TΓ-semi group. Hence M is trio TΓ-semi group.

Th 3.10: Every trio TΓ-semi group is a duo TΓ-semi group.

Proof: Let Q be a trio TΓ-semi group. Then by theorem 3.9, $x\Gamma Q\Gamma Q\Gamma x = Q\Gamma x\Gamma Q\ \forall\ x \in Q$. Therefore, Q is a duo TΓ-semi group.

Note 3.11: The converse of the theorem 3.10, need not necessarily true. i.e., every duo TΓ-semi group need not be trio TΓ-semi group.

Example 3.12: Consider the set T = { 0, - *s*, - *t*, - *r*} and Γ = T with the following compositions:

.	0	-*s*	-*t*	-*r*
0	0	0	0	0
s	0	-*s*	-*t*	-*r*
t	0	0	0	0
r	0	-*s*	-*t*	-*r*

.	0	-*s*	-*t*	-*r*
0	0	0	0	0
-*s*	0	*s*	*t*	*r*
-*t*	0	0	0	0
-*r*	0	*s*	*t*	*r*

Clearly M is a TΓ-semi group. Let P = {0. – *t*} is a L-TΓ-ideal and R-TΓ-ideal of M, but not La-TΓ-ideal of M and hence M is a duo TΓ-semi group but not trio TΓ-semi group. Since let $-t \in$ P and $-a, -c \in$ M ⇒ $(-s)\alpha(-t)\beta(-r) \in M\Gamma P\Gamma M$ and $(-s)\alpha(-t)\beta(-r) = -r \notin P$ and hence $M\Gamma P\Gamma M \nsubseteq P$. Therefore P is not La-TΓ-ideal of M and hence M is not trio TΓ-semi group.

Th 3.13: Every commutative TΓ-semi group is a trio TΓ-semi group.

Proof **:** Let T is a commutative TΓ-semi group Therefore $x\Gamma T\Gamma T = T\Gamma T\Gamma x = T\Gamma x\Gamma T\ \forall\ x \in T$.. By theorem 3.9, T is a trio TΓ-semi group.

Th 3.14: Every quasi commutative TΓ-semi group is a trio TΓ-semi group.

Proof **:** Suppose that T is a quasi commutative TΓ-semi group. Then for each *a*, *b*, *c*∈ T, there exists a $n \in N$ such that $a\alpha b\beta c = b^n\ \alpha a\beta c = b\alpha c\beta a = c^n\ \alpha b\beta a = c\alpha a\beta b = a^n\ \alpha c\beta b$. suppose Q be a L-TΓ-ideal of T. Therefore, $T\Gamma T\Gamma A \subseteq Q$. Let $a\in Q$ and *s*, *t* ∈ T. Since T is a quasi commutative TΓ-semi group, there exist a odd *n* such that $a\alpha s\beta t = t^n\alpha s\beta a \in T\Gamma T\Gamma Q \subseteq Q$. Therefore, $a\alpha s\beta t \in Q$ for all $a \in Q$ and $s, t \in T$, $\alpha, \beta \in \Gamma$ and hence $Q\Gamma T\Gamma T \subseteq Q$. Thus Q is R-TΓ-ideal of T. Now $a\alpha s\beta t = t\alpha a\beta s \in T\Gamma Q\Gamma T \subseteq Q\ \forall\ a \in Q$, $s, t \in T$ and $\alpha, \beta \in \Gamma$ and hence $Q\Gamma T\Gamma T \subseteq Q$. Thus Q is a La-TΓ-ideal of T. Therefore T is a L-trio TΓ-semi group. Similarly, we can prove that T is a La-trio TΓ-semi group and T is a R-trio TΓ-semi group. Therefore, every quasi commutative TΓ-semi group is a trio TΓ-semi group.

Def 3.15: An element *a* of a TΓ-semi group T is said to be ***regular*** provided $x, y \in T$ and $\alpha, \beta, \gamma. \delta \in \Gamma$ such that $a\alpha x\beta a\gamma y\delta a = a$. The ternary semi group T called ***regular TΓ-semi group***

Some authors may define the regular element in TΓ-semi group if there exist an element $x \in T$, $\alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$. But obviously both the conditions are same.

Th 3.16: Every idempotent element in a TΓ-semi group is regular.

Def 3.17: An element *a* of a TΓ-semi group T is said to be ***left regular*** if there exist $x, y\in T$ and $\alpha, \beta \in \Gamma$ such that $a = (a\alpha)^3 x\beta y$.

Def 3.18: An element *a* of a TΓ-semi group T is said to be ***lateral regular*** if there exist $x, y\in T$ and $\alpha, \beta \in \Gamma$ such that $a = x\alpha\,(a\beta)^3 y$.

Def 3.19: An element *a* of a TΓ-semi group T is said to be ***right regular*** if there exist $x, y\in T$ and $\alpha, \beta \in \Gamma$ such that $a = x\alpha y\beta\,(a\gamma)^3$.

Def 3.20: An element *a* of a TΓ-semi group T is said to be ***intra regular*** if there exist $x, y\in T$ such that $a = x\alpha(a\beta)^5 y$.

Def 3.21: An element *a* of a TΓ-semi group M is said to be ***semi-simple*** if $q\in (<q>\Gamma)^2 <q>$ i.e. $(<q>\Gamma)^2 <q> = <q>$.

Th 3.22: An element *a* of a TΓ-semi group M is said to be *semi simple* if $q \in (<q>\Gamma)^{m-1} <q>$ i.e. $(<q>\Gamma)^{m-1} <q>$ = <*q*> ∀ odd *m*.

Def 3.23 : A TΓ-semi group M is called ***semi simple TΓ*-semi group** provided every element in M is semi simple.

Th 3.24: If T is a trio TΓ-semi group with identity, then the following are equivalent for any element *a* ∈ T.

1) ***a* is regular.**
2) ***a* is left regular.**
3) ***a* is right regular.**
4) ***a* is lateral regular**
5) ***a* is intra regular.**
6) ***a* is semi simple.**

Proof **:** Since T is trio TΓ-semigroup, $a\Gamma T\Gamma T = T\Gamma a\Gamma T = T\Gamma T\Gamma T\Gamma a\Gamma T\Gamma T = T\Gamma T\Gamma a$.

We have $a\Gamma T\Gamma a = a\Gamma a\Gamma T = T\Gamma a\Gamma a = T\Gamma T\Gamma a\Gamma T\Gamma T = (a\Gamma)^2 T\Gamma(a\Gamma)^2 = a\Gamma T\Gamma a\Gamma T\Gamma a = a\Gamma a\Gamma a\Gamma T\Gamma T = T\Gamma T\Gamma a\Gamma a\Gamma a = T\Gamma a\Gamma a\Gamma a\Gamma T = <(a\Gamma)^2 a> = <a>\Gamma<a>\Gamma<a>$.

(1) ⇒ (2) : Let *a* is regular. Then $a = a\alpha x\beta a\gamma y\delta a$ for some $x, y \in T$ and $\alpha, \beta, \gamma. \delta \in \Gamma$. Therefore $a \in a\Gamma T\Gamma a\Gamma T\Gamma a = a\Gamma a\Gamma a\Gamma T\Gamma T = a\Gamma a\Gamma T \Rightarrow a = (a\alpha)^3 x\beta y$ for some $x, y \in T$ and $\alpha, \beta \in \Gamma$. Therefore *a* is left regular.

(2) ⇒ (3) : let *a* is left regular. Then $a = (a\alpha)^3 x\beta y$ for some for some $x, y \in T$ and $\alpha, \beta \in \Gamma$. Therefore $a \in a\Gamma a\Gamma a\Gamma T\Gamma T = T\Gamma T\Gamma a\Gamma a\Gamma a \Rightarrow a = x\alpha y\beta\,(a\gamma)^3$ for some for some for some $x, y \in T$ and $\alpha, \beta, \gamma. \in \Gamma$. Therefore *a* is right regular.

(3) ⇒ (4): Let *a* is right regular. Then for some $x, y \in T$, and $\alpha, \beta \in \Gamma$, $a = (a\alpha)^3 x\beta y$. Therefore $a \in T\Gamma T\Gamma a\Gamma a\Gamma a = <(a\Gamma)^2 a> \Rightarrow a = x\alpha\,(a\beta)^3 y$ for some $x, y \in T$ and $\alpha, \beta \in \Gamma$. Therefore *a* is lateral regular.

(4) ⇒ (5): Let *a* is lateral regular. Then for some $x, y \in T$, $\alpha, \beta \in \Gamma$, $a = x\alpha\,(a\beta)^3 y$. Therefore $a \in T\Gamma a\Gamma a\Gamma a\Gamma T = T\Gamma(a\Gamma)^5 T = <(a\Gamma)^4 a> \Rightarrow a = x\alpha(a\beta)^5 y$ for some $x, y \in T$.

Therefore *a* is intra regular.

(5) ⇒ (6): Let *a* is intra regular. Then $a = x\alpha(a\beta)^5 y$ for some $x, y \in T$. Therefore $a \in (<a>\Gamma)^4 <a>$. Therefore *a* is semi simple.

Def 3.25: An element *a* of a TΓ-semi group T is said to be an ***idempotent*** element provided $(a\Gamma)^2 a = a$.

Def 3.26: An element *a* of a TΓ-semi group T is said to be ***zero*** of T if $a\Gamma b\Gamma c = b\Gamma a\Gamma c = b\Gamma c\Gamma a = a\ \forall\ b,c,x \in T$.

Notation 3.27: For any TΓ-semi group T, let E_T denotes the set of all idempotents of T together with the binary relation denoted by $e \le f$ if and only if $e = e\Gamma e\Gamma f = f\Gamma e\Gamma e$ for $e, f \in E_T$.

Def 3.28: A TΓ-ideal Q of a TΓ-semi group M is said to be ***semi-primary*** provided √Q is a prime TΓ-ideal of M.

Def 3.29: A TΓ-semi group M is said to be a ***semi primary TΓ-semi group*** provided every TΓ-ideal of M is a semi primary TΓ-ideal

Th 3.30: Let T is a trio semi primary TΓ-semi group, then the idempotents of T form a chain under natural ordering.

4. Conclusion

In this paper we are introducing the concept of trio TΓ-ideals in TΓ-semi group. Previously, many of the researchers studied about duo ideals in different algebraic structures.

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