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Research paper

A Study on Pseudo Symmetric Γ-Ideals in Ternary Γ-Semigroups

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Abstract

In this paper section 1 reflects, the terms, '***PSTΓ-ideals***' of a ternary Γ-semigroup and '***PSTΓ-semi group***' are introduced can characterized ***PSTΓ-semi group***. In section 2, the terms, '***SPSTΓ-ideals***, '***SPSTΓ-semi group***' are introduced and classified these ***SPSTΓ-ideals***.

Keywords: *PSTΓ-ideal, SPSTΓ-ideal, prime, semiprime, Archimedean.*

1. Introduction

The notions of PSTΓ-ideals in semi groups, *SPST**Γ**-semi group* and some classes of *PST**Γ**-semi group* was introduced by Ramakotaiah and Anjaneyulu . In this thesis we introduce and made a study on *PST**Γ**-* ideals and *SPST**Γ**-ideals* *SPST**Γ**-semi groups* and obtained KRULL's theorem for *SPST**Γ**-semi group* in ternary semi groups.

2. Preliminaries

Note 2.1 : For preliminaries refer to the references and their references.

Note 2.2: Throughout this paper PSTΓ-Ideal, SPSTΓ-ideal, CPTΓ-ideal and CSTΓ-ideal means pseudo-symmetric ternary Γ-ideal, semi pseudo-symmetric Γ-ideal, completely prime ternary Γ-ideals and completely semi prime ternary Γ-ideal respectively unless otherwise stated.

3. PSTΓ-Ideals

We now introduce the notion of a PSTΓ-Ideal of a TΓ-semi group.

Def 3.1 : A TΓ-ideal P of a TΓ-semi group T is called PSTΓ-Ideal if $x, y, z \in T$, $x\Gamma y\Gamma z \subseteq P \Rightarrow x\Gamma s\Gamma y\Gamma t\Gamma z \subseteq P$ for all $s, t \in T$.

Note 3.2: A TΓ-ideal P of a TΓ-semi group T is PSTΓ-Ideal iff $x, y, z \in T$, $x\Gamma y\Gamma z \subseteq P$ implies $x\Gamma T^1\Gamma y\Gamma T^1\Gamma z \subseteq P$.

Ex 3.3: Let $Z = \{u, v, w\}$ and $\Gamma = \{i, j, k\}$. Define a ternary operation **'.'** in T as shown in the following table:

.	u	v	w
u	u	u	u
v	u	u	u
w	u	v	w

Define a mapping T × Γ × T × Γ × T → T by $uivjw = uvw$. It is easy to see that T is a TΓ-semi group. The TΓ-ideals of T are { *u* }, { *u* , *v* }, { *u, v,wr* } which are PSTΓ-Ideal.

Th 3.4 : Let P be a PSTΓ-Ideal in a T**Γ**-semi group T and $p, q, r \in T$. Then $p\Gamma q\Gamma r \subseteq P$ iff $<p>\Gamma<q>\Gamma<r> \subseteq P$.

Cor 3.5 : Let P be any PSTΓ-Ideal in a T**Γ**-semi group T and $p_1, p_2, \ldots, p_n \in T$ where n is an odd $n \in N$. Then $p_1\Gamma p_2\Gamma \ldots \Gamma p_n \subseteq A$ iff $<p_1>\Gamma<p_2>\Gamma\ldots\Gamma<p_n> \subseteq A$.

Cor 3.6: Let P is a PSTΓ-Ideal in a T**Γ**-semi group T. Then for any odd $m \in N$, $(p\Gamma)^{m-1}p \subseteq P$ implies $(<p>\Gamma)^{m-1}<p> \subseteq P$.

Cor 3.7 : Let P be a PSTΓ-Ideal in a T**Γ**-semi group T. If $(a\Gamma)^{n-1}a \subseteq P$, for some odd $m \in N$, then $(<a\Gamma s\Gamma t\Gamma>^{m-1}<a\Gamma s\Gamma t>) \subseteq P$, $(<s\Gamma t\Gamma a\Gamma>^{m-1}<s\Gamma t\Gamma a>) \subseteq P$, and $(<s\Gamma a\Gamma t\Gamma>^{n-1}<s\Gamma a\Gamma t>) \subseteq P$ for all $s, t \in T$.

Th 3.8: Let Q_r be a PSTΓ-Ideal of M, then $\bigcap_{r=1}^{n} Q_r \neq \varnothing$ of a TΓ-semi group M is a PSTΓ-Ideal of M.

Th 3.9 : Every CSTΓ-ideal A in a TΓ-semi group M is a PSTΓ-Ideal.

***Proof* :** Let Q be a CSTΓ-ideal of the TΓ-semi group M.
Let $x, y, z \in T$ and $x\Gamma y\Gamma z \subseteq Q$. $x\Gamma y\Gamma z \subseteq Q$ implies
$(y\Gamma z\Gamma x\Gamma)^2 y\Gamma z\Gamma x = (y\Gamma z\Gamma x)\Gamma(y\Gamma z\Gamma x)\Gamma(y\Gamma z\Gamma x)$
$= y\Gamma z\Gamma(x\Gamma y\Gamma z)\Gamma(x\Gamma y\Gamma z)\Gamma x \subseteq Q$.
$(y\Gamma z\Gamma x\Gamma)^2 y\Gamma z\Gamma x \subseteq Q$, Q is CSTΓ-ideal $\Longrightarrow y\Gamma z\Gamma x \subseteq Q$.
Similarly $(z\Gamma x\Gamma y\Gamma)^2 z\Gamma x\Gamma y = (z\Gamma x\Gamma y)\Gamma(z\Gamma x\Gamma y)\Gamma(z\Gamma x\Gamma y)$
$= z\Gamma(x\Gamma y\Gamma z)\Gamma(x\Gamma y\Gamma z)\Gamma x\Gamma y \subseteq Q$.
$(z\Gamma x\Gamma y\Gamma)^2 z\Gamma x\Gamma y \subseteq Q$, Q is CSTΓ-ideal $\Rightarrow z\Gamma x\Gamma y \subseteq Q$.
If $s, t \in T^1$, then
$(x\Gamma s\Gamma y\Gamma t\Gamma z\Gamma)^2 x\Gamma s\Gamma y\Gamma t\Gamma z$
$= (x\Gamma s\Gamma y\Gamma t\Gamma z)\Gamma(x\Gamma s\Gamma y\Gamma t\Gamma z)\Gamma(x\Gamma s\Gamma y\Gamma t\Gamma z)$

$= x\Gamma s\Gamma y\Gamma t\Gamma[z\Gamma x\Gamma(s\Gamma y\Gamma t)\Gamma(z\Gamma x\Gamma s)\Gamma y]\Gamma t\Gamma z \subseteq Q$.
$(x\Gamma s\Gamma y\Gamma t\Gamma z\Gamma)^2 x\Gamma s\Gamma y\Gamma t\Gamma z \subseteq Q$, Q is CSTΓ-ideal
$\Rightarrow x\Gamma s\Gamma y\Gamma t\Gamma z \subseteq Q$. Therefore Q is a PSTΓ-Ideal.

Note 3.10 : The converse of theorem 3.9, is not true,

Ex 3.11 : Consider the TΓ-semi group in example 3.3. P = {*p*} is a PSTΓ-Ideal in the ternary **Γ**-semi group T, and it is not completely semi prime, since $q\alpha q\alpha q = p \in P$, but $q \notin A$.

Th 3.12 : If A is a PSTΓ-Ideal of a TΓ-semi group M then $P_2 = P_4$.
Proof **:** Obviously, $P_4 \subseteq P_2$. Let $p \in P_2$. Then for some odd $m \in N$ we have $(p\Gamma)^{m-1}x \subseteq P$. Since P is PSTΓ-Ideal, $(p\Gamma)^{n-1}p \subseteq P \Rightarrow (< p>\Gamma)^{n-1}< p > \subseteq P \Rightarrow p \in P_4$. Hence, $P_2 \subseteq P_4$ and hence $P_2 = P_4$.

Th 3.13 : If P is a PSTΓ-Ideal of a TΓ-semi group M then P_2 = {x : $(x\Gamma)^{n-1}x \subseteq A$ for some odd $n \in N$ } is a minimal CSTΓ-ideal of T.
Proof **:** Obviously, $P \subseteq P_2$ and hence P_2 (≠∅) subset of M. Let $x \in P_2$ and $s, t \in M$.
Now $x \in P_2 \Rightarrow (x\Gamma)^{n-1}x \subseteq A$ for some odd *n*. $(x\Gamma)^{n-1}x \subseteq P$, *s, t* ∈ T, A is a PSTΓ-Ideal of T $\Rightarrow (x\Gamma s\Gamma t\Gamma)^{n-1}x\Gamma s\Gamma t \subseteq P$, $(s\Gamma x\Gamma t\Gamma)^{n-1}s\Gamma x\Gamma t \subseteq P$, $(s\Gamma t\Gamma x\Gamma)^{n-1}s\Gamma t\Gamma x \subseteq P \Rightarrow x\Gamma s\Gamma t \subseteq P_2$, $s\Gamma x\Gamma t \subseteq P_2$, $s\Gamma t\Gamma x \subseteq P_2$. Therefore P_2 is a TΓ-ideal of M. Let $x \in M$ and$(x\Gamma)^2x \subseteq P_2$. Now $(x\Gamma)^2x \subseteq P_2 \Rightarrow (x\Gamma)^2x\Gamma)^{n-1}(x\Gamma)^2x \subseteq P$ for some odd $n \Rightarrow (x\Gamma)^{3n-1}x \subseteq P \Rightarrow x \in P_2$. So P_2 is a CSTΓ-ideal of T. Let A be any CSTΓ-ideal of T containing P. Let $x \in P_2$. Then $(x\Gamma)^{n-1}x \subseteq P$ for some odd *n*. By corollary 3.6, $(x\Gamma)^{n-1}x \subseteq P \Rightarrow (< x>\Gamma)^{n-1}< x > \subseteq P \subseteq A$. Since P is CSTΓ-ideal, $(< x>\Gamma)^{n-1}< x > \subseteq A \Rightarrow x \in A$. Therefore P_2 is the minimal CSTΓ-ideal of M contains P.

Th 3.14 : If P is a PSTΓ-Ideal of a TΓ-semi group M then P_4 = {x : $(< x>\Gamma)^{n-1}< x > \subseteq P$ for some odd n} is the minimal semi prime TΓ-ideal of M contains P.
Proof **:** Clearly $P \subseteq P_4$ and hence P_4 is a nonempty subset of M. Let $x \in P_4$ and $s, t \in T$.
Since $x \in P_4$, $(< x>\Gamma m^{m-1}< x > \subseteq P$ for some odd *m*. Now $(< x\Gamma s\Gamma t>\Gamma)^{m-1}< x\Gamma s\Gamma t > \subseteq (< x>\Gamma)^{m-1} < x > \subseteq P$, $(< s\Gamma x\Gamma t >\Gamma)^{m-1}< s\Gamma x\Gamma t >$ and $(< s\Gamma t\Gamma x>\Gamma)^{m-1} < s\Gamma t\Gamma x > \subseteq (< x>\Gamma)^{m-1} < x > \subseteq P$ $\Rightarrow x\Gamma s\Gamma t, s\Gamma x\Gamma t, s\Gamma t\Gamma x \subseteq P_4$. Then P_4 is a TΓ-ideal of M containing P. Let $x \in T$ such that $(< x>\Gamma)^2< x > \subseteq P_4$. Then $((< x>\Gamma)^2< x >\Gamma)^{m-1}(< x>\Gamma)^2< x > \subseteq P \Rightarrow (< x >\Gamma)^{3m-1} < x > \subseteq P$ $\Rightarrow x \in P_4$. Therefore P_4 is a semi prime TΓ-ideal of M containing P. Let P is a semi prime TΓ-ideal of T containing P. Suppose $x \in P_4$. Then $(< x >\Gamma)^{m-1}< x > \subseteq P \subseteq A$. Since A is a semi prime TΓ-ideal of T, $(< x >\Gamma)^{n-1}< x > \subseteq A$ for some odd number *n* $\Rightarrow x \in A$. ∴ $P_4 \subseteq A$. Hence P_4 is the minimal semi prime TΓ-ideal of M containing P.

Th 3.15 : Each prime TΓ-ideal A minimal relative to containing a PSTΓ-Ideal P in a TΓ-semi group M is CPTΓ-ideal.

Cor 3.16 : Each prime TΓ-ideal A minimal relative to containing a CSTΓ-ideal P in a TΓ-semi group M is CPTΓ-ideal.

Th 3.17 : Let Q be a TΓ-ideal of a TΓ-semi group M. Then Q is CPTΓ-ideal iff Q is prime and PSTΓ-Ideal.

Proof **:** Suppose Q is a completely prime TΓ-ideal of T. Therefore Q is prime. Let *p, q, r* ∈ T and $p\Gamma q\Gamma r \subseteq Q$. $p\Gamma q\Gamma r \subseteq Q$, Q is completely prime ⇒ $p \in Q$ or $q \in Q$ or $r \in Q \Rightarrow p\Gamma s\Gamma q\Gamma t\Gamma r \subseteq Q$ for all *s, t* ∈ T. Hence Q is a PSTΓ-Ideal.
Conversely, let Q is prime and PSTΓ-Ideal. Let *p, q, r* ∈ T and $p\Gamma q\Gamma r \subseteq Q$. $p\Gamma q\Gamma r \subseteq Q$, Q is a PSTΓ-Ideal ⇒ $< p >\Gamma< q >\Gamma< r > \subseteq Q \Rightarrow < p > \subseteq Q$ or $< q > \subseteq Q$ or $< r > \subseteq Q \Rightarrow p \in A$ or $q \in Q$ or $r \in Q$. Therefore Q is CPTΓ-ideal.

Cor 3.18: Let Q be a TΓ-ideal of a TΓ-semi group M. Then Q is CPTΓ-ideal iff Q is prime and CSTΓ-ideal.

Cor 3.19 : Let Q be a TΓ-ideal of a TΓ-semi group M. Then Q is CSTΓ-ideal **iff Q is semi prime and PSTΓ-Ideal.**

Th 3.20 : Let Q be a PSTΓ-ideal of a TΓ-semi group M and P_r be the CPTΓ-ideal of M, P_q be the minimal CPTΓ-ideal of M and P_t be the minimal CSTΓ-ideal of M. Then the following are equivalent.

1) **$Q_1 = \bigcap_{r=1}^{n} P_r$ containing Q.**
2) **$Q_1^1 = \bigcap_{q=1}^{n} P_q$ containing Q.**
3) **$Q_1^{11} = \bigcap_{t=1}^{n} P_t$ relative to containing Q.**
4) **Q_2 = {$x \in T : (x\Gamma)^{m-1}x \subseteq Q$ for some odd m}**
5) **Q_3 = The intersection of all prime TΓ-ideals of T containing Q.**
6) **Q_3^1 = The intersection of all minimal prime TΓ-ideals of T containing Q.**
7) **Q_3^{11} = The minimal semi prime TΓ-ideal of T relative to containing Q.**
8) **Q_4 = {$x \in T : (< x >\Gamma)^{m-1}< x > \subseteq Q$ for some odd m}.**

Def 3.21 : A TΓ-semi group T is said to be a **PSTΓ-** ***semi group*** if every TΓ-ideal in T is a PSTΓ-Ideal.

Th 3.22 : Every commutative TΓ-semi group is a PSTΓ- ***semi group*****.**

Proof **:** Suppose M is commutative TΓ-semi group. Then $p\Gamma q\Gamma r = q\Gamma r\Gamma p = r\Gamma p\Gamma q = q\Gamma p\Gamma r = r\Gamma q\Gamma p = p\Gamma r\Gamma q$ for all *p, q, r* ∈ T. Let Q be a TΓ-ideal of T. Suppose *p, q, r* ∈ T, $p\Gamma q\Gamma r \subseteq Q$ and *s, t* ∈ T. Then $p\Gamma s\Gamma q\Gamma t\Gamma r = p\Gamma q\Gamma s\Gamma t\Gamma r = p\Gamma q\Gamma s\Gamma r\Gamma t = p\Gamma q\Gamma r\Gamma s\Gamma t \subseteq Q$. Hence Q is a PSTΓ-ideal and hence M is a PSTΓ-semi group.

Th 3.23 : Every pseudo commutative TΓ-semi group is a PSTΓ- ***semi group*****.**

Proof **:** Let T be a pseudo commutative TΓ-semi group and Q be any TΓ-ideal of T. Suppose *p, q, r* ∈ T, $p\Gamma q\Gamma r \subseteq Q$. If *s, t* ∈ T. Then $p\Gamma s\Gamma q\Gamma t\Gamma r = p\Gamma q\Gamma p\Gamma t\Gamma r = s\Gamma q\Gamma r\Gamma p\Gamma t = s\Gamma(p\Gamma q\Gamma r)\Gamma t \subseteq Q$. Therefore, $p\Gamma s\Gamma q\Gamma t\Gamma r \subseteq Q$ for all *s, t* ∈ T. Hence Q is a PSTΓ-*ideal*. Hence, T is a PSTΓ- *semi group*.

Th 3.24 : If M is a TΓ-semi group in which every element is a mid-unit, then M is a PSTΓ- ***semi group*****.**
Proof **:** Let M be a TΓ-semi group in which every element is a mid-unit and Q be any TΓ-ideal of M. Let p, *q, r* ∈ T and $p\Gamma q\Gamma r \subseteq Q$. If *s* ∈ T, then *s* is a mid-unit and hence, $p\Gamma s\Gamma q\Gamma s\Gamma r = p\Gamma q\Gamma r \subseteq Q$. Hence Q is a PSTΓ-*ideal*. Hence M is a PSTΓ- *semi group*.

4. SPSTΓ-ideals:

We now introduce the notion of SPSTΓ-ideals of a TΓ-semi group

Def 4.1 : A TΓ-ideal Q in a TΓ-semi group M is said to be ***SPSTΓ-ideal*** if for any odd *m, x* ∈ T, $(x\Gamma)^{m-1}x \subseteq Q \Rightarrow (< x >\Gamma)^{m-1} < x> \subseteq Q$.

Th 4.2 : Every PSTΓ-*ideal* of a TΓ-semi group is a SPSTΓ-ideals.

Note 4.3 : The converse of the above theorem, is not true.

Example 4.4 : Let T be a free TΓ-semi group over the alphabet {*p, q, r, s, t*}. Let Q = < *p*Γ*q*Γ*r* > ∪ < *q*Γ*r*Γ*p* > ∪ < *r*Γ*p*Γ*q* >. Since *p*Γ*q*Γ*r* ⊆ Q and *p*Γ*s*Γ*q*Γ*t*Γ*r* ⊈ Q, Q is not PSTΓ-*ideal*. Suppose $(x\Gamma)^{n-1}x \subseteq$ Q for some odd *n*. Now the word *x* contains $p\alpha q\beta r$ or $q\beta r\alpha p$ or $r\gamma p\alpha q$ for some $\alpha, \beta, \gamma \in \Gamma$ and hence $(< x >\Gamma)^{n-1}< x > \subseteq$ Q. Therefore $(x\Gamma)^{n-1}x \subseteq$ Q for some odd number $n \Rightarrow (< x >\Gamma)^{n-1}< x > \subseteq$ Q. Therefore Q is a SPSTΓ-ideals.

Th 4.5 : Each semi prime TΓ-ideal P minimal relative to containing a SPSTΓ-ideal A in a TΓ-semi group T is CSTΓ-ideal.

Cor 4.6 : Each prime TΓ-ideal P in a TΓ-semi group T minimal relative to containing a SPSTΓ-ideal A is CPTΓ-ideal.

Cor 4.7 : Each prime TΓ-ideal minimal relative to containing a PSTΓ-ideal A in a TΓ-semi group T is CPTΓ-ideal.

Th 4.8 : If Q is a TΓ-ideal in a TΓ-semi group T, then
1) Q is CSTΓ-ideal.
2) Q is semi-prime as well as PSTΓ-ideal.
3) Q is semi-prime as well as SPSTΓ-ideal are equivalent.

Proof **:** (1) ⇒(2) : Let Q is a CSTΓ-ideal of T ⟹ Q is a semi prime TΓ-ideal of T and by th 3.19, Q is a PSTΓ-ideal of T.

(2) ⇒(3) : Let Q is semi prime and PSTΓ-ideal. By th 4.2, Q is a SPSTΓ-ideal. Therefore, Q is semi prime and SPSTΓ-ideal.

(3) ⇒ (1) : Let Q is semi prime and SPSTΓ-ideal.
Let $p \in$ T, $(p\Gamma)^2p \subseteq$ Q. Since Q is SPSTΓ-ideal, $p \in$ T, $(p\Gamma)^2p \subseteq$ Q ⇒ $(< p >\Gamma)^2< p > \subseteq$ Q. Since Q is semi prime, by th 2.10, $(< p >\Gamma)^2< p > \subseteq$ Q ⇒ $< p > \subseteq$ Q. ∴ Q is completely semi prime.

Th 4.9 : If Q is a TΓ-ideal of a semi simple TΓ-semi group M, then the conditions
1) Q is CSTΓ-ideal.
2) Q is PSTΓ-ideal.
3) Q is SPSTΓ-ideal are equivalent.
Proof : (1) ⟹(2) : Let Q is CSTΓ-idea. By cor 3.19, Q is PSTΓ-ideal.
(2) ⟹ (3) :Let Q is PSTΓ-ideal. By theorem 4.2, Q is SPSTΓ-ideal.
(3) ⟹ (1) : Suppose that Q is SPSTΓ-ideal. Let $q \in$ T, $(q\Gamma)^2q \subseteq$ Q. Since Q is SPSTΓ-ideal, $(q\Gamma)^2q \subseteq$ Q ⟹ $(< q >\Gamma)^2< q > \subseteq$ Q. Since T is semi simple, *q* is a semi simple element. Therefore $q \in (< q >\Gamma)^2< q > \subseteq$ Q ⊆ Q. Thus Q is completely semi prime.

Th 4.10 : If Q is a TΓ-ideal of a TΓ-semi group M, then the conditions.
1) Q is CPTΓ-ideal.
2) Q is prime as well as PSTΓ-ideal.
3) Q is prime as well as SPSTΓ-ideal are equivalent.

Proof **:** (1) ⇒ (2) : Let Q is completely prime. By theorem 3.17, A is prime as well as PSTΓ-ideal.
(2) ⇒ (3) : Let Q is prime as well as PSTΓ-ideal. Since Q is PSTΓ-ideal by th 4.2, Q is SPSTΓ-ideal.
(3) ⇒ (1) : Let Q is prime as well as SPSTΓ-ideal. Since Q is prime then we have, Q is semi prime. Since Q is semi prime and SPSTΓ-ideal, by theorem 3.8, A is CPTΓ-idea. Since Q is prime and CSTΓ-idea then we have, Q is CPTΓ-idea.

The following theorem is an analogue of KRULL's Theorem.

Th 4.11 : Let Q be a SPSTΓ-ideal of a TΓ-semi group M and Let Q be a PSTΓ-ideal of a TΓ-semi group M and P_r be the CPTΓ-ideal of M, P_q be the minimal CPTΓ-ideal of M and P_t be the minimal CSTΓ-ideal of M. Then the following are equivalent.

1) $Q_1 = \bigcap_{r=1}^{n} P_r$ **containing Q.**

2) $Q_1^1 = \bigcap_{q=1}^{n} P_q$ **containing Q.**

3) $Q_1^{11} = \bigcap_{t=1}^{n} P_t$ **relative to containing Q.**

4) $Q_2 = \{x \in T : (x\Gamma)^{m-1}x \subseteq Q$ **for some odd m}**

5) Q_3**= The intersection of all prime TΓ-ideals of T containing Q.**

6) Q_3^1 **= The intersection of all minimal prime TΓ-ideals of T containing Q.**

7) Q_3^{11} **= The minimal semi prime TΓ-ideal of T relative to containing Q.**

8) $Q_4 = \{x \in T : (<x>\Gamma)^{m-1}< x > \subseteq A$ **for some odd *n*}**

We now present some of the consequences of the above theorem.

Th 4.12 : If P is a maximal TΓ-ideal of a TΓ-semi group M with $P_4 \neq M$, then
1) P is CPTΓ-ideal.
2) P is CSTΓ-ideal.
3) P is PSTΓ-ideal.
4) P is SPSTΓ-ideal are equivalent.
Proof **:** (1) ⇒ (2) : Let P is CPTΓ-ideal. Then we have, P is CSTΓ-ideal.
(2) ⇒ (3) : Let P is is CSTΓ-ideal. By th 3.9, P is PSTΓ-ideal.
(3) ⇒ (4) : Let P is PSTΓ-ideal. By th 4.2, P is SPSTΓ-ideal.
(4) ⇒ (1) : Let P is SPSTΓ-ideal. By the th 3.11, $P \subseteq P_4 \subseteq M$. Since P is maximal TΓ-ideal and $P_4 \neq T$, it implies that $P = P_4$. Let $x \in$ M, $(x\Gamma)^2x \subseteq$ P. Since P is SPSTΓ-ideal, $(< x >\Gamma)^2< x > \subseteq$ P. Then $x \in P_4 =$ P. ∴ P is CSTΓ-ideal. Let *x, y* ∈ M, *x*Γ*y* ⊆ P. Since P is CSTΓ-ideal, by cor 2.8, *x*Γ*y*Γ*z* ⊆ P ⇒ < *x* > Γ< *y* >Γ < *z* > ⊆ P. If possible *x* ∉ P, *y* ∉ P, *z* ∉ P. Then P ∪ < *x* >, P ∪ < *y* >, P ∪ < *z* > are TΓ-ideals of T and P ∪ < *x* > = P ∪ < *y* > = P ∪ < z > = M, Since P is maximal, *y, z* ∈ P ∪ < *x* >, *x, z* ∈ P ∪ < *y* > and *x, y* ∈ P ∪ < *z* > ⇒ *y, z* ∈ < *x* >, *x, z* ∈ < *y* >, *x, y* ∈ < *z* > ⇒ < *x* > = < *y* > = < *z* >. Now < *x* > Γ< *y* >Γ < *z* > ⊆ P ⇒ < *x* >Γ < *y* >Γ< *z* > = $(< x >\Gamma)^2< x > \subseteq$ P ⇒ $(x\Gamma)^2x \subseteq$ P ⇒ *x* ∈ P. It is a contradiction. ∴ either *x* ∈ P or *y* ∈ P or *z* ∈ P. ∴ P is CPTΓ-ideal.

We now introduce the notion of a SPSTΓ-semi group.

Defi 4.13 : A TΓ-semi group M is said to be a ***SPSTΓ-semi group*** if every TΓ-ideal of T is SPSTΓ-semi group.

Th 4.14 : A TΓ-semi group M is *SPSTΓ-semi group* iff every principal TΓ-ideal is *SPSTΓ-ideal*.

Th 4.15 : In a *SPSTΓ-semi group* M, an element *a* is semi simple iff *a* is lateral regular.

Th 4.16 : If M is a *SPSTΓ-semi group*, then
1) S = {*p* ∈ T : √<*p*> ≠ M} is empty or a CPTΓ-ideal.
2) M\S is empty or an Archimedean TΓ-sub-semi group of M are true.

Proof **:** (1) suppose S = ∅, then nothing to prove. If S ≠ ∅, then clearly S is a TΓ-ideal of M. Let *p, q, r* ∈ M and *p*Γ*q*Γ*r* ⊆ S. If possible *p* ∉ S, *q* ∉ S, *r* ∈ S, then √<*p*> = M, √<*q*> = M and √ < *r* > = M. ∵ *p*Γ*q*Γ*r* ⊆ S, then √<*p*Γ*q*Γ*r*> ≠ M. Now M = √<*p*> ∩ √<*q*> ∩ √<*r*> = √<*p*Γ*q*Γ*r*> ≠ M. It is a contradiction. Hence *p* ∈ S or *q* ∈ S or *r* ∈ S. ∴ S is a CPTΓ-ideal.
(2) ∵ S is a CPTΓ-ideal, M\S is either empty or a TΓ-sub-semi group of M. Let *p, q, r* ∈ T\S. Then √< *p* > = √< *q* > = √< *r* > = M. Now *q, r* ∈√<*p*>, *r, p* ∈ √< *q* >, *r, p* ∈ √ < *r* > therefore we have, $(q\Gamma)^{n-1}q \subseteq < p >$ for some odd *n*. So $(q\Gamma)^{n+1}q \subseteq M\Gamma p\Gamma M$ ⇒

$(q\Gamma)^{n+1}q = s\Gamma p\Gamma t$ for some $s, t \in$ M. If either s or $t \in$ S, then $(q\Gamma)^{n+1}q \subseteq$ S and hence $q \in$ S. It is a contradiction. Hence $s, t \in$ M\S. Now $(q\Gamma)^{n+1}q = s\Gamma p\Gamma t \subseteq$ (M\S)ΓpΓ(M\S). Hence M\S is an Archimedean TΓ-sub semi group of M.

Th 4.17 :If M is a *SPSTΓ-semi group*, then
1) M is a strongly Archimedean TΓ-semi group.
2) M is an Archimedean TΓ-semi group.
3) M has no proper CPTΓ-ideals.
4) M has no proper CSTΓ-ideals.
5) M has no proper prime TΓ-ideals.
6) M has no proper semi prime TΓ-ideals are equivalent.

Th 4.18 : If P is a nontrivial maximal TΓ-ideal of a *SPSTΓ-semi group* M then P is prime.

Proof : Let P is not prime. Then $\exists\ p, q, r \in$ M\P $\ni$ $< p >\Gamma< q >\Gamma< r > \subseteq$ P. Now any $u \in$ M\P, we have M = P ∪ < b > = P ∪ < c > = P ∪ < u >. Since $q, r, u \in$ T\P, we have $q, r \in$ < u > and $u \in$ < q >, $u \in$ < r >. So < q > = < r > = < u>. Therefore $(<q>\Gamma)^2< q > \subseteq$ P, $(<r>\Gamma)^2< r > \subseteq$ P. If $p \neq q$, then $p = s\alpha q\beta t$ for some $s, t \in$ M^1 and $\alpha, \beta \in \Gamma$. So $p \in$ < s >Γ< q >Γ< t >. If either $s \in$ P or $t \in$ P then $p \in$ P. It is a contradiction. If $s \notin$ P and $t \notin$ P, then < s >Γ< q >Γ< t > ⊆ $(< q >\Gamma)^2< q > \subseteq$ P. ∴ $p \in$ < s >Γ< q >Γ < t > ⊆ P. ∴ $a \in$ P. It is a contradiction. Hence $p = q$ and hence P is trivial, which is not true. Therefore P is prime.

Th 4.19 : If M is a *SPSTΓ-semi group* and contains a nontrivial maximal TΓ-ideal then M contains semi simple elements.

Th 4.20 : Let M be a *SPS*-Archimedean TΓ-semi group. Then a TΓ-ideal P is maximal iff it is trivial, as well as M has no maximal TΓ-ideals if M = $(M\Gamma)^2$M.

Th 4.21 : Let M be a *SPSTΓ-semi group* containing maximal TΓ-ideals. If either M has no semi simple elements or M is an Archimedean TΓ-semi group, then M ≠ $(M\Gamma)^2$M as well as $(M\Gamma)^2M = P^*$ where P^* denotes the intersection of all maximal TΓ-ideals.

4. Conclusion

D. M. Rao studied about PSΓ- ideals in Γ-semigroups. Further D. M. Rao and A .A. extended the same results to T-semi groups. Here mainly we study PSTΓ-ideals and extended the results to TΓ-semi groups.

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