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Research paper

Analysis of Means (ANOM) based on the Size Biased Lomax Distribution

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Abstract

A life time random variable which assumes a size biased Lomax model is considered as a measurable quality characteristic. In this paper, Decision lines are estimated using Analysis of Means (ANOM) technique for size biased Lomax distribution. Results are discussed through examples based on real data. Also, the results are compared with that of Shewart control charts.

Keywords: *ANOM; Control chart; In control; Q-Q plot*

1. Introduction

Many researchers of statistical quality control will generally use the well-known control charts proposed by Shewart. Shewart developed decision lines under the supposition that the quality characteristic assumes normal distribution. These constants are not advisable to adopt if the underlined quality characteristic is proposed to follow any non- symmetric distribution. A process which is an alternative to normal is to be considered.

It was noticed from the earlier research investigations that size biased Lomax distribution is one such type of skewed distribution that was not concentrated by researchers to construct the decision lines.It is also observed from the earlier studies that for reliability and life testing studies Size biased Lomax distribution (SBLD) is also a better model. Therefore, construction of control charts using SBLD is desirable, if a the lifetime random variable assuming the data of quality nature. In view of that an attempt is made to construct control charts for Analysis of Means (ANOM). Non-normal probability model to establish quality control procedures are originated by different researchers.

The research investigations in this direction are Edgeman (1989) [1] derived Inverse Gaussian control charts, Chan and Cui (2003) [2] designed Skewness correction $\bar{X}$ and R charts for skewed distributions, Rao R. S and Kantam. (2008) [3] chosen the double exponential probability model and find the variable decision lines for process mean. Various references of control charts for ANOM includes ([4] - [17]).

In this research paper, an attempt is made to discuss the notion of control charts for individual observations is made use to develop a graphical technique called analysis of means (ANOM) is presented in Section 2. Construction of Control Limits for Analysis of Means (ANOM) using SBLD is established in Section 3. A comparative study of ANOM with normal population is also made for some examples in Section 4. Section 5 deals with Summary and conclusions.

2. Analysis of Means (ANOM)

The common tool for quality control practitioner is the Shewart control chart. The presence of assignable cause indicates a possibility of an improvement in the process if the corrective is known. Suppose the corrective is not known, it is an indication of the non-homogeneity for which a control chart is to be established for that particular statistic of the subgroup. For example, if the proposed statistic is sample mean, this leads to non-homogeneity of process mean representing departures from target mean. This type of investigation is generally studied with the help of means (ANOM) to split the subgroup means into various homogeneous sub categories and those are non-homogeneous among the groups, under an assumption that the probability model of the variate is normal.

We have already noticed that any statistical method if needs to be applied for a non-normal data separate evaluation is essential. Ott (1967) [18] identified that for comparing to observe weather the overall mean deviates significantly from a group of treatment means. In this process decision lines are constructed to compare the overall mean with the sample mean values. If all the sample means are within the decision lines it is regarded as the grand mean not significantly differ the sample means. The grand mean is said to be differ significantly from the sample mean if some values are outside the decision lines.

One can assess simultaneously the significance of samples as well as the statistical significance through ANOM chart; of course, conceptually it is also like a control chart which portrays decision lines.
The notion of control chart for averages for adopting ANOM procedure will be taken in the other direction, grouping of plotted means to fall outside the control limits or within the decision lines.

There is an indication of non-homogeneity of means, if all the means fall outside the decision lines, or else we may say there is an existence of homogeneity among the means. In this paper, we consider the data variate which is supposed to assume Size Biased Lomax Distribution and construct ANOM procedure suggested by Ott (1967).

Suppose if we take the confidence coefficient as (1- α), the probability that the subgroup averages spread between the decision lines should be (1-α). The probability statement becomes n[th] power of the probability that a subgroup average will fall between the decision lines if we assumes the independence of the subgroup. Also, it is an indication that, the confidence interval for mean to stay between two probabilities as: $P(x_i \le L) = \alpha/2$ and $P(x_n \ge U) = \alpha/2$, for the sampling distribution $\bar{\bar{x}}$. By considering SBLD as a underlined probability distribution, we implemented the same procedure to construct the control limits.

3. Control Limits for Analysis of Means (ANOM)

The pdf and cdf of the size biased Lomax distribution are respectively given in equations 3.1 and 3.2 below:

$$f(t) = \frac{\alpha(\alpha-1)}{\sigma}\frac{t}{\sigma}\left(1+\frac{t}{\sigma}\right)^{-(\alpha+1)}; t \ge 0, \alpha > 1, \sigma > 0 \quad (3.1)$$

$$F(t) = 1 - \left(1+\frac{\alpha t}{\sigma}\right)\left(1+\frac{t}{\sigma}\right)^{-\alpha}; t \ge 0, \alpha > 1, \sigma > 0 \quad (3.2)$$

where α and σ are shape and scale parameters respectively.

Consider a sample of size n from the SBLD which are the means of the k subgroups, say, $\bar{x}_1, \bar{x}_2, \ldots\ldots \bar{x}_k$.

Using this probability model construct the decision lines as that we prepare control charts for normal population. Take (1-α) as confidence coefficient along with the probability statements (3.3) and (3.4) we may construct the control chart constants.

$$P\{ LCL < \bar{x}_i \; \square \; i = 1 \text{ to } k < UCL\} \quad (3.3)$$

If subgroups becomes independent

$$P\{LCL < \bar{x}_i < UCL\} = (1-\alpha)^{1/k} \quad (3.4)$$

One can find the two constants L* and U* with equi-tailed probability for each subgroup average, we can see that

$$P\{\bar{x}_i < L^*\} = P\{\bar{x}_i > U^*\} = \frac{1-(1-\alpha)^k}{2}$$

Because of the symmetric nature of normal distribution, $U^* = -L^*$, whereas in case of skewed populations U^* and L^* to be calculated individually from the sampling distribution of means, $\bar{x}_i$. Therefore, for our SBLD these two limits found separately and it depends on the number of subgroups 'k' and the subgroup size 'n'. For a given values of 'n' and 'k', at different level of significances, applying the equations 3.3 and 3.4, we obtained the constants L^* and U^* and are presented in Tables 1, 2 and 3.

A quality chart for means saying 'In Control' decision indicates that all the subgroup means though vary among themselves are homogenous with respect to any type. This is exactly the null hypothesis in an ANOVA technique. Therefore, the values in Tables 1, 2 and 3 will be applied as an alternative to ANOVA process. We have chosen some examples for testing the similarity of averages involved in each of them. These examples are verified for the goodness of fit for SBLD using Q-Q plot technique.

While dealing with the numerical examples we are finding the upper decision line (UDL) and lower decision line (LDL) to accept or reject the samples, where $LDL = LCL \times \bar{\bar{x}}$ and $UDL = UCL \times \bar{\bar{x}}$

4. Comparative Study: ANOM of SBLD VS Normal Distribution

Example 1: A metal manufacturer observed differences in iron content of 5 suppliers that are given in the following table 4. From each of the supplier five ingots were selected randomly. The data of the table indicates the iron determinations on each ingot in percent by weight. Test the given 25 observations weather the five suppliers maintaining the same content of the material.

Table 4: Variations in iron content of raw material by 5 suppliers

Suppliers				
1	2	3	4	5
3.46	3.59	3.51	3.38	3.29
3.48	3.46	3.64	3.40	3.46
3.56	3.42	3.46	3.37	3.37
3.39	3.49	3.52	3.46	3.32
3.40	3.50	3.49	3.39	3.38

Example 2: Three brands of batteries are under study. It is suspected that the life (in weeks) of the three brands is different. Five batteries of each brand are tested with the following results given in Table 5. Test whether the lives of these brands of batteries are different at 5 %level of significance.

Table 5: life (in weeks) of the three brands of batteries

Weeks of life		
Brand 1	Brand 2	Brand 3
100	76	108
96	80	100
92	75	96
96	84	98
92	82	100

Example 3: The investigations on concentrations obtained by 4 catalysts that result the concentration of one in a 3 component liquid mixture are shown in table 6. Observe the concentration at 5% los, that the 4 Catalysts have the same effect on concentration.

Table 6: Concentrations of Four Catalysts

Catalyst			
1	2	3	4
58.2	56.3	50.1	52.9
57.2	54.5	54.2	49.9
58.4	57.0	55.4	50.0
55.8	55.3	54.9	51.7

Goodness of fit: The Q-Q plot (correlation coefficient) technique was adopted for finding the goodness of fit to these three examples given in Table 7 below. It is noticed that there is a significant linear relationship between sample and population quantiles. Therefore SBLD is proved as a better probability model.

Table 7: Correlation Coefficient Values

	Size biased Lomax distribution	Normal distribution
Example 1 (n = 25)	0.9024	0.206750
Example 2 (n = 15)	0.8098	0.414920
Example 3 (n = 16)	0.8015	0.444710

The control limits are constructed for these observations in each table as a single sample, for both Normal and SBLD populations and verified the homogeneity of means.

By using the above three examples we found the LDL, UDL values and the coverage probabilities for both normal distribution as well as SBLD are evaluated and are presented in the following Table 8 and Table 9. The conclusions are discussed in section 5.

Table 8: Coverage Probabilities with respect to Normal distribution

	(LDL , UDL)	No. of subgroups fall		
		Withinthe decision lines	Coverageprob ability	Outside the decision
Example 1 n = 5, k = 5,α =0.05	(3.379 , 3.517)	3	0.6	2
Example 2 n = 5, k = 3,α =0.05	(87.82 , 95.52)	2	0.7	1
Example 3 n = 4, k = 4,α =0.05	(26.14 , 82.84)	2	0.5	2

Table 9: Coverage probabilities with respect to SBLD

	(LDL , UDL)	No. of subgroups fall		
		Withinthe decision lines	Coverage probability	Outside the decision lines
Example 1 n = 5, k = 5, α =0.05	(2.272,127.286)	5	1.0	0
Example 2 n = 5, k = 3, α =0.05	(67.15,2594.77)	3	1.0	0
Example 3 n = 4, k = 4, α =0.05	(29.41,1595.19)	4	1.0	0

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5. Summary and Conclusions

Analysis of Means control chart is clearly varies from the ordinary control lines because Shewart chart procedure is an usual test of hypothesis where as ANOM chart is used to discriminate the variation between specific causes of variation and common causes of variation. To evaluate the competence of the data, these two approaches are complementing each other. Using ANOM technique, the percentiles of sampling distribution of means in the samples from SBLD are calculated. It is also observed that Size biased Lomax distribution is a preferable when compared with the Normal by the Q-Q plot correlation coefficient of each data set with normal as well as SBLD distinctly which showed in Table 7 of correlation coefficients.

Hence we may conclude that, all the means to be homogeneous with the help of SBLD (Since no observation is found outside the decision lines) is a better decision than some means to be away from homogeneity while applying normal, Therefore Analysis of Means procedure is preferable.

Table 1: Size biased Lomax distribution constants for analysis of means (σ=1, 1-α =0.90)

n	k														
	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50
2	0.44093	0.33955	0.29393	0.26619	0.24692	0.23244	0.22101	0.21166	0.20381	0.19708	0.17358	0.15892	0.14066	0.1292	0.12101
	6.28785	9.39221	11.78111	13.79737	15.57487	17.18249	18.66127	20.03796	21.33118	22.55449	27.91332	32.4321	40.01372	46.406	52.03827
3	0.69877	0.56154	0.49857	0.45979	0.43256	0.41195	0.39556	0.38209	0.37071	0.36092	0.32634	0.30444	0.27673	0.259	0.24628
	7.97047	11.7811	14.71075	17.18249	19.36106	21.33118	23.1s4323	24.83007	26.41456	27.91332	34.4783	40.01372	49.30056	57.13	64.02881
4	0.93015	0.76394	0.68698	0.63933	0.60572	0.58019	0.55984	0.54306	0.52886	0.51662	0.47318	0.44548	0.41021	0.3875	0.37111
	9.39221	13.79737	17.18249	20.03796	22.55449	24.83008	26.92298	28.87121	30.70118	32.43211	40.01373	46.4061	57.13034	66.172	74.13783
5	1.14087	0.94973	0.8608	0.80555	0.7665	0.73677	0.71304	0.69344	0.67684	0.66251	0.61152	0.5789	0.53721	0.5103	0.49077
	10.64637	15.57487	19.36107	22.55449	25.36869	27.91332	30.25363	32.43211	34.47831	36.41375	44.89096	52.03827	64.02883	74.138	83.04427
6	1.33536	1.22052	1.02248	0.96051	0.91662	0.88319	0.85646	0.83436	0.81564	0.79995	0.74182	0.704861	0.65751	0.6269	0.60461
	11.78111	17.1825	21.33119	24.83008	27.91332	30.70118	33.26511	35.65173	37.89341	40.01373	49.30056	57.13034	70.2657	81.34	91.09644
7	1.51678	1.2833	1.17408	1.106	1.05775	1.02095	0.99151	0.96715	0.94651	0.92866	0.86501	0.80224	0.77168	0.7377	0.71295
	12.82518	18.66127	23.14323	26.92298	30.25363	33.26511	36.03468	38.61269	41.03412	43.32444	53.35575	61.81312	76.00122	87.962	98.50126
8	1.6874	1.43532	1.3172	1.24349	1.19121	1.15131	1.11938	1.09295	1.070544	1.05117	0.98197	0.93748	0.89835	0.8433	0.81629
	13.79737	20.03797	24.83008	28.87121	32.43211	35.65173	38.61269	41.36883	43.95755	46.4061	57.13034	66.17184	81.33977	94.127	105.3936
9	1.84893	1.57948	1.45306	1.37411	1.31808	1.27531	1.2414	1.2127	1.18864	1.16783	1.0935	1.04568	0.9842	0.9443	0.91518
	14.71076	21.33119	26.41456	30.70118	34.47831	37.89341	41.03412	43.95755	46.7034	49.30056	60.67559	70.2657	86.35391	99.913	111.867
10	2.0027	1.71688	1.58277	1.49879	1.43924	1.39375	1.35732	1.32716	1.30156	1.27941	1.20025	1.14929	1.08374	1.0411	1.010071
	15.57487	22.55449	27.91332	32.43211	36.41375	40.01373	43.32444	46.4061	49.30056	52.03827	64.02883	74.13783	91.09644	105.39	117.9897

Table 2: Size biased Lomax distribution constants for analysis of means (σ=1, 1-α =0.95)

n	k														
	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50
2	0.33628	0.26379	0.23041	0.20985	0.19542	0.18452	0.17587	0.16876	0.16278	0.15763	0.13954	0.12817	0.11389	0.10486	0.09841
	9.53485	13.9995	17.43033	20.3243	22.87466	25.18086	27.30191	29.27634	31.13091	32.88511	40.56863	47.0469	57.91523	67.07817	75.15121
3	0.55706	0.45641	0.40905	0.37947	0.3585	0.24251	0.32974	0.31918	0.31023	0.30251	0.27502	0.25745	0.23506	0.22065	0.21022
	11.95607	17.43033	21.63489	25.1809	28.30557	31.13091	33.72931	36.14802	38.41982	40.56863	49.98024	57.91523	71.22705	82.44985	92.33756
4	0.75847	0.63512	0.57659	0.53979	0.51359	0.49354	0.47746	0.46413	0.45282	0.44303	0.40803	0.38551	0.35659	0.33784	0.3242
	13.99957	20.32426	25.18086	29.2763	32.88511	36.14802	39.14877	41.94195	44.56546	47.0469	57.91523	67.07817	82.44985	95.40913	106.8266
5	0.94343	0.80072	0.73258	0.68963	0.65896	0.63544	0.61656	0.60088	0.58756	0.57601	0.53463	0.5079	0.47345	0.45104	0.43469
	15.80104	22.87466	28.30557	32.8851	36.92028	40.56863	43.92384	47.0469	49.98024	52.75474	64.90638	75.15121	92.33756	106.8266	119.5921
6	1.115	0.95474	0.90948	0.83007	0.79546	0.76889	0.74752	0.72977	0.71467	0.70158	0.65458	0.62415	0.58484	0.55922	0.54046
	17.43033	25.18086	31.13091	36.148	40.56863	44.56546	48.24109	51.66237	54.87582	57.91523	71.22705	82.44985	101.2769	117.1491	131.1329
7	1.27558	1.10003	1.01575	0.96242	0.92424	0.89491	0.8713	0.85168	0.83498	0.82049	0.76842	0.73465	0.69097	0.66242	0.64153
	18.92903	27.30191	33.72932	39.1488	43.92384	48.24109	52.21135	55.9069	59.37789	62.66092	77.03962	89.16174	109.4974	126.6414	141.7457
8	1.42697	1.23702	1.14567	1.08781	1.04636	1.01448	0.98882	0.96748	0.94931	0.93354	0.87681	0.83999	0.79228	0.76108	0.73823
	20.32426	29.27634	36.14802	41.942	47.0469	51.66237	55.9069	59.8578	63.56844	67.07817	82.44985	95.40913	117.1491	135.4768	151.6243
9	1.57055	1.36719	1.26926	1.20718	1.16267	1.12844	1.10086	1.07793	1.05839	1.04143	0.98038	0.94071	0.88929	0.85562	0.83095
	21.63489	31.13091	38.41982	44.5655	49.98024	54.87582	59.37789	63.56844	67.50433	71.22705	87.53137	101.2769	124.3357	143.7755	160.902
10	1.70739	1.49143	1.49143	1.32128	1.27391	1.23746	1.20809	1.18366	1.16283	1.14476	1.07966	1.03733	0.98242	0.94644	0.92007
	22.87466	32.88511	40.56863	47.0469	52.75474	57.91522	62.66092	67.07817	71.22705	75.15121	92.33756	106.8266	131.1329	151.6243	169.6779

Table 3: Size biased Lomax distribution constants for analysis of means (σ=1, 1-α =0.99)

n	k														
	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50
2	0.19416	0.15666	0.13869	0.12739	0.11936	0.11322	0.10831	0.10425	0.10081	0.09784	0.08729	0.08057	0.07204	0.06659	0.06267
	23.12378	33.23759	41.00039	47.5455	53.31221	58.52594	63.32055	67.78338	71.97505	75.93961	93.30329	107.9418	132.49853	153.20152	171.44109
3	0.35665	0.31665	0.27372	0.25626	0.24369	0.23399	0.22617	0.21966	0.21411	0.20929	0.19194	0.18069	0.16616	0.15669	0.14979
	28.61077	41.00039	50.50909	58.52594	65.58919	71.97505	77.84753	83.31363	88.44753	93.30329	114.56933	132.49853	162.57511	187.93066	210.26956
4	0.51127	0.44116	0.40636	0.38397	0.36776	0.3552	0.34504	0.33655	0.32929	0.32299	0.30015	0.28523	0.26579	0.25303	0.2436
	33.23759	47.54551	58.52594	67.78338	75.93961	83.31363	90.09476	96.40649	102.33476	107.9418	132.49853	153.20152	187.93066	217.20907	243.0039
5	0.65625	0.5738	0.53266	0.50607	0.48678	0.4718	0.45966	0.44947	0.4408	0.43323	0.40573	0.38769	0.36411	0.34856	0.33712
	37.31439	53.31221	65.58919	75.93961	85.05876	93.30329	100.88408	107.9418	114.56983	120.83869	148.29473	171.44109	210.26956	243.0039	271.84504
6	0.79239	0.69908	0.65233	0.62207	0.60006	0.58296	0.56907	0.55744	0.54748	0.5388	0.50721	0.48644	0.45922	0.44123	0.42796
	41.00039	58.52594	71.97505	83.31363	93.30329	102.33476	110.64016	118.37067	125.63135	132.49853	162.57511	187.93066	230.46561	266.32428	297.91623
7	0.92086	0.81772	0.76593	0.73234	0.70789	0.68887	0.67341	0.66046	0.64935	0.63967	0.60442	0.58119	0.55071	0.53051	0.51561
	44.39022	63.32055	77.84753	90.09476	100.88408	110.64016	119.61099	127.96106	135.80366	143.22108	175.70735	203.09473	249.03766	287.76958	321.89322
8	1.04268	0.93052	0.87409	0.83746	0.81077	0.78999	0.77309	0.75893	0.74679	0.73619	0.69758	0.67211	0.63863	0.61643	0.60003
	47.54551	67.78338	83.31363	96.40649	107.9418	118.37067	127.96106	136.88759	145.27165	153.20152	187.93066	217.20907	266.32428	307.73038	344.21011
9	1.15873	1.03818	0.97745	0.93799	0.90922	0.88681	0.8685	0.85329	0.84019	0.82874	0.78702	0.75947	0.72322	0.69917	0.68138
	50.50909	71.97505	88.44753	102.3347	114.56982	125.63135	135.80366	145.27165	154.16384	162.5751	199.41105	230.46561	282.56001	326.47804	365.17071
10	1.26971	1.14129	1.07654	1.03443	1.00371	0.97978	0.96029	0.94396	0.92995	0.91771	0.87307	0.84358	0.80475	0.77896	0.75988
	53.31221	75.93961	93.30329	107.9418	120.83869	132.49853	143.22108	153.20152	162.57511	171.44109	210.26956	243.0039	297.91623	344.21011	384.99513

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