International Journal of Engineering & Technology, 7 (3.27) (2018) 481-487

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Portfolio Selection and Post Optimality Test Using Goal Programming

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Abstract

In a practical portfolio planning process the investment decision to be taken by an investor is not simple and is influenced by several other constraints like stock price, co-moment with market, return with respect to risk, past performance and so many. In this purview, a hybrid approach is employed for portfolio selection which combines multiple methodologies like investor topology, cluster analysis, analytical hierarchy process (AHP) for ranking the assets and biogeographic-based optimization (BBO). Furthermore, with the help of goal programming (GP), performing post optimality test for betterment the result which is obtained by BBO. In the goal programming, objective is to be minimizing the weighted deviations of desire goals. Weighted deviation is known as achievement, which has two branches namely over achievement and under achievement.

Keywords. Analytical hierarchy process, Cluster analysis, Biogeographical-based optimization, Post optimality analysis, Goal programing.
MSC. 65K10, 62P05, 94D05.

1. Introduction

Selection of stock is not an easy task. The investment of stock does not guarantee since the decision requires to be made today with missing information about future price. To resolving the problem of portfolio selection, numerous models have been introduced.

The problem of portfolio selection was initially presented by Professor Harry Markowitz [9]. He proposed a Markowitz model for portfolio selection that is because investing in more than one stock is less risky than investing in a single stock. Konno and Yamazaki [10] introduced an improved version of Markowitz model both computationally and theoretically, and the risk is calculated as mean absolute deviation (MAD). Speranza [13] presented a linear programming model. In this model, the risk has been measured by semi absolute deviation method.

Hasuike and Katagiri [18] considered a portfolio selection problem using investor's subjectivity and then applied sensitivity analysis for changing the subjectivity. Portfolio selection is done using fuzzy programming problem. Stoyan and Kwon [16] addressed a complex stochastic goal mixed-integer programming model for stock and bond portfolio. Masmoudi and Abdelaziz [14] presented a bi-objective stochastic programming, portfolio optimization model, which is solved by goal programming with the objectives return and risk. Ghahtarani and Najafi [3] presented robust optimization goal programing for portfolio selection problem. Siew and Hoe [12] applied a goal programming model using mean return and tracking error for optimizing portfolio. Tamiz and Azmi et. al. [15] proposed the extended factors of stocks and applied goal programming for portfolio selection. They applied three alternatives of goal programming namely weighted, lexicographic and minimax approach.

From the literature survey, we conclude that goal programing is the most widely used optimization technique and applied for portfolio optimization. Sensitivity analysis or post optimality test has not been considered much for portfolio optimization while it is important for real situation.

In this approach, a portfolio selection with multiple methodology and post optimality test is employed. Firstly, X-means algorithm is applied for clustering the stocks into three different clusters such as high return stock, less risky stock and liquid stock as investors are divided into three main categories according to investor behavioral survey [4]. In X-means, there is no need to specify the number of clusters. Then by applying AHP, socks are valuated under some new features such as relative strength index (RSI), coefficient of variation (CV) and some basic features return, risk, liquidity, alpha and beta. The optimization is done using biogeographic-based optimization with eight objective functions return, risk, liquidity, relative strength index (RSI), coefficient of variation (CV), alpha, beta and AHP weighted score. After optimization post, optimality test performed using goal programming. The daily closing price, number of shares and turnover rate for all the selected 15 stocks are taken from BSE, Bombay stock exchange, Mumbai, India, (from February-15 to January-16).

This paper is organized as follows: Segment 2 contains a description of research methodology, BBO algorithm and its working process with reference to each of the eight objectives, namely return, risk, liquidity, relative strength index, coefficient of variation, alpha, beta and AHP weighted score. Segment 3, presents the numerical illustration and post optimality test, concluding remarks are given in segment 4.

2. Methodology

Following systematic strategy used for solving the multi-objective linear programming problem

A. Investor behavior pattern

Generally, investor is focused only on basic factors like return, risk and liquidity. However, there are some new and important factors to consider before selecting the stocks. These factors are alpha, beta, relative strength index (RSI) and coefficient of variation (CV).

- J.Welles Wilder introduced relative strength index in 1978, it calculates the present and past performance of a stock because of today's closing prices. RSI generally belongs to the range 30-70.
- Coefficient of variation helps to evaluate the value of instability relative to the return rate.
- Alpha-coefficient compares return with respect to risk.
- Beta-coefficient shows volatility or systematic risk of a stock or portfolio as compare to the market. Beta 1 shows that the stock's price changes with the market. Beta Greater than 1 shows higher volatility and less than 1 shows less volatility than the market.

B. Cluster

Different investor has different approach towards selecting stocks. Stocks divided into three groups' namely high return stocks, less risky stocks and liquid stocks according to the investors' interest. Cluster analysis is a technique to grouping similar data that is different from another group data. X-means [6] algorithm is used for clustering. It is an extended version of K-means, which attempts to automatically determine the number of clusters. It starts with just one centroids and then iteratively increases the centroid as required.

C. AHP

AHP technique developed by Thomas L. Saaty [17], which is a multi-criterion decision-making (MCDM) tool. It has a particular application in group decision making. There are three main steps of AHP for ranking the object:

1. Hierarchy structure design
2. Weight analysis
3. Consistency proof

Figure 1 shows the four level hierarchy structure of AHP. Firstly, form a pairwise comparison matrix for each criterion with respect to its parent criteria.

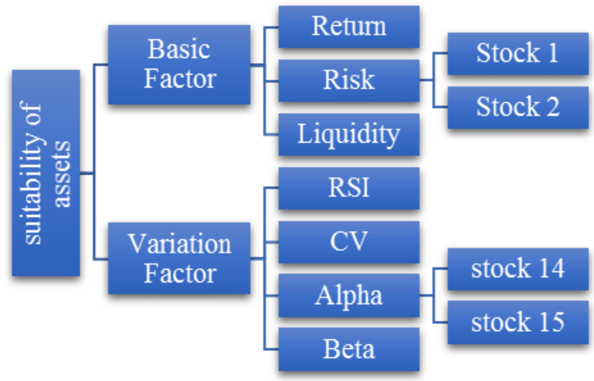

Fig.1: Hierarchy Structure

The judgmental matrix A is formed by as follows:

$$A = \begin{bmatrix} w1/w1 & w1/w2 & \cdots & w1/wn \\ w2/w1 & w2/w2 & \cdots & w2/wn \\ \vdots & & \ddots & \vdots \\ wn/w1 & wn/w2 & \cdots & wn/wn \end{bmatrix}$$

Where 'w' is known as weight of the objects. The consistency index (CI) for each n^{th} order matrix is calculated as:

CI = $(\lambda_{max}$- n)/ (n -1)

λ_{max} is the highest Eigen value of the matrix A.

The consistency ratio (CR) is calculated as:

CR = CI/RI

Where RI the random index be determined by on the order of the matrix.

The acceptable is CR is less than or equal to 0.10. Inconsistencies exist and pairwise comparisons need revision when CR > 0.01.

D. Portfolio selection model

For stock selection and optimization Biogeography-based optimization algorithm (BBO) is used, which has been explained in [8]. BBO is population based algorithm and introduced by Dan Simon in 2008 [7]. It is evolutionary algorithm based on the concept of migration and mutation.

For solving multi-objective programming problem following parameter are used:

n = 15 stocks (population size)

H= 50 (habitat)

H_j = [SIV_{j1}, SIV_{j2}, …., SIV_{j15}], j=1, 2, …, 50 (here SIVs represents proportion of the stocks)

For HSI calculate below equation for each H

$$HSI_j = [\sum_{i=1}^{n} r_i x_i + \sum_{i=1}^{n} k_i x_i + \sum_{i=1}^{n} L_i x_i + \sum_{i=1}^{n} R_i x_i + \sum_{i=1}^{n} C_i x_i + \sum_{i=1}^{n} \alpha_i x_i + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} W_{AHP_i} x_i$$

], j=1, 2, …, 50

Set maximum immigration rate (I) = 1(assume habitat is empty) and

Maximum emigration rate (E) = 1 (assume habitat contains all 15 stock).

Immigration rate λ_s = I (1-s/15)

Emigration rate μ_s = E (s/ 15)

Where **s** is the number of stock and λ_s decides whether to improve each SIV in i^{th} solution and μ_s of other habitat decides which solution should migrate that particular selected SIV. After that process, apply mutation for improving SIVs because of habitat's probability.

Mutation rate having s species: $m(s) = m_{max}((1-P_s)/P_{max})$
Where m_{max} = maximum mutation rate which is user defined parameter.

P_{max}= maximum probability of species
P_s= probability of HSI exactly having s species

$$P_i = \begin{cases} -(\lambda_i+\mu_i)P_i+\mu_{i+1}P_{i+1}, & i=0 \\ -(\lambda_i+\mu_i)P_i+\lambda_{i-1}P_{i-1}+\mu_{i+1}P_{i+1}, & 1\le i\le 15 \\ -(\lambda_i+\mu_i)P_i+\lambda_{i-1}P_{i-1}, & i=15 \end{cases}$$

Now first iteration is complete, loop can be terminated after defined number of iteration.

Portfolio Selection Problem

The multi-objective portfolio selection problem with eight objective functions such as return, risk, relative strength index, coefficient of variation, earning yield, price to earnings growth ratio, AHP weight and some notations are introduced as follows:

r_i : return of the i^{th} stock,
L_i : liquidity of the i^{th} stock,
x_i : the proportion of the total fund invested in the i^{th} stock,
b_i : the binary variable indicating whether the i^{th} stock, contained in the portfolio or not,
i.e. $b_i = \begin{cases} 1, if\ ith\ stock\ contained\ in\ portfolio \\ 0, if\ not\ containing\ in\ portfolio \end{cases}$
k_i : risk of the i^{th} stock,
R_i : relative strength index of the i^{th} stock,
C_i : coefficient of variation of the i^{th} stock,
w_{AHPi} :the AHP weight of the i^{th} stock,
α_i : alpha-coefficient of the i^{th} stock,
β_i : beta-coefficient of the i^{th} stock,
u_i : the maximum fraction of the i^{th} stock,
l_i : the minimum fraction of the i^{th} stock,
n : total number of stocks in the cluster,
a : number of stocks in the selected portfolio

Objective functions are as follows:

1) Return
The return of the portfolio is written as:
$$f_1(x) = \sum_{i=1}^{n} r_i x_i$$
Where $r_i = \frac{1}{12}\sum_{i=1}^{12} r_{it}$.

2) Risk
The semi-absolute deviation of return of the portfolio below the expected return over the past period t,
t = 1, 2, …T, can be written as:
$$k_t(x) = \left| min\left\{0, \sum_{i=1}^{n}(r_{it}-r_i)\right\} x_i \right| = \frac{|\sum_{i=1}^{n}(r_{it}-r_i)x_i| + \sum_{i=1}^{n}(r_i - r_{it})x_i}{2}$$
Consequently, the expected semi-absolute deviation of return of the portfolio x=$(x_1, x_2, x_3, \dots, x_n)$ below the expected return becomes
$$f_2(x) = k(x) = \frac{1}{T}\sum_{t=1}^{T} k_t(x) = \sum_{t=1}^{T} \frac{|\sum_{i=1}^{n}(r_{it}-r_i)x_i| + \sum_{i=1}^{n}(r_i - r_{it})x_i}{2T},$$
Where k (x) represents portfolio risk.

3) Liquidity
Liquidity is measured as the possibility of transformation of an investment into cash without affecting the asset's price. It is a known fact that turnover rates of assets cannot be exactly forecasted. For this, we use the idea of possibility theory, which was introduced by Zadeh [11] and improved by Dubois and Prade [5]
Fuzzy number F is known as trapezoidal with tolerance interval [a, b, α, β], if its membership function defined as:

$$\mu_F = \begin{cases} 1-\frac{a-t}{\alpha} & if\ (a-\alpha)\le t\le a \\ 1 & if\ a\le t\le b \\ 1-\frac{t-b}{\beta} & if\ b\le t\le(b+\beta) \\ 0 & otherwise. \end{cases}$$

Let the turnover rate of the i^{th} stock is denoted by a trapezoidal fuzzy number $L_i = (L_{ai}, L_{bi}, \alpha_i, \beta_i)$. Then, the portfolio turnover rate is given as $\sum_{i=1}^{n} L_i x_i$. The turnover rate of the portfolio by the fuzzy extension principle [11] is given as
$f_3(x) = \sum_{i=1}^{n} L_i x_i = \sum_{i=1}^{n}(\frac{L_{ai}+L_{bi}}{2} + \frac{\beta_i - \alpha_i}{6})x_i$.

4) Relative strength index (RSI)
The RSI of the portfolio is written as:
$f_4(x) = \sum_{i=1}^{n} R_i x_i$,
Where $R_i = 100 - \frac{100}{1+avg_i}$ and $avg_i = \frac{avg\ gain}{avg\ loss}$.

5) Coefficient of variation (CV)
The CV of the portfolio is written as:
$f_5(x) = \sum_{i=1}^{n} C_i x_i$,
Where $C_i = \frac{SD_i}{return_i}$ of the i^{th} stock.

6) Alpha-coefficient (α)
The α of the portfolio is written as:
$f_6(x) = \sum_{i=1}^{n} \alpha_i x_i$,
Where α_i is the alpha-coefficient of the i^{th} stock.

7) Beta-coefficient (β)
The β of the portfolio is written as:
$f_7(x) = \sum_{i=1}^{n} \beta_i x_i$,
Where β_i is the beta-coefficient of the i^{th} stock.

8) AHP weight
The AHP weight of the portfolio is written as:
$$f_8(x) = \sum_{i=1}^{n} w_{AHPi} x_i$$
Where w_{AHPi} is weight of i^{th} stock.

Constraints:

Investment economical restriction on the stocks:

9) Sum of proportion of stocks should be 1
$$\sum_{i=1}^{n} x_i = 1$$

10) Number of stocks held in a portfolio:
$$\sum_{i=1}^{n} b_i = a$$

11) The maximum percentage of the investment which can be invested in a stock:
$x_i \le u_i b_i, \quad i = 1, 2, \dots, n,$

12) The minimum percentage of the investment which can be invested in a stock:
$x_i \ge l_i b_i, \quad i = 1, 2, \dots, n,$

The upper and lower bounds have been taken to avoid too many large investments and in the same manner too many small investments.

The decision problem:

$$\max f_1(x) = \sum_{i=1}^{n} r_i x_i \quad (1)$$

$$\min f_2(x) = \sum_{i=1}^{n} k_i x_i \quad (2)$$

$$\max f_3(x) = \sum_{i=1}^{n} L_i x_i \quad (3)$$

$$\max f_4(x) = \sum_{i=1}^{n} R_i x_i \quad (4)$$

$$\max f_5(x) = \sum_{i=1}^{n} C_i x_i \quad (5)$$

$$\max f_6(x) = \sum_{i=1}^{n} \alpha_i x_i \quad (6)$$

$$\max f_7(x) = \sum_{i=1}^{n} \beta_i x_i \quad (7)$$

$$\max f_8(x) = \sum_{i=1}^{n} w_{AHPi} x_i \quad (8)$$

subject to

$$\sum_{i=1}^{n} x_i = 1 \quad (9)$$

$$\sum_{i=1}^{n} b_i = a, \quad (10)$$

$$x_i \le u_i b_i, i = 1, 2, \ldots, n, \quad (11)$$

$$x_i \ge l_i b_i, i = 1, 2, \ldots, n, \quad (12)$$

$$x_i \ge 0, i = 1, 2, \ldots, n, \quad (13)$$

$$b_i \epsilon \{0, 1\}, i = 1, 2, \ldots, n. \quad (14)$$

3. Numerical Illustration

The results of an experimental study built on the data set of 147 assets registered in BSE, Mumbai, India, (from February-15 to January-16) are as follows:

A. Cluster analysis

For performing cluster analysis**,** X-means tool of the Rapid Miner version 5.2 software are used. The initial distribution of first centroid is performed by k-means clustering. The result of the X-means algorithm is shown in table 1.

Table1. Cluster result

Parameters	**Cluster 1 (46 stocks)**	**Cluster 2 (78 stocks)**	**Cluster 3 (23stocks)**
average return	0.0441	0.0154	0.0731
average risk	0.0547	0.0344	0.0744
turnover rate	0.0010	0.0005	0.0010
Category	Liquid	less risky	high return

As per investors' behavior about risk, return and liquidity stocks divided into 3 categories.

1) Cluster 1: Stocks having high liquidity as compare to other cluster grouped in this cluster. This cluster is for those investors who are interested in liquid stocks and medium risk.

2) Cluster 2: contains high return stock as compared to other clusters. This cluster is for those investors who focused only on maximum return.

3) Cluster 3: contain less risky stocks as compared to other clusters. This cluster is for those investors who are risk averse.

Symbolic representations of stocks from each cluster are shown in table 2.

Table 2: Stocks for each cluster

Symbol	**Cluster 1**	**Cluster 2**	**Cluster 3**
S1	Whbrady	Blue Star	Kinetic Eng.
S2	Nelco Ltd.	Great Estate	Tokyo Plast
S3	Nocil Ltd	Swaraj Engine	Force Motor
S4	Ceat Limited	Bajfinance	Kg Denim
S5	Nucleus S/w Exports Ltd.	Finolex Ind.	Zenith Fiber
S6	Sauras.Cem.	Bharat Pet.	Jenson Nicolson
S7	Fedder.Llyod	Lakshmi Mill	NIIT Ltd.
S8	Dcw Ltd.	Jsw steel	Tata Elxsi
S9	Eveready Ind. India Ltd.	Pel	Century Ext
S10	Himachal Fertilizer	Swan Eng	Jasch Indust
S11	Timex Group	Pfizer Ltd.	Medi-caps
S12	Camph.& All	Sri Adhikari Brothers Tel. Net. Ltd.	Pas.Acrylon
S13	Andhra Petro	Kajaria Cer.	Modi Rubber
S14	Sha Eng Pla	Asian Paints	Mafatlal Ind
S15	Majestic Aut	Lic Housing Finance	Panyam Cement

B. Numerical calculation of AHP weights

In this segment under the criteria and sub-criteria in AHP, stocks ranked according to the investor preference. The weights are given in table 3.

Table 3. Weight of criteria and sub-criteria

Criteria	**Weight**	**sub-criteria**	**Weight**
Basic factor	0.6500	Risk	0.2321
		Return	0.1857
		Liquidity	0.2321
		Relative Strength Index	0.0700
Valuation factor	0.3500	Coefficient of Variation	0.0700
		Alpha	0.1050
		Beta	0.1050

Table 4-6 represents the input data for all three clusters.

Table 4: Input data for cluster 1

Stocks	Return	Risk	RSI	CV	Alpha	Beta	Liquidity	AHP-weight
S1	0.0628	0.0402	56.8973	1.5707	0.0864	2.3939	0.0173	0.1884
S2	0.0296	0.0638	50.6893	5.2433	0.0515	2.2247	0.0021	0.0598
S3	0.0369	0.0471	52.5398	3.0668	0.0615	2.4884	0.0019	0.0639
S4	0.0314	0.0544	50.6261	4.8066	0.0382	0.6942	0.0016	0.0524
S5	0.0326	0.0534	51.3058	3.7950	0.0542	2.1909	0.0011	0.0538
S6	0.0512	0.0620	51.4297	3.8625	0.0955	4.4973	0.0010	0.0677
S7	0.0323	0.0661	50.8229	4.9407	0.0711	3.9288	0.0009	0.0582
S8	0.0417	0.0366	52.1835	2.2204	0.0641	2.2632	0.0009	0.0608
S9	0.0371	0.0534	54.1593	3.4756	0.0615	2.4742	0.0008	0.0544
S10	0.0301	0.0522	50.6999	3.8591	0.0482	1.8273	0.0008	0.0495
S11	0.0673	0.0483	55.1665	1.9584	0.0753	0.8048	0.0007	0.0581
S12	0.0653	0.0486	53.9227	1.7890	0.0876	2.2667	0.0007	0.0624
S13	0.0308	0.0592	49.3334	4.6172	0.0668	3.6529	0.0006	0.0553
S14	0.0628	0.0402	56.8973	1.5707	0.0864	2.3939	0.0006	0.0649
S15	0.0556	0.0669	49.7316	3.1074	0.0657	1.0232	0.0006	0.0504

Table 5: Input data for cluster 2

Stocks	Return	Risk	RSI	CV	Alpha	Beta	Liquidity	AHP-weight
S1	0.0110	0.0151	51.2555	3.7010	0.0165	0.5584	0.0000	0.0499
S2	0.0008	0.0161	50.6686	55.5904	0.0038	0.3029	0.0001	0.0595
S3	0.0111	0.0172	51.0755	3.8100	0.0199	0.8866	0.0001	0.0547
S4	0.0342	0.0174	44.0169	1.3198	0.0402	0.6067	0.0001	0.0774
S5	0.0048	0.0188	49.7030	10.3879	0.0137	0.9080	0.0001	0.0488
S6	0.0192	0.0190	52.7992	2.5065	0.0284	0.9364	0.0001	0.0660
S7	0.0017	0.0197	49.6072	31.8555	0.0089	0.7298	0.0001	0.0523
S8	0.0058	0.0203	50.0006	9.4639	0.0030	-0.2801	0.0004	0.0583
S9	0.0102	0.0213	51.6400	5.1282	0.0178	0.7695	0.0000	0.0461
S10	0.0186	0.0215	52.3783	2.9564	0.0257	0.7256	0.0013	0.1370
S11	0.0128	0.0218	50.8221	4.1963	0.0161	0.3333	0.0001	0.0465
S12	0.0349	0.0220	56.2961	1.7178	0.0455	1.0740	0.0005	0.1059
S13	0.0207	0.0229	53.7769	2.9862	0.0281	0.7542	0.0001	0.0633
S14	0.0073	0.0232	51.0264	7.8238	0.0215	1.4368	0.0001	0.0563
S15	0.0052	0.0235	50.4360	12.5956	0.0241	1.9144	0.0004	0.0779

Table 6: Input data for cluster 3

Stocks	Return	Risk	RSI	CV	Alpha	Beta	Liquidity	AHP-weight
S1	0.0924	0.0668	53.8029	1.7958	0.1109	1.8737	0.0006	0.0565
S2	0.0915	0.0721	53.6389	1.8909	0.1050	1.3704	0.0008	0.0556
S3	0.0907	0.0873	55.4821	2.4825	0.1368	4.6748	0.0065	0.1258
S4	0.0892	0.0650	53.3348	1.7777	0.1285	3.9862	0.0017	0.0740
S5	0.0883	0.0478	59.4048	1.2688	0.1133	2.5360	0.0007	0.0643
S6	0.0860	0.0669	51.9850	2.2301	0.1070	2.1276	0.0011	0.0620
S7	0.0807	0.0797	54.2527	2.4463	0.1315	5.1498	0.0014	0.0712
S8	0.0805	0.0647	56.4970	1.8872	0.0946	1.4356	0.0049	0.0996
S9	0.0779	0.1038	49.7346	3.1470	0.0907	1.3028	0.0003	0.0444
S10	0.1027	0.0758	54.4138	1.9894	0.1281	2.2159	0.0007	0.0590
S11	0.0740	0.0602	52.2760	2.4143	0.0845	1.0600	0.0010	0.0569
S12	0.0734	0.0689	50.5282	4.4138	0.0980	2.4928	0.0004	0.0563
S13	0.0704	0.0790	52.5589	2.9460	0.0972	2.7164	0.0000	0.0478
S14	0.0701	0.0537	55.4624	1.8392	0.0988	2.9051	0.0004	0.0567
S15	0.0698	0.0794	52.6300	3.1823	0.1308	6.1775	0.0011	0.0700

C. Assets Allocation

The numerical results for each cluster are shown in table 7.

Table7: Results for each cluster

Stock	cluster 1	cluster 2	cluster 3
S1	0.0000	0.0000	0.3994
S2	0.0000	0.0000	0.0635
S3	0.0000	0.0000	0.0725
S4	0.0000	0.0000	0.0622
S5	0.0000	0.0000	0.4024
S6	0.0000	0.0000	0.0000
S7	0.0000	0.0000	0.0000
S8	0.0000	0.0000	0.0000
S9	0.0000	0.0000	0.0000

S10	0.0000	0.0000	0.0000
S11	0.2153	0.0651	0.0000
S12	0.0354	0.5473	0.0000
S13	0.1188	0.1495	0.0000
S14	0.4398	0.2051	0.0000
S15	0.1907	0.0330	0.0000

4. Post-optimality test

Post optimality test is done using goal programing. Goal programing (GP) is an optimization technique, which was first applied by Charnes, Cooper and Ferguson in 1955 [1] while the actual name first appeared in 1961 by Charnes and Cooper [2]. It is also known as extension of linear programing to solve multiple. It has wide application [19] in finance and for solving portfolio selection problem.

Standard form of GP is as follows:

Minimize Z $=\sum_{j=1} P_i\,(d_i^- + d_i^+)$

Subject to

$$\sum_{j=1}^{n} a_{ji} x_j + d_1^- - d_1^+ = b_i$$

$x_j,\ d^+,\ d^- \geq 0$, for all i and j.

$d^+ * d^- = 0.$

Where

Z = sum of deviations of all desire goals.

P_i = priority weights of goals according to their rank

d^+= over achievement deviation (slack variable)

d^-= under achievement deviation (surplus variable).

In this portfolio selection problem, goals are return and risk, the reason behind for taking above-mentioned constraints is that these are basic factors as investor concern about only return and risk. So that the priority weight of goals return and risk having equal value ($P_1 = P_2 = 1$).

Goal 1: to produce maximum return.

Goal 2: to avoid maximum risk.

Since goal is to maximize return and minimize risk so that under achievement (d_2^-) is not allowed in the case of return and over achievement (d_2^+) is not allow in the case of risk.

GP model for all three clusters:

min= $d_1^- + d_2^+$;

Subject to

$\sum_{i=1}^{n} r_i x_i + d_1^- - d_1^+ = f_1(x)$; (return)

$\sum_{i=1}^{n} k_i\, x_i + d_2^- - d_2^+ = f_2(x)$; (risk)

$\sum_{i=1}^{n} x_i = 1$;

$\sum_{i=1}^{n} b_i = a$;

$x_i \leq u_i b_i, \quad i = 1, 2, \ldots, n,$

$x_i \geq l_i b_i, \quad i = 1, 2, \ldots, n,$

$b_i \epsilon \{0, 1\}, \quad i = 1, 2, \ldots, n.$

$d_1^- * d_1^+ = 0;$

$d_2^- * d_2^+ = 0;$

$x_i,\ b_i,\ d_1^-, d_1^+, d_2^-, d_2^+ \geq 0.$

For solving above linear programing, Lingo 12.0 is used. The results are shown in table 8.

Selected stocks	**Improved ratio**		
	Cluster 1	Cluster 2	Cluster 3
X1	0.3770	0.3707	0.4537
X2	0.0225	0.5484	0.0225
X3	0.0225	0.0225	0.0898
X4	0.5555	0.0225	0.0225
X5	0.0225	0.0359	0.4115

From the above result, it is observed that the new portfolio with improved ratio gives better solution. The result of the clusters is improved by 9.40%, 0.89% and 0.06% respectively as compared to those results, which has been obtained from BBO.

5. Conclusion

This paper develops a hybrid approach for portfolio selection and presented a post optimality test for improving obtained results. Methodology for portfolio selection is involved Behavior Survey, Cluster Analysis, AHP, BBO algorithm which takes less execution time as compare to other optimization technique. Cluster analysis done by X-means algorithm, which generates actual number of cluster according to the data. Because in X-means there is no need to define number of cluster, it decides itself. [20]
The purpose of this paper is to address goal programing for post optimality test that can be improve the result. After portfolio selection, an optimality test applied on the selected stocks to get new and improved proportion of stocks.[21] However, in every case, improvement in the proportion is not necessarily. If the selected portfolio is optimum, then there is no change in the solution otherwise better proportion of the stocks can be evaluated by the proposed optimality test.

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