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Research paper

ALzughair Transform of Differentiation and Integration

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Abstract

With partial derivatives non-line differential equations by many از natural phenomena and because of this, they are difficult or non-existent to solve the analysis. There are no equations for complete solving this type of equation. The aim of this paper is to introduce Alzughair transform of differentiation and integration.

Keywords: Differential Equations, Lee Group and Convergences, Lee Symmetry, Optimal Device

1. Introduction

Alzughair transform has a vital effect on solving ODE and PDE with variable parameters, this transform appeared for the first time at 2017 (1)

2. Definition

Which is an effective way to find the exact answers to the dynamical systems of the method of symmetry is described. With partial derivatives nonlinear differential equations. The answer can be used to reduce the symmetry of differential equations, and therefore the class Accurately found. This means that, in terms of the definition of symmetry, each symmetry answers the answer to it. The definite answer to other solutions is the differential equation with the ability to be used by using symmetry achieved.

Ordinary differential equations are divided into linear and nonlinear groups. The solutions of a linear ordinary differential equation can be multiplied by a fixed number of sum or a constant number. These categories of equations are fully and precisely known and analyzed and there are analytical packets for them. In contrast to the nonlinear ordinary differential equations, there is an incompatibility property for their solutions. Solving these equations is more complicated in general, and rarely can they find a closed answer based on mathematical primitive functions. Instead, for such equations, solutions can be found in series or in the form of integral. In addition, it is possible to estimate the answer to nonlinear differential equations by using graphical numerical methods, which can be implemented manually or in a computer. These estimating methods can provide useful information in the absence of analytical and closed answers.

2.1 Definition [1](2):

Let f is defined function at a period (a,b) then integral transform for f is given by

$$F(s) = \int_a^b K(s,x) f(x)\,dx$$

Where K is a fixed function of 2 parameters, named the kernel of the transform and a,b are real numbers or $\pm\infty$, such that the above integral is convergent.

2.2 Definition [2](1):

Alzughair transform $[z(f(x))]$ for the function $f(x)$ where $x \in [1,e]$ is defined by the following integral

$$z(f(x)) = \int_1^e \frac{(lnx)^s}{x} f(x)dx = F(s)$$

Such that this integral is convergent, s is positive constant. From the above definition we can write

$$z(u(x,t)) = \int_1^e \frac{(lnt)^s}{t} u(x,t)\,dt = v(x,s)$$

Such that $u(x,t)$ is a function of x and t

2.3 Property [1] (1): (Linear property)

This transformation is characterized by the linear property, that is

$$Z[Au_1(x,t) \pm Bu_2(x,t)] = A\,Z[u_1(x,t)] \pm BZ[u_2(x,t)]$$

,where A and B are constants , the functions $u_1(x,t)$ and $u_2(x,t)$ are defined when $t \in [1,e]$

3. ALzughair Transform of some fundamental Functions:

ID	Function, $f(x)$	$F(s)=\int_1^e \frac{(lnx)^s}{x} f(x)dx = Z[f(x)]$	Regional of Convergence
1	k, k is constant	$\frac{k}{s+1}$	$s>-1$
2	,n∈ $R (lnx)^n$	$\frac{1}{s+(n+1)}$	$s>-(n+1)$
3	$lnlnx$	$\frac{-1}{(s+1)^2}$	$s>-1$
4	$(lnlnx)^n$	$\frac{(-1)^n\, n!}{(s+1)^{n+1}}$, $n=1,2,3,\dots$	$s>-1$
5	$sin(alnlnx)$	$\frac{-a}{(s+1)^2+a^2}$	$s>-1$
6	$cos(alnlnx)$	$\frac{s+1}{(s+1)^2+a^2}$	$s>-1$
7	$sinh(alnlnx)$	$\frac{-a}{(s+1)^2-a^2}$	$\|p+1\|>a$
8	$cosh(alnlnx)$	$\frac{s+1}{(s+1)^2-a^2}$	$\|p+1\|>a$

3.1 Theorem [1](1):

If $Z[u(x,t)] = v(x,s)$ and a is constant , then $Z[(lnt)^a u(x,t)] = v(x,s+a)$

3.2 Definition [3](1):

Let $f(x)$ be a function where $t\in[1,e]$ and $Z[u(x,t)] = v(x,t)$, $f(x)$ is said to be an inverse for Alzughair transform and written as :

$Z^{-1}[F(p)] = f(x)$ where Z^{-1} returns the transform to the original function.

Note(1):
if

$Z^{-1}(v_1(x,s)) = u_1(x,t), Z^{-1}(v_2(x,s)) = u_2(x,t), \dots, Z^{-1}(v_n(x,s)) = u_n(x,t)$

, where $a_1, a_2, \dots, a_n$ are constants then,

$$Z^{-1}[a_1v_1(x,s) \pm a_2v_2(x,s) \pm \cdots \pm a_nv_n(x,s)]$$

$$= a_1Z^{-1}[v_1(x,s)] \pm a_2Z^{-1}[v_2(x,s)] \pm \cdots \pm a_nZ^{-1}[v_n(x,s)]$$

$$= a_1u_1(x,t) \pm a_2u_2(x,t) \pm \cdots \pm a_nu_n(x,t)$$

3.3 Theorem[2](1):

If $Z^{-1}[v(x,s)] = u(x,t)$ and a is constant , then $Z^{-1}[v(x,s+a)] = (lnt)^a u(x,t)$

4. ALzughair Transform of Differentiation

4.1 Theorem[3]:

let f be piecewise continuous function on $R\times$ [1,e] and $z[f(x,t)] = F(s); s$ is positive constant then:

$$\frac{d^k}{ds^k} z[f(x,t)] = z[(ln(lnt)^k f(x,t)]; \quad k=1,2,3,\dots..$$

Proof:

$$z[f(x,t)] = \int_1^e \frac{(lnt)^s}{t} f(x,t)dt$$

$$\rightarrow \frac{d}{ds}Z[f(x,t)] = \frac{d}{ds}\int_1^e \frac{(lnt)^s}{t} f(x,t)dt = \int_1^e \frac{\partial}{\partial s}\frac{(lnt)^s}{t} f(x,t)dt$$

Note

$$\frac{d}{ds}(lnt)^s;\ lnt\ is\ constant = (lnt)^s\, ln(lnt)$$

$$= \int_1^e \frac{(lnt)^s}{t} ln(lnt) f(x,t)dt$$

$$= \int_1^e ln(lnt)\frac{(lnt)^s}{t} f(x,t)dt$$

Hence

$$\frac{d}{ds}\, z\,[f(x,t)] = z[ln(lnt).f(x,t)]$$

Also,

$$\frac{d^2}{ds^2}\, z[f(x_1t)] = \frac{d}{ds}\left[\frac{d}{ds} z[f(x,t)]\right]$$

$$= \frac{d}{ds}[z\ (ln(lnt)f(x,t)]$$

$$= \frac{d}{ds}\int_1^e ln\,(lnt)\,\frac{(lnt)^s}{t}\ f(x,t)\,dt$$

$$= \int_1^e \frac{(lnt)^s}{t}\ ln(lnt)\ ln(lnt)f(x,t)\,dt$$

$$= \int_1^e \frac{(lnt)^s}{t}(ln\,(lnt))^2\ f\ (x,t)\,dt$$

So,

$$\frac{d^2}{ds^2}\ Z\ [f(x,t)] = Z\ [(ln(lnt)^2\ f(x,t)]$$

And so on ⋮

⋮

⋮

$$\frac{d^k}{ds^k}\ Z\,[f(x,t)] = \int_1^e \frac{\partial}{\partial s}\left(ln(lnt)\right)^{k-1}\frac{(lnt)^s}{t}\ f(x,t)\,dt$$

By applying the above relationship:

$$z[x(lnlnt)^2\sin(2lnlnt)] = \frac{d^2}{ds^2}\left[\frac{-2x}{(s+1)^2+4}\right] = \frac{d}{ds}\left[\frac{4x(s+1)}{((s+1)^2+4)^2}\right] = \frac{4x((s+1)^2+4)^2-16x((s+1)^2+4)(s+1)^2}{((s+1)^2+4)^4}$$

$$\frac{[(s+1)^2+4][4x((s+1)^2+4)-16x(s+1)^2]}{((s+1)^2+4)^4}) =$$

$$=(\frac{-12x(s+1)^2+64x}{((s+1)^2+4)^3}$$

5. ALzughair Transform of Integration

In partial differential equations, it is also necessary to compute Alzughair transform of the integrals. Consider,

$$u(x,lnt) = \int_e^t f\ (x,v)\,dv\,;u_t\ (x,lnt) = f(x,lnt)\ and\ u(x,lne) = 0$$

To find $Z[u(x,lnt)]$, we write

$$= \int_1^e \left(ln(lnt)\right)^{k-1}\frac{(lnt)^s}{t}\,ln(lnt)f(x,t)\,dt$$

$$= \int_1^e \left(ln(lnt)\right)^{k}\frac{(lnt)^s}{t}\ f(x,t)\,dt$$

$$= Z\left[\left(ln(lnt)\right)^k\ f(x,t)\right]$$

$$\frac{d^k}{ds^k}\ Z\,[f(x,t)] = Z\left[\left(ln(lnt)\right)^k f(x,t)\right];k = 1,2,3,\ldots..$$

$$\rightarrow Z\left[\left(ln(lnt)\right)^k f(x,t)\right] = \frac{d^k}{ds^k}\ Z\,[f(x,t)];k = 1,2,3,\ldots.$$

4.2 Example[1]: To find $z[(lnlnt)^3\ t^2\cosh x]$ by applying above relationship:

$$z[(lnlnt)^3\ t^2\cosh x] = \frac{d^3}{ds^3}\left[\frac{\cosh x}{s+3}\right] = \frac{d^2}{ds^2}\left[\frac{-\cosh x}{(s+3)^2}\right] = \frac{d}{ds}\left[\frac{2\cosh x}{(s+3)^3}\right] = \frac{-6\cosh x}{(s+3)^3}$$

By using the usual method , we also get

$$z[(lnlnt)^3\ t^2\cosh x] = \frac{-3!\cosh x}{(s+3)^3}$$

4.3 Example[2]: To find $z[x(lnlnt)^2\sin(2lnlnt)]$

$$Z[u(x,lnt)] = \int_1^e \frac{(lnt)^s}{t}\ u(x,lnt)\,dt$$

$$Z[u(x,lnt)] = \frac{(lnt)^{s+1}}{s+1}\ u(x,lnt)\ \Big|_1^e - \frac{1}{s+1}$$

$$\int_1^e (lnt)^{s+1}\ u_t(x,lnt)\,\frac{1}{t}\ dt$$

$$= \frac{-1}{s+1} \int_1^{\varepsilon} \frac{(lnt)^{s+1}}{t} f(x, lnt)\, dt$$

$$= \frac{-1}{s+1} Z[lnt\ f(x, lnt)]$$

By taking Z^{-1} to both sides we get

$$u(x, lnt) = Z^{-1}\left[\frac{-1}{s+1} Z\left(lnt\ f(x, lnt)\right)\right]$$

5.1 Example[3]: To find $Z^{-1}\ [(\frac{\sin(alnlnx)}{(s+1)(s+4)}]$

We note that

$$Z^{-1}\ [(\frac{\sin(alnlnx)}{(s+1)(s+4)} = Z^{-1}\left[\frac{\frac{\sin(alnlnx)}{(s+4)}}{(s+1)}\right]$$

Applying the above relationship we get:

$$= Z^{-1}\left[\frac{Z[(lnt)^3 \sin(alnlnx)]}{s+1}\right]$$

$$= -Z^{-1}\ \left[\frac{(-Z[lnt.(lnt)^2 \sin(alnlnx)]}{(s+1)}\right]$$

$$= \int_{\varepsilon}^{t} -(lnu)^2 \sin(alnlnx) \frac{du}{u}$$

$$= \sin(alnlnx) \int_{\varepsilon}^{t} -\frac{(lnu)^2}{u} du$$

$$= -\frac{(lnt)^3 \sin(alnlnx)}{3} + \frac{\sin(alnlnx)}{3}$$

5.2 Example[4]: To find $Z^{-1}\left[\frac{lnx}{(s+3)(s+6)}\right]$

We note that (s+1) is not find in a denominator so,

$$Z^{-1}\left[\frac{lnx}{(s+3)(s+6)} \frac{s+1}{s+1}\right] = lnx\ Z^{-1}\left[\frac{\frac{(s+1)}{(s+3)(s+6)}}{(s+1)}\right]$$

$$\frac{lnx\ (s+1)}{(s+3)(s+6)} = lnx \frac{2}{(s+6)} - \ lnx \frac{1}{(s+3)}$$

$$= Z^{-1}\left[\frac{\frac{lnx\ (s+1)}{(s+3)(s+6)}}{(s+1)}\right]$$

$$= Z^{-1}\left[\frac{-Z[lnx.2(lnt)^5] - Z[lnx(lnt)^2]}{(s+1)}\right]$$

$$= Z^{-1}\left[\frac{-Z[lnx\ lnt\ (2(lnt)^4 - lnt)]}{(s+1)}\right]$$

Applying the above relationship we get:

$$= lnx \int_{\varepsilon}^{t} [2\ \ (lnu)^4 - lnu] \frac{du}{u}$$

$$= \frac{2lnx}{5} (lnt)^5 - \frac{lnx}{2} (lnt)^2$$

6. Conclusion

Sonoluminescence converts sound waves into light. Since the discovery of this phenomenon, the theoretical justification has always been considered. In conventional approaches to simulating this phenomenon, usually solving ordinary differential equations (ODEs) is due to a homogeneous hydrochemical model for the evolution of the bubble. To perform simulations based on ODE models, the governing equation governing the motion of the bubble wall, the Rail-Plast equation, is solved simultaneously with the homogeneous equations for the evolution of gas inside the bubble, and by calculating the temperature and pressure of the bubble at the end of its erosion, the sonoluminescence radiation is calculated. But the high velocity of the bubble wall, as well as the presence of different gases inside the bubble, can lead to heterogeneity within the bubble. A complete simulation of the evolution of the bubble is presented by solving partial differential equations (PDE) arising from the Euler equations. In simulation, two sets of equations are solved simultaneously. 1. Boundary equations that are the ODE equations of Rail-Polst and are used to solve the Runge-Kuta method. 2. Euler equations that have PDE equations for the evolution of gas into the bubble and dissolved by the Lex-Frederick method. Also, for the first time in this simulation, the surface evaporation of water in the bubble wall is accompanied by chemical reactions related to water vapor in the simulation of the sonoluminescence bubble. The program's results indicate that the temperature and pressure in the bubble are not uniform. So solving the ODE, which considers the evolution of the gas inside the bubble uniformly, can lead to an incorrect answer. In this solution, an improved form of the rail-polar equation, called the Clare-McSSI equation, is used. In addition, the dependence of non-homogeneity inside the bubble on the magnitude of the particle is also studied in the simulation results. The dissertation also has a laboratory section. In this section, the purpose of the dissertation is to produce SL bubbles in vitro in water and sulfuric acid. Using a HZ 0/5 accuracy signal generated through an oscilloscope, bubble trapping was sustained at the center of the cavity, which corresponds to the production of radiation. Also, how the spatial instability and bubble shape were qualitatively investigated in the laboratory system.

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