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Research paper

Collective Value-At-Risk (Colvar) In Life Insurance Collection

Abdul Talib Bon[1*)], Muhammad Iqbal Al-Banna Ismail[2)], Sukono[3)], Adhitya Ronnie Effendie[4)]

[1,2] Universiti Tun Hussein Onn, Malaysia [3]Padjadjaran University, Indonesia [4] Gadjah Mada University, Indonesia
Corresponding auther E-mail: talib@uthm.edu.my iiqbalismail@gmail.com, fsukono@yahoo.com adhityaronnie@yahoo.com

Abstract

Analysis of risk in life insurance claims is very important to do by the insurance company actuary. Risk in life insurance claims are generally measured using the standard deviation or variance. The problem is, that the standard deviation or variance which is used as a measure of the risk of a claim can not accommodate any claims of risk events. Therefore, in this study developed a model called risk measures *Collective Modified Value-at-Risk.* Model development is done for several models of the distribution of the number of claims and the distribution of the value of the claim. *Collective* results of model development *Modified Value-at-Risk* is expected to accommodate any claims of risk events, when given a certain level of significance

Keywords*: group life insurance, the risk of a claim, the collective risk, Value-at-Risk,*

1.1 Introduction

Insurance is one of the techniques to manage risk, which is quite widely used. Insurance can be viewed as a tool in which an individual can transfer the risk to another party, in which the insurance company to accumulate funds from individuals to meet the financial needs related to damages (Dickson, 2005). As organizations take control and recipient risk, the insurance company would have to take into account the risk if there were multiple claims, because if not, will result in losses that could make the insurance company went bankrupt (Bowers et al., 1997). In risk management, the insurance company must know the character of those to predict the risk of loss that will occur in the future. Character of these risks can be studied in a model of the distribution of claims (Riaman et al., 2012). There are two standard approaches for distribution claim modeling during the insurance period is the collective risk modeling and individual risk modeling (Arkin & Shorgin, 2001).

In the collective risk modeling, claims that appear every risk occurs is called the individual claim, the accumulation of individual claims during the period of insurance claims referred to as aggregation (Dickson, 2005; Kahn, 1992). Distribution model can be formed from the aggregation of the claims of the models and the number of individual claims, so as to form a model of the distribution of the aggregation claims must first be determined and the model of the distribution of the individual claim amount (Heckman & Meyers, 2001; Bowers et al., 1997). Collective Risk collective, usually measured using variance. But often the variance risk measure can not accommodate any event risk, because there is a risk of claims beyond the amount of variance (Dickson, 2005).

Therefore, in this study developed a model of collective risk measure, called the *Collective Modified Value-at-Risk (ColMVaR).* Development of this model is based on the collective risk model contained in research Dickson in 2005 and Khan in 1992. The goal is to formulate a model as one alternative for measuring the collective risk. *Collective Modified Value-at-Risk (ColMVaR)* result of this development is expected to accommodate any event collective risk, when given a certain level of significance. As a numerical illustration, *ColMVaR* models were used to analyze the simulation data that has risk characteristics of claims incurred.

1.2 Research Background

Insurance companies in risk management should carefully consider the risks that may occur during the period of insurance. Because the insurance company as a guarantor risk of a loss of the insured must be able to bear the possibility of a claim from the insured to the insurer. Insurers should know the risks of risk characteristics, which can be studied in a model of distribution of claims. Of the distribution function claims, insurers can determine the price of underwriting risks insured. Underwriting price is intended to prevent insurers from greater losses (Pramdsti, 2011). Insurers take into account of risk not only individuals, but also the risk of aggregation collective.

1.3 Research Questions

Based on the description of the background of the above problems, in general, the problem in this research is "How can developes *Modified Collective value-at-Risk* and its application in risk analysis group life insurance". From this general problem is decomposed into several specific issues as follows:

1. How to model individual risk and collective risk in life insurance, as well as its application?
2. How to estimate the model parameters of individual risk and collective risk in life insurance, for some models the distribution of the number and the value of the claim?
3. How the development model of *Collective Value-at-Risk* and *Collective Modified Value-at-Risk* in life insurance, for a number of claims distribution model and the value of the claim?
4. How do you compare the results of calculations *Collective Risk, Collective Value-at-Risk,* and *Collective Modified Value-at-Risk* in the simulation data and the value of the number of claims life insurance claims?

1.4 Research Objectives

Based on the above formulation of the problem, the general purpose of this research is "Formulating the model of *Collective Modified value-at-Risk* and data simulation apply to life insurance claims". Based on general purpose we explain into the following specific objectives :

1. To modeling the individual risk and collective risk in life insurance, as well as its application.
2. To estimating the model parameters of individual risk and collective risk in life insurance, for some models the distribution of the number and the value of the claim.
3. To developing the model of *Collective Value-at-Risk* and *Collective Modified Value-at-Risk* in life insurance, for a number of claims distribution model and the value of the claim.
4. To comparing the results of calculations *Collective Risk, Collective Value-at-Risk,* and *Collective Modified Value-at-Risk* in the simulation data and the value of the number of claims life insurance claims.

2. Methodology

Individual risk can be viewed as individual claims units *to-i* (i = 1, 2, ..., *N),* and denoted by X_i, So that $X = \{X_i\}_{i=1,2,\ldots,N}$ where *N* is the lot of a claim. X_i which can be assumed to be continuous or discrete distribution, which is a random variable are independent and identically distributed. While the collective risk is the sum of *N* individual claims, namely (Pramdsti, 2011):

$$S = \sum_{i=1}^{N} X_i \qquad (2.1)$$

The main benefit of the collective risk model is that it is an efficient computational model, which is also closer to reality. However, the collective model, some ignored the policy information (Mahmoudvand & Edalati, 2009). Referring Pramdsti (2011), Mahmoudv and & Edalati (2009) and Dickson (2005), that the average amount of collective claims can be expressed as:

$$E[S] = E[E[S \mid N = n]]$$
$$= \sum_{N} E[S \mid N = n]P(N = n) = E[X]E[N]$$

While the variance as a measure of risk aggregation claims collectively can be determined by the following equation:

$$V[S] = E[E[S^2 \mid N = n]] - (E[S])^2$$
$$= \sum_{N} \left(nV[X] + n^2(E[X])^2\right)P(N = n) - (E[S])^2$$
$$= V[X]E[N] + (E[x])^2 V[N]$$

Referring Andreas de Vries (2000), because the variance or standard division is a measure of the average deviation, which is often not able to accommodate all events deviation (risk). Therefore, the idea emerged to quantify the risk carried by quintile or better known as *Value-at-Risk (VaR).*
Based on the research results Pramdsti (2011), Mahmoudvand & Edalati (2009) and Dickson (2005), and Andreas de Vries (2000), in this study will develop a model of *Collective Modified Value-at-Risk (ColMVaR).*

2.1 Risk Model Claims set

Is defined S as the sum of a collection of random variables total of claims incurred within one year of the risk. Suppose the N random variable indicates the number of claims of risk this year, and let the random variableX_i to declare the amount of the claim. Aggregate claim amount is the sum of the number of individual claims, therefore can write as in equation (2.1) with the understanding $S = 0$that when $N = 0$. (If there is no claim, then the aggregate claim amount is zero). In this paper, modeling number of individual claims as a non-negative random variable with mean positive (Dickson, 2005).

Now made two important assumptions. First, it is assumed $\{X_i\}_{i=1}^{\infty}$ a that the random variable is a sequence of iid (independent and identically distributed), and second, it is assumed that the random variable N independent of $\{X_i\}_{i=1}^{\infty}$. These assumptions reveals that the amount of any claim does not depend on the number of other claims, and that the distribution of the number of claims did not change throughout the year. The assumption also states that the number of claims has no effect on the amount of the claim (Dickson, 2005; Bowers et al., 1997).

In particular, the risk of a portfolio of insurance policies, and collective risk model appears from the testimony that in this study considered the overall risk. In particular, count the number of claims of the portfolio, and not from individual policyholders (Kahn, 1992).

2.2 Collective Risk

Starting with some notation. Suppose that $G(x) = \Pr(S \leq x)$the distribution function shows a collection of claims $F(x) = \Pr(X_1 \leq x)$, stating the distribution function of the number of individual claims, and suppose that $p_n = \Pr(N = n)$ a $\{p_n\}_{n=1}^{\infty}$ probability function for the number of claims.
Therefore, can be obtained from the distribution S function by noting that the incident $\{S \leq x\}$ occurred when a n claim occurs, n = 0,1,2, ..., and the amount of the n claim is not over x. Therefore, it can be shown the events $\{S \leq x\}$ as a unity of mutually exclusive events $\{S \leq x$ dan $N = n\}$ and, thus (Dickson, 2005; Heckman & Meyers, 1983):

$$\{S \leq x\} = \bigcup_{n=0}^{\infty} \{S \leq x \text{ dan } N = n\}$$

Therefore

$$G(x) = \Pr(S \le x) = \sum_{n=0}^{\infty} \Pr(S \le x \text{ dan } N = n)$$

Now

$$\Pr(S \le x \text{ dan } N = n) = \Pr(S \le x \mid N = n)\Pr(N = n)$$

and

$$\Pr(S \le x \mid N = n) = \Pr\left(\sum_{i=1}^{n} X_i \le x\right) = F^{n*}(x)$$

So, for $x \ge 0$,

$$G(x) = \sum_{n=0}^{\infty} p_n F^{n*}(x) \quad (2.2)$$

remember that $F^{0*}(x)$ *is* defined to be 1, and zero if (Bowers et al., 1997).
In principle, equation (1) can result in the calculation of *the mean* of the distribution of aggregate claims. However, complexity is not present in a form suitable for most individual claim amount distribution of practice such as Pareto and lognormal. Though in cases when no appropriate forms, the distribution function in equation (3.2) remains to be evaluated as a finite sum.

Using similar arguments, the case when the number of individual claims distributed on a positive integer with probability function

$$f_j = F(j) - F(j-1)$$

to $j = 1, 2, 3, \ldots$, probability *of* functions $\{g_x\}_{x=0}^{\infty}$ provided by S, and $g_0 = p_0$ for $x = 1, 2, 3, \ldots$,

$$g(x) = \sum_{n=1}^{\infty} p_n f_x^{n*} \quad (2.3)$$

where $f_x^{n*} = \Pr(\sum_{i=1}^{n} X_i = x)$. Formula (2.3) not more beneficial than formula (2.2). However, on certain distributions to N, g_x can be calculated recursively for $x = 1,2,3, \ldots$, using g_0 as initial values for the recursive calculation, and this paper does not specifically addressed this issue (Dickson, 2005).

2.3 Modeling Collective Value-at-Risk

Moments and the moment generating function of S can be calculated using conditional expectation argument. The key result is that for any two random variables Y and Z, there is a relevant moment (Dickson, 2005; Bowers et al., 1997):

$$E[Y]E = E[E(Y|Z)] \quad (2.4)$$

and

$$V[Y] = E[V(Y|Z)] + V[E(Y|Z)] \quad (2.5)$$

As the closest application of equation (3.4) is obtained

$$E[S] = E[E(S|N)]$$

Now suppose $m_k = E[X_1^k]$ for $k = 1,2,3, \ldots$ the moment is to *k.* Therefore, obtained

$$E[S|N = n] = E\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} E[X_i] = nm_1$$

and since it was formed for $n = 0,1,2, \ldots$, $E[S|N] = Nm_1$ and therefore

$$E[S] = E[Nm_1] = E[N]m_1 \quad (2.6)$$

This is a very interesting result, because it suggests that the expected number of claims collection is the product of the expected number of claims and the expectations of the amount of each claim (Dickson, 2005).

Using a similar method, using the information that $\{X_i\}_{i=1}^{\infty}$ is independent random variables,

$$V[S|N = n] = V\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} V[X_i] = \; n(m_2 - m_1^2)$$

thus $V[S|N] = N(m_2 - m_1^2)$. Then, by applying equation (2.5) is obtained

$$\begin{aligned} V[S] &= E[V(S|N)] + V[E(S|N)] \\ &= E[N(m_2 - m_1^2)] + V[Nm_1] \\ &= E[N](m_2 - m_1^2) + V[N]m_1^2 \end{aligned} \quad (2.7)$$

Equation (2.7) does not have the same type on the basis of interpretation as equation (2.6), but it shows that the variance is shown in the form of *mean* and variance in the second distribution of the number of claims and the distribution of the number of individual claims (Meng-Yi Li, 2000).
Further, that the *Collective Value-at-Risk* is defined as

$$ColVaR = -\tilde{N}\{E(S) + z_{\alpha}(V[S])^{1/2}\} \quad (2.8)$$

with $\tilde{N}$ many claim they want to know the level of risk, and z_{α} percentile of the standard normal distribution when given level of significance α. Due to the risk of claims related to the issue, the value of z_{α} selected which is located on the left tail (Andreas de Vries, 2000; Casiglio et al., 2002; Manganelli and Engle, 2001).

When equation (2.6) and (2.7) is substituted into equation (2.8), the model obtained *Collective Value-at-Risk* with the equation as

$$ColVaR = -\tilde{N}\left\{E[N]m_1 + z_{\alpha}\left(E[N](m_2 - m_1^2) + V[N]m_1^2\right)^{1/2}\right\} \quad (2.9)$$

2.4 Analysis Stages

The data used in this study is simulated data generated by the characteristics of some life insurance claims data for the actual credit. To process the data will be performed using **R Software.** The methodology used in this study was based on the following stages:

1. Studying on individual risk models and collective risk models in general are often used for the analysis of life insurance risks.
2. Studying on the collective risk model, where the number of insurance claims Negative Binomial distribution.

3. Conduct studies estimate parameters include *the mean*, variance, *skewness*, and *kurtosis*, for some model distribution of the number of claims and claims.
4. To develop models *Collective Value-at-Risk* model development and *Collective Modified Value-at-Risk*, for some models the distribution of many of the claims and claims.
5. Implementing the simulation data and comparative analysis of the results of the calculation of *collective risk*, *collective Value-at-Risk*, and *Collective Modified Value-at-Risk*, the credit life insurance simulated data.
6. Conclusion.

References

[1] Albrecher, H. 2008. Risk Modelling in Insurance. *Working Paper*. Radon Institute, Austrian Academy of Sciences and University of Linz, Austria. September 3, 2008.

[2] Andersen, T.M. 2010. Collective risk sharing: The social safety net and employment. First and preliminary draft. *Working Paper*. School of Economics and Management, Aarhus University. November 2010.

[3] Andreas de Vries. (2000). Value-at-Risk. *Working Paper*. F.H. Sudwestfalen University of Applied Sciences. May 19, 2000.

[4] Arkin, V.I. & Shorgin, S.Y. (2001). Models and Method of Actuarial Mathematics. *Mathematical Models of Life Support System* – Vol. II.

[5] Bowers, N.L., Gerber, H.U., Hickman, J.C., Jones, D.C. & Nesbitt, C.J. (1997). *Actuarial Mathematics.* The Society of Actuaries, Schaumberg, Illinois.

[6] Constantinescu, C. 2013. Applications of Probability Theory to Insurance Mathematics. *Working Paper*. Nankai University, July 8-18, 2013.

[7] Cosiglio, A., Percorella, A. & Zenios, S.A. (2002). A Conditional Value-at-Risk Model for Insurance Product with Guarantee. *Working Paper*. University of Palermo.

[8] Dickson, D.C. (2005). *Insurance Risk and Ruin.* Cambridge University Press.

[9] Djuric, Z. (2013). Collective Risk Model in Non-Life Insurance. *Economic Horizons*, May - August 2013, Volume *15*, Number 2, 167 – 175, UDC: 33 eISSN 2217-9232

[10] Heckman, P.E. & Meyers, G.G. (1983). The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions. *Proceedings of the Causality Actuarial Society, 70, 22-61*.

[11] Kahn, P.M. (1992). An Introduction to Collective Risk Theory and Its Application to Stop-Loss Reinsurance. *Transactions of Society of Actuaries*, 1992 Vol. 14 PT. 1 No. 40.

[12] Mahmoudvand, R. & Edalati, A. (2009). On the Distribution of Discounted Collective Risk Model. *Journal of Statist. Res. Iran* 6 (2009): 193-207.

[13]]Manganelli, S. and Engle, R.F. (2001). Value-at-Risk Models in Finance. *Working Paper No. 75*. European Central Bank Working Paper Series, August 2001.

[14] Meng-Yi Li. (2000). Value-at-Risk for the Reserves of Multi-Product Life Insurance. *Working Paper*. National Chengchi University.

[15] Pramesti, G. (2011). Distribusi Rayleigh Untuk Klaim Agregasi. *Jurnal Media Statistik*, Vol. 4, No. 2, Desember 2011: 105-112.

[16] Riaman, Supena, Y., Lesmana, E., Sukono & Firdaus, R. (2012). Analisis Model Risiko Kolektif pad Asuransi Jiwa Kredit Menggunakan Model Klaim Agregasi. *Paper*. Jurusan Matematika FMIPA Universitas Padjadjaran.

3. Conclusion

This is to propose and develope Modified Collective Value At Risk. Some distribution such as Poisson, Lognormal and Binomial distribution will be use for combining and mixture to develop the propose model.