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Research paper

Type-2 pentagonal fuzzy numbers and its application to get equivalent proverbs in two different languages

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Abstract

Uncertainties in words or phrases are usually represented by fuzzy sets and the quantification of it is represented by fuzzy number.Type-2 fuzzy set is the generalization made from Zadeh's fuzzy set which resolve the uncertainty of the membership function of type-1 fuzzy set. In this paper, we introduce a new fuzzy number called Type-2 Pentagonal Fuzzy Number (T2PFN). We study its algebraic properties. We define different type of Type-2 Pentagonal Fuzzy Numbers. Also use it to analyze the complex structural representation and categorization of words used in the proverbs from two Indian Languages (IL) Hindi and Tamil. Proverbs used in communication might have different meanings, emotions and intentions in different cultures and contexts. Getting equivalent proverbs in two different languages is a challenge to the translators. Proverbs are popularly utilized in oral contexts which reflect the lifestyle of the common people representing their culture, tradition, emotion and experience gained from forefathers. We try to grasp the core meaning of a proverb expressed through context defined words. Getting equivalent proverbs in two different languages requires the understanding of the context, meaning in that particular cultural environment and timing used in the conversation. Type-2 fuzzy set is employed to study the similarity relation between the texts of two languages with the help of Matlab toolbox. An illustration is made with an example by analyzing the semantically similar proverbs from the two languages.

Keywords: *Emotion; Indian Language; Proverb; Similarity Measure; Type-2 Fuzzy Set; Type-2 Pentagonal Fuzzy Number.*

1. Introduction

Type-2 fuzzy sets are used to express uncertainty and inaccuracy in a better way. Type-2 fuzzy set theory has been developed from ordinary fuzzy set which was introduced by L.A Zadeh[8]. Type-2 fuzzy set is characterized by a membership function on [0-1] and the membership functions are themselves fuzzy. Type-2 fuzzy set was modified and made simple by Jerry M. Mendel and Robert I. Bob John [7].

Fuzzy numbers are used to resolve the uncertainty that occurs in decision making problems. T. Pathinathan and K. Ponnivalavan have introduced Pentagonal Fuzzy Numbers in the year 2014 [11]. In the same year T. Pathinathan and K. Ponnivalavan also discussed Diamond Fuzzy numbers [10]. T. Pathinathan and K. Ponnivalavan studied about the Reverse Order Triangular, Trapezoidal and Pentagonal Fuzzy Numbers in 2014[12].

In this paper we introduce Type-2 Pentagonal Fuzzy Number (T2PFN) and use it to solve the complexity that arises in translating a proverb form one Indian language into another. The new tool gives us more exact results.

The paper is organized as follows: Section two comprises the required preliminaries. In section three we define the Type-2 Pentagonal Fuzzy Numbers and Type-2 Pentagonal Fuzzy Matrices. In section four we discuss the application of the T2PFNs and propose a model to help the translators to get an equivalent proverb when compared with two different languages. The application of Type-2 Pentagonal Fuzzy Number was verified in section five followed by conclusion.

2. Preliminaries

2.1. Pentagonal fuzzy number [11]

A Pentagonal Fuzzy Number (PNF) of a fuzzy set $\underset{\sim}{A}$ is defined as $\underset{\sim}{A}_p = (a,b,c,d,e)$ and its membership function is given by,

$$\underset{\sim}{A}_p = \begin{cases} 0 & for\ x < a, \\ \dfrac{(x-a)}{(b-a)} & for\ a \le x \le b \\ \dfrac{(x-b)}{(c-b)} & for\ b \le x \le c \\ 1 & x = c \\ \dfrac{(d-x)}{(d-c)} & for\ c \le x \le d \\ \dfrac{(e-x)}{(e-d)} & for\ d \le x \le e \\ 0 & for\ x > e \end{cases}$$

In addition, $\underset{\sim}{A}_p$ should satisfy the following conditions:

1) $\mu_{\underset{\sim}{A}_p}(x)$ Is a continuous functions in the interval $[0,1]$.
2) $\mu_{\underset{\sim}{A}_p}(x)$ Is strictly increasing and continuous function on $[a,b]$ and $[b,c]$.

3) $\mu_{A_p}(x)$ is strictly decreasing and continuous on $[c,d]$ and $[d,e]$.

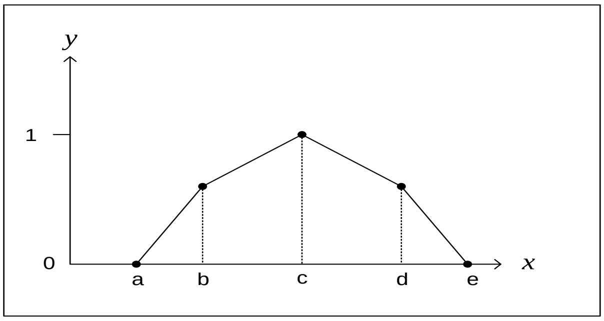

Fig. 1: Graphical Representation of Pentagonal Fuzzy Number (PFN)

2.2. Type-2 fuzzy set [4], [6], and [7]

A type-2 fuzzy set, denoted $\underset{\sim}{A}$, is a characterized by a type-2 membership function $\mu_{\underset{\sim}{A}}(x,u)$, where $x \in X$ and $u \in J_x \subseteq [0,1]$, That is, $\underset{\sim}{A} = \left\{ \left((x,u), \mu_{\underset{\sim}{A}}(x,u) \right) \mid \forall x \in X, \forall u \in J_X \subseteq [0,1] \right\}$ in which $0 \le \mu_{\underset{\sim}{A}}(x,u) \le 1$.

2.3. Type-2 fuzzy numbers [3], [5]

Let $\underset{\sim}{A}$ be a type-2 fuzzy set defined in the universe of discourse R, if it satisfies the following axioms:

1) $\underset{\sim}{A}$ is Normal,
2) $\underset{\sim}{A}$ is a convex set,
3) In addition, the support of $\underset{\sim}{A}$ is closed and bounded, then $\underset{\sim}{A}$ is called a type-2 fuzzy number.

3. Type-2 pentagonal fuzzy numbers

3.1. Definition: type-2 pentagonal fuzzy numbers

A type-2 Pentagonal Fuzzy Number $\underset{\sim}{A}$ on R is agreed,

$\underset{\sim}{A} = \left\{ x, \left(\mu_A^1(x), \mu_A^2(x), \mu_A^3(x), \mu_A^4(x) \mu_A^5(x) \right); x \in R \right\}$ In addition $\mu_A^1(x) \le \mu_A^2(x) \le \mu_A^3(x) \le \mu_A^4(x) \le \mu_A^5(x)$, for all, $x \in R$.

$\underset{\sim}{A} = (\underset{\sim}{A}_1, \underset{\sim}{A}_2, \underset{\sim}{A}_3, \underset{\sim}{A}_4, \underset{\sim}{A}_5)$

Where

$\underset{\sim}{A}_1 = \left(A_1^{L_1}, A_1^{L_2}, A_1^M, A_1^{U_1}, A_1^{U_2} \right)$, $\underset{\sim}{A}_2 = \left(A_2^{L_1}, A_2^{L_2}, A_2^M, A_2^{U_1}, A_2^{U_2} \right)$, $\underset{\sim}{A}_3 = \left(A_3^{L_1}, A_3^{L_2}, A_3^M, A_3^{U_1}, A_3^{U_2} \right)$, $\underset{\sim}{A}_4 = \left(A_4^{L_1}, A_4^{L_2}, A_4^M, A_4^{U_1}, A_4^{U_2} \right)$ And $\underset{\sim}{A}_5 = \left(A_5^{L_1}, A_5^{L_2}, A_5^M, A_5^{U_1}, A_5^{U_2} \right)$ Are same type of fuzzy numbers?

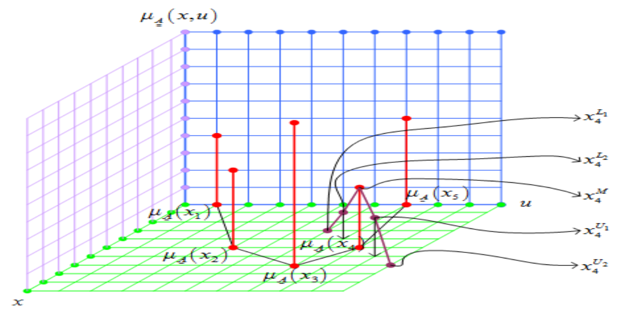

Fig. 1: Graphical Representation of Type-2 Pentagonal Fuzzy Number (PFN).

3.2. Arithmetic operations on type-2 pentagonal fuzzy numbers

Let us consider two sub sets $\underset{\approx}{x}$ and $\underset{\approx}{y}$ as,

$$\underset{\approx}{x} = \left(\underset{\sim}{x}_1, \underset{\sim}{x}_2, \underset{\sim}{x}_3, \underset{\sim}{x}_4, \underset{\sim}{x}_5 \right)$$

$$= \begin{pmatrix} \left(x_1^{L_1}, x_1^{L_2}, x_1^M, x_1^{U_1}, x_1^{U_2} \right), \left(x_2^{L_1}, x_2^{L_2}, x_2^M, x_2^{U_1}, x_2^{U_2} \right), \\ \left(x_3^{L_1}, x_3^{L_2}, x_3^M, x_3^{U_1}, x_3^{U_2} \right), \left(x_4^{L_1}, x_4^{L_2}, x_4^M, x_4^{U_1}, x_4^{U_2} \right), \\ \left(x_5^{L_1}, x_5^{L_2}, x_5^M, x_5^{U_1}, x_5^{U_2} \right) \end{pmatrix}$$

$$\underset{\approx}{y} = \left(\underset{\sim}{y}_1, \underset{\sim}{y}_2, \underset{\sim}{y}_3, \underset{\sim}{y}_4, \underset{\sim}{y}_5 \right)$$

$$= \begin{pmatrix} \left(y_1^{L_1}, y_1^{L_2}, y_1^M, y_1^{U_1}, y_1^{U_2} \right), \left(y_2^{L_1}, y_2^{L_2}, y_2^M, y_2^{U_1}, y_2^{U_2} \right), \\ \left(y_3^{L_1}, y_3^{L_2}, y_3^M, y_3^{U_1}, y_3^{U_2} \right), \left(y_4^{L_1}, y_4^{L_2}, y_4^M, y_4^{U_1}, y_4^{U_2} \right), \\ \left(y_5^{L_1}, y_5^{L_2}, y_5^M, y_5^{U_1}, y_5^{U_2} \right) \end{pmatrix}$$

be two type-2 pentagonal fuzzy number. Then we define some arithmetic operations as follow.

3.2.1. Addition

$$\underset{\approx}{x} + \underset{\approx}{y} = \begin{pmatrix} \left(x_1^{L_1} + y_1^{L_1}, x_1^{L_2} + y_1^{L_2}, x_1^M + y_1^M, x_1^{U_1} + y_1^{U_1}, x_1^{U_2} + y_1^{U_2} \right), \\ \left(x_2^{L_1} + y_2^{L_1}, x_2^{L_2} + y_2^{L_2}, x_2^M + y_2^M, x_2^{U_1} + y_2^{U_1}, x_2^{U_2} + y_2^{U_2} \right), \\ \left(x_3^{L_1} + y_3^{L_1}, x_3^{L_2} + y_3^{L_2}, x_3^M + y_3^M, x_3^{U_1} + y_3^{U_1}, x_3^{U_2} + y_3^{U_2} \right), \\ \left(x_4^{L_1} + y_4^{L_1}, x_4^{L_2} + y_4^{L_2}, x_4^M + y_4^M, x_4^{U_1} + y_4^{U_1}, x_4^{U_2} + y_4^{U_2} \right), \\ \left(x_5^{L_1} + y_5^{L_1}, x_5^{L_2} + y_5^{L_2}, x_5^M + y_5^M, x_5^{U_1} + y_5^{U_1}, x_5^{U_2} + y_5^{U_2} \right) \end{pmatrix}$$

3.2.2. Subtraction

$$\underset{\approx}{x} - \underset{\approx}{y} = \begin{pmatrix} \left(x_1^{L_1} - y_5^{U_2}, x_1^{L_2} - y_5^{U_1}, x_1^M - y_5^M, x_1^{U_1} - y_5^{L_2}, x_1^{U_2} - y_5^{L_1} \right), \\ \left(x_2^{L_1} - y_4^{U_2}, x_2^{L_2} - y_4^{U_1}, x_2^M - y_4^M, x_2^{U_1} - y_4^{L_2}, x_2^{U_2} - y_4^{L_1} \right), \\ \left(x_3^{L_1} - y_3^{U_2}, x_3^{L_2} - y_3^{U_1}, x_3^M - y_3^M, x_3^{U_1} - y_3^{L_2}, x_3^{U_2} - y_3^{L_1} \right), \\ \left(x_4^{L_1} - y_2^{U_2}, x_4^{L_2} - y_2^{U_1}, x_4^M - y_2^M, x_4^{U_1} - y_2^{L_2}, x_4^{U_2} - y_2^{L_1} \right), \\ \left(x_5^{L_1} - y_1^{U_2}, x_5^{L_2} - y_1^{U_1}, x_5^M - y_1^M, x_5^{U_1} - y_1^{L_2}, x_5^{U_2} - y_1^{L_1} \right) \end{pmatrix}$$

3.2.3. Scalar multiplication

If $K \ge 0$ and $K \in R$ then

$$k\underset{\approx}{x} = \begin{pmatrix} \left(kx_1^{L_1}, kx_1^{L_2}, kx_1^M, kx_1^{U_1}, kx_1^{U_2} \right), \left(kx_2^{L_1}, kx_2^{L_2}, kx_2^M, kx_2^{U_1}, kx_2^{U_2} \right), \\ \left(kx_3^{L_1}, kx_3^{L_2}, kx_3^M, kx_3^{U_1}, kx_3^{U_2} \right), \left(kx_4^{L_1}, kx_4^{L_2}, kx_4^M, kx_4^{U_1}, kx_4^{U_2} \right), \\ \left(kx_5^{L_1}, kx_5^{L_2}, kx_5^M, kx_5^{U_1}, kx_5^{U_2} \right) \end{pmatrix}$$

And if $K < 0$ and $K \in R$

then, $k\underset{\approx}{x} = \begin{pmatrix} \left(kx_5^{U_2}, kx_5^{U_1}, kx_5^M, kx_5^{L_2}, kx_5^{L_1} \right), \left(kx_4^{U_2}, kx_4^{U_1}, kx_4^M, kx_4^{L_2}, kx_4^{L_1} \right), \\ \left(kx_3^{U_2}, kx_3^{U_1}, kx_3^M, kx_3^{L_2}, kx_3^{L_1} \right), \left(kx_2^{U_2}, kx_2^{U_1}, kx_2^M, kx_2^{L_2}, kx_2^{L_1} \right), \\ \left(kx_1^{U_2}, kx_1^{U_1}, kx_1^M, kx_1^{L_2}, kx_1^{L_1} \right) \end{pmatrix}$.

3.2.4. Multiplication

Define, $\sigma y = \begin{pmatrix} y_1^{L_1} + y_1^{L_2} + y_1^M + y_1^{U_1} + y_1^{U_2} + y_2^{L_1} + y_2^{L_2} + y_2^M + y_2^{U_1} + \\ y_2^{U_2} + y_3^{L_1} + y_3^{L_2} + y_3^M + y_3^{U_1} + y_3^{U_2} + y_4^{L_1} + y_4^{L_2} + y_4^M + \\ y_4^{U_1} + y_4^{U_2} + y_5^{L_1} + y_5^{L_2} + y_5^M + y_5^{U_1} + y_5^{U_2} \end{pmatrix}$.

If $\sigma y \geq 0$, then $\underset{\approx}{x} \times \underset{\approx}{y} = \left(\begin{array}{l} \left(\frac{x_1^{L_1}\sigma y}{25}, \frac{x_1^{L_2}\sigma y}{25}, \frac{x_1^{M}\sigma y}{25}, \frac{x_1^{U_1}\sigma y}{25}, \frac{x_1^{U_2}\sigma y}{25} \right), \\ \left(\frac{x_2^{L_1}\sigma y}{25}, \frac{x_2^{L_2}\sigma y}{25}, \frac{x_2^{M}\sigma y}{25}, \frac{x_2^{U_1}\sigma y}{25}, \frac{x_2^{U_2}\sigma y}{25} \right), \\ \left(\frac{x_3^{L_1}\sigma y}{25}, \frac{x_3^{L_2}\sigma y}{25}, \frac{x_3^{M}\sigma y}{25}, \frac{x_3^{U_1}\sigma y}{25}, \frac{x_3^{U_2}\sigma y}{25} \right), \\ \left(\frac{x_4^{L_1}\sigma y}{25}, \frac{x_4^{L_2}\sigma y}{25}, \frac{x_4^{M}\sigma y}{25}, \frac{x_4^{U_1}\sigma y}{25}, \frac{x_4^{U_2}\sigma y}{25} \right), \\ \left(\frac{x_5^{L_1}\sigma y}{25}, \frac{x_5^{L_2}\sigma y}{25}, \frac{x_5^{M}\sigma y}{25}, \frac{x_5^{U_1}\sigma y}{25}, \frac{x_5^{U_2}\sigma y}{25} \right) \end{array} \right)$.

If $\sigma y < 0$, then $\underset{\approx}{x} \times \underset{\approx}{y} = \left(\begin{array}{l} \left(\frac{x_5^{U_2}\sigma y}{25}, \frac{x_5^{U_1}\sigma y}{25}, \frac{x_5^{M}\sigma y}{25}, \frac{x_5^{L_2}\sigma y}{25}, \frac{x_5^{L_1}\sigma y}{25} \right), \\ \left(\frac{x_4^{U_2}\sigma y}{25}, \frac{x_4^{U_1}\sigma y}{25}, \frac{x_4^{M}\sigma y}{25}, \frac{x_4^{L_2}\sigma y}{25}, \frac{x_4^{L_1}\sigma y}{25} \right), \\ \left(\frac{x_3^{U_2}\sigma y}{25}, \frac{x_3^{U_1}\sigma y}{25}, \frac{x_3^{M}\sigma y}{25}, \frac{x_3^{L_2}\sigma y}{25}, \frac{x_3^{L_1}\sigma y}{25} \right), \\ \left(\frac{x_2^{U_2}\sigma y}{25}, \frac{x_2^{U_1}\sigma y}{25}, \frac{x_2^{M}\sigma y}{25}, \frac{x_2^{L_2}\sigma y}{25}, \frac{x_2^{L_1}\sigma y}{25} \right), \\ \left(\frac{x_1^{U_2}\sigma y}{25}, \frac{x_1^{U_1}\sigma y}{25}, \frac{x_1^{M}\sigma y}{25}, \frac{x_1^{L_2}\sigma y}{25}, \frac{x_1^{L_1}\sigma y}{25} \right) \end{array} \right)$.

3.2.5. Division

If $\sigma y \neq 0$, then we define division as follow:

If $\sigma y > 0$, then $\frac{\underset{\approx}{x}}{\underset{\approx}{y}} = \left(\begin{array}{l} \left(\frac{25x_1^{L_1}}{\sigma y}, \frac{25x_1^{L_2}}{\sigma y}, \frac{25x_1^{M}}{\sigma y}, \frac{25x_1^{U_1}}{\sigma y}, \frac{25x_1^{U_2}}{\sigma y} \right), \\ \left(\frac{25x_2^{L_1}}{\sigma y}, \frac{25x_2^{L_2}}{\sigma y}, \frac{25x_2^{M}}{\sigma y}, \frac{25x_2^{U_1}}{\sigma y}, \frac{25x_2^{U_2}}{\sigma y} \right), \\ \left(\frac{25x_3^{L_1}}{\sigma y}, \frac{25x_3^{L_2}}{\sigma y}, \frac{25x_3^{M}}{\sigma y}, \frac{25x_3^{U_1}}{\sigma y}, \frac{25x_3^{U_2}}{\sigma y} \right), \\ \left(\frac{25x_4^{L_1}}{\sigma y}, \frac{25x_4^{L_2}}{\sigma y}, \frac{25x_4^{M}}{\sigma y}, \frac{25x_4^{U_1}}{\sigma y}, \frac{25x_4^{U_2}}{\sigma y} \right), \\ \left(\frac{25x_5^{L_1}}{\sigma y}, \frac{25x_5^{L_2}}{\sigma y}, \frac{25x_5^{M}}{\sigma y}, \frac{25x_5^{U_1}}{\sigma y}, \frac{25x_5^{U_2}}{\sigma y} \right) \end{array} \right)$.

If $\sigma y < 0$, then $\frac{\underset{\approx}{x}}{\underset{\approx}{y}} = \left(\begin{array}{l} \left(\frac{25x_5^{U_2}}{\sigma y}, \frac{25x_5^{U_1}}{\sigma y}, \frac{25x_5^{M}}{\sigma y}, \frac{25x_5^{L_2}}{\sigma y}, \frac{25x_5^{L_1}}{\sigma y} \right), \\ \left(\frac{25x_4^{U_2}}{\sigma y}, \frac{25x_4^{U_1}}{\sigma y}, \frac{25x_4^{M}}{\sigma y}, \frac{25x_4^{L_2}}{\sigma y}, \frac{25x_4^{L_1}}{\sigma y} \right), \\ \left(\frac{25x_3^{U_2}}{\sigma y}, \frac{25x_3^{U_1}}{\sigma y}, \frac{25x_3^{M}}{\sigma y}, \frac{25x_3^{L_2}}{\sigma y}, \frac{25x_3^{L_1}}{\sigma y} \right), \\ \left(\frac{25x_2^{U_2}}{\sigma y}, \frac{25x_2^{U_1}}{\sigma y}, \frac{25x_2^{M}}{\sigma y}, \frac{25x_2^{L_2}}{\sigma y}, \frac{25x_2^{L_1}}{\sigma y} \right), \\ \left(\frac{25x_1^{U_2}}{\sigma y}, \frac{25x_1^{U_1}}{\sigma y}, \frac{25x_1^{M}}{\sigma y}, \frac{25x_1^{L_2}}{\sigma y}, \frac{25x_1^{L_1}}{\sigma y} \right) \end{array} \right)$.

3.3. Some numerical examples on arithmetic operations

Let $\underset{\approx}{A} = \left\{ \mu_{\underset{\approx}{A}}(0.2), \mu_{\underset{\approx}{A}}(0.3), \mu_{\underset{\approx}{A}}(0.4), \mu_{\underset{\approx}{A}}(0.5), \mu_{\underset{\approx}{A}}(0.6) \right\}$

$$= \left\{ \begin{array}{l} (0.2,(0.5,0.6,0.7,0.8,0.9)),(0.3,(0.0,0.1,0.2,0.3,0.4)), \\ (0.4,(0.7,0.8,0.9,1,1)),(0.5,(0.1,0.2,0.3,0.4,0.5)), \\ (0.6,(0.3,0.5,0.6,0.9,0.9)) \end{array} \right\}$$

$$\underset{\approx}{A} = \left\{ \begin{array}{l} (0.5,0.6,0.7,0.8,0.9),(0.0,0.1,0.2,0.3,0.4), \\ (0.7,0.8,0.9,1,1),(0.1,0.2,0.3,0.4,0.5),(0.3,0.5,0.6,0.9,0.9) \end{array} \right\}$$

$\underset{\approx}{B} = \left\{ \mu_{\underset{\approx}{A}}(0.5), \mu_{\underset{\approx}{A}}(0.6), \mu_{\underset{\approx}{A}}(0.7), \mu_{\underset{\approx}{A}}(0.8), \mu_{\underset{\approx}{A}}(0.9) \right\}$

$$= \left\{ \begin{array}{l} (0.5,(0.2,0.5,0.6,0.7,0.8)),(0.6,(0.5,0.6,0.7,0.8,1)), \\ (0.7,(0.5,0.6,0.7,0.8,1)),(0.8,(0.3,0.4,0.5,0.6,0.7)), \\ (0.9,(0.6,0.7,0.8,0.9,1)) \end{array} \right\}$$

$$\underset{\approx}{B} = \left\{ \begin{array}{l} (0.2,0.5,0.6,0.7,0.8),(0.5,0.6,0.7,0.8,1), \\ (0.5,0.6,0.7,0.8,1),(0.3,0.4,0.5,0.6,0.7),(0.6,0.7,0.8,0.9,1) \end{array} \right\}$$

3.3.1. Addition

$$\underset{\approx}{A} + \underset{\approx}{B} = \left\{ \begin{array}{l} (0.7,1.1,1.3,1.5,1.7),(0.5,0.7,0.9,1.1,1.4), \\ (1.2,1.4,1.6,1.8,2),(0.4,0.6,0.8,1,1.2), \\ (0.9,1.2,1.4,1.8,1.9) \end{array} \right\}$$

3.3.2. Subtraction

$$\underset{\approx}{A} - \underset{\approx}{B} = \left\{ \begin{array}{l} (-0.5,-0.3,-0.1,0.1,0.3),(-0.7,-0.5,-0.3,-0.1,0.1), \\ (-0.3,0,0.2,0.3,0.5),(-0.9,-0.6,-0.4,-0.2,0), \\ (-0.5,-0.2,0,0.4,0.7) \end{array} \right\}$$

3.3.3. Scalar multiplication

If $k = 3$, then $k \times \underset{\approx}{A} = \left\{ \begin{array}{l} (1.5,1.8,2.1,2.4,2.7),(0.0,0.3,0.6,0.9,1.2), \\ (2.1,2.4,2.7,3,3),(0.3,0.6,0.9,1.2,1.5), \\ (0.9,1.5,1.8,2.7,2.7) \end{array} \right\}$

If $k = -3$, then $k \times \underset{\approx}{A} = \left\{ \begin{array}{l} (-2.7,-2.7,-1.8,-1.5,-0.9), \\ (-1.5,-1.2,-0.9,-0.6,-0.3), \\ (-3,-3,-2.7,-2.4,-2.1), \\ (-1.2,-0.9,-0.6,-0.3,0), \\ (-2.7,-2.4,-2.1,-1.8,-1.5) \end{array} \right\}$

If $k = 0$, then $\underset{\approx}{A}$ become as below;

$$\underset{\approx}{A} = \left\{ \begin{array}{l} (0,0,0,0,0,),(0,0,0,0,0,),(0,0,0,0,0,), \\ (0,0,0,0,0,),(0,0,0,0,0,) \end{array} \right\}$$

Then $\underset{\approx}{A}$ said to be type-2 zero pentagonal fuzzy number. Its denoted by 0.

3.3.4. Multiplication

Let, $\sigma \underset{\approx}{B} = \left(\begin{array}{l} 0.2+0.5+0.6+0.7+0.8+0.5+0.6+0.7+ \\ 0.8+1+0.5+0.6+0.7+0.8+1+0.3+0.4+ \\ 0.5+0.6+0.7+0.6+0.7+0.8+0.9+1 \end{array} \right)$, $\sigma \underset{\approx}{B} = 16.5$,

since $\sigma y \geq 0$, then $\underset{\approx}{x} \times \underset{\approx}{y} = \left\{ \begin{array}{l} (0.33,0.396,0.462,0.528,0.594), \\ (0,0.066,0.132,0.198,0.264), \\ (0.462,0.528,0.594,0.66,0.66), \\ (0.066,0.132,0.198,0.264,0.33), \\ (0.198,0.33,0.396,0.594) \end{array} \right\}$.

3.2.5. Division

Here $\sigma \underset{\approx}{B} = 16.5$, hence $\sigma y \neq 0$, and $\sigma y > 0$, therefore the division is,

$$\underset{\approx}{\frac{x}{y}} = \begin{pmatrix} (0.7575, 0.9090, 1.0606, 1.2121, 1.3636), \\ (0, 0.1515, 0.3030, 0.4545, 0.6060), \\ (1.0606, 1.2121, 1.3636, 1.5151, 1.5151), \\ (0.1515, 0.3030, 0.4545, 0.6060, 0.7575), \\ (0.4545, 0.7575, 0.9090, 1.3636, 1.3636) \end{pmatrix}.$$

3.4. Matrix representation on type-2 pentagonal fuzzy number

3.4.1. Definition

Let X said to by type-2 pentagonal fuzzy matrix (T2PFM), and order $m \times n$, then it is define as $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ where the ij^{th} element $\underset{\approx}{x}_{ij}$ of X is the type-2 pentagonal fuzzy number.

3.4.2. Some operations on type-2 pentagonal fuzzy matrices (T2PFMs)

We define the following operations on type-2 Pentagonal Fuzzy Matrices into classical matrices. Let $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ and $Y = \left(\underset{\approx}{y}_{ij}\right)_{m\times n}$ be two T2PFMs of same order. Then, the following operations are satisfied:

1) $X + Y = \left(\underset{\approx}{x}_{ij} + \underset{\approx}{y}_{ij}\right)$.
2) $X - Y = \left(\underset{\approx}{x}_{ij} - \underset{\approx}{y}_{ij}\right)$.
3) For $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ and $Y = \left(\underset{\approx}{y}_{ij}\right)_{m\times n}$ then $XY = \left(\underset{\approx}{z}_{ij}\right)_{m\times n}$ where $\underset{\approx}{z}_{ij} = \sum_{k=1}^{n} \underset{\approx}{x}_{ik} \cdot \underset{\approx}{y}_{kj}$, $i = 1, 2, ..., l$ and $j = 1, 2, ..., m$.
4) X^T Alternatively $X' = \left(\underset{\approx}{x}_{ji}\right)$.

3.4.3. Types of type-2 pentagonal fuzzy matrices

1) Definition: Square T2PFM

Let $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ is said to be a square T2PFM if the number of rows is equal to the number of columns, i.e. $m \times n$. Otherwise it is called non-square T2PFM.

2) Definition: Symmetric T2PFM

Let $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ is said to be a symmetric T2PFM about the principal diagonal if $\underset{\approx}{x}_{ij} = \underset{\approx}{x}_{ji}$ for all $i, j = 1, 2, ..., n$. i.e. $A^T = A$.

3) Definition: Skew-Symmetric T2PFM

Let $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ is said to be a skew-symmetric T2PFM if T2PFM is square matrix and $\underset{\approx}{x}_{ij} = -\underset{\approx}{x}_{ji}$ for all $i, j = 1, 2, ...n$. and $x_{ij} = 0$. i.e. $A^T = -A$.

4) Definition: Skew-Symmetric equivalent T2PFM

A square T2PFM $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ is said to be a skew-symmetric equivalent T2PFM if $\underset{\approx}{x}_{ij} = -\underset{\approx}{x}_{ji}$ for all $i, j = 1, 2, ...n$. and $x_{ij} = 0$. i.e. $A^T = -A$.

5) Definition: Diagonal T2PFM

Let $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ is said to be a diagonal T2TFM if T2PFM is square and all the elements outside the principal diagonal are 0.

6) Definition: Diagonal-equivalent T2PFM

Let $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ is a square T2PFM and it said to be a diagonal-equivalent T2PFM if all the elements outside the principal diagonal are zero.

7) Definition: Scalar T2PFM

Let a diagonal T2PFM $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ is said to be a scalar T2PFM if every entry $\underset{\approx}{x}_{ij}$ in the principal diagonal are the same.

8) Definition: Scalar–equivalent T2PFM

Let a diagonal-equivalent T2PFM $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ is said to be a scalar-equivalent T2PFM if the value of $\underset{\sim}{R}(\underset{\approx}{x}_{ij})$ is the same for every entry $\underset{\approx}{x}_{ij}$ in the principal diagonal.

9) Definition: Unit T2PFM or Identity T2PFM

Let $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ is a scalar T2PFM and it said to be an unit T2PFM or identity T2PFM if $\underset{\approx}{x}_{ij} = 1$ for every entry $\underset{\approx}{x}_{ij}$ in the principal diagonal. It is denoted by I.

10) Definition: Unit-equivalent T2PFM or Identity-equivalent T2PFM

Let $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ is a scalar-equivalent T2PFM and said to be an unit-equivalent T2PFM or identity-equivalent T2PFM if $\underset{\approx}{x}_{ij} = \underset{\approx}{1}$ for every entry $\underset{\approx}{x}_{ij}$ in the principal diagonal. It is denoted by $\hat{I}$.

11) Definition: Null T2PFM or Zero T2PFM

The $m \times n$ T2PFM with each entry 0 is called the null T2PFM or zero T2PFM. It is denoted by $\hat{O}$.

12) Definition: Null-equivalent T2PFM or Zero-equivalent T2PFM

The $m \times n$ T2PFM with each entry $\underset{\approx}{0}$ is called the null-equivalent T2PFM or zero-equivalent T2PFM. It is denoted by $\hat{O}$.

13) Definition: Upper triangular T2PFM

Let a square T2PFM $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ is called an upper triangular T2PFM if all the entries below the principal diagonal are 0.

14) Definition: Upper triangular-equivalent T2PFM

Let a square T2PFM $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ is called an upper triangular-equivalent T2PFM if all the entries below the principal diagonal are $\underset{\approx}{0}$.

15) Definition: Lower triangular T2PFM

Let a square T2PFM $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ is called a lower triangular T2PFM if all the entries above the principal diagonal are 0.

16) Definition: Lower triangular-equivalent T2PFM

Let a square T2PFM $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ is called a lower triangular equivalent T2PFM if all the entries above the principal diagonal are $\underset{\approx}{0}$.

17) Definition: Row T2PFM

Let a square T2PFM $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ and $m = 1$, i.e.) $1 \times n$ T2PFM is called a row T2PFM.

18) Definition: Column T2PFM

Let a square T2PFM $X = \left(\underset{\approx}{x}_{ij}\right)_{m\times n}$ and $n = 1$, i.e.) $m \times 1$ T2PFM is called a column T2PFM.

3.5. Some properties of type-2 pentagonal fuzzy matrices

3.5.1. Property

For any three T2PFMs, $A = \left(\underset{\approx}{a}_{ij}\right)$, $B = \left(\underset{\approx}{b}_{ij}\right)$ and $C = \left(\underset{\approx}{c}_{ij}\right)$ of order $m \times n$, then it holds the following:

1) $A + B = B + A$
2) $A + (B + C) = (A + B) + C$
3) $A + A = 2A$
4) $A - A = 0$ i.e. a null-equivalent T2PFM
5) $A + 0 = 0 + A = A$
6) $A - 0 = A$.

3.5.2. Property

Let A of B be two T2TFMs of the same order and p, q be two scalars, then:

1) $p(qA) = (pq)A$
2) $p(A+B) = pA + pB$
3) $p(A - B) = pA - pB$
4) $(p+q)A = pA + qA \ if \ p,q \geq 0.$

3.5.3. Property

If A and B are two T2PFMs and p, q are two scalars then:

1) $(A')' = A$
2) $(A+B)' = A' + B'$
3) $(pA)' = pA'$
4) $(pA+qB)' = pA' + qB'$

3.5.4. Property

1) The product of two upper triangular T2PFMs of order $n \times n$ is also an upper triangular T2PFM.
2) The product of two lower triangular T2PFMs of order $n \times n$ is also a lower triangular T2PFM.

4. Application on type-2 pentagonal fuzzy numbers (T2PFN)

In any language, proverbs are context based and culture conditioned. The same idea may be expressed in two different languages through two different approaches. As proverbs could have different shades of meaning, when it is being translated, uncertainty occurs. We propose a new algorithm to reduce this uncertainty and try to get an equivalent proverb in Hindi for a Tamil proverb. In the process, we use English as a medium to pass from Tamil to Hindi.

4.1. Proposed algorithm

Step 1: We chose a Tamil proverb and get its core meaning. The core meaning is expressed in terms of context meaning words. Through context meaning words, we identify the key words of the proverb and get the equivalent translation of these key words into English, if need be, with the help of synonyms. We get one or more cluster of key words.
Step 2: Using fuzzy linguistic scale, we assign primary membership value for translation and secondary membership value for context meaning words as Type-2 Pentagonal Fuzzy Numbers (T-2PFN) value.
Step 3: We use max-min composition and projection [1-2] to check the relation between two key words from different clusters. We get either one or more equivalent English proverbs through the key words which get the maximum value of the global projection. We recheck the core meaning of the translated English proverb which is equivalent to Tamil.
Step 4: Again, we proceed as in Step1, Step 2 and Step 3 to identify the equivalent core meaning of English proverb with context meaning words in Hindi, with the Hindi words or its synonymous, we arrive at different either one or more equivalent Hindi Proverbs. We choose proverb whose core meaning is equivalent to its English core meaning.
Step 5: We examine the proverbs in Tamil, English and Hindi for its core meaning. We have checked at every stage the corresponding core meanings. Thus the core meaning for Hindi proverb is the same for core meaning of the Tamil proverb.
Step 6: Finally, we verify our result using transitivity relation with the same membership value of the Type-2 Pentagonal Fuzzy Numbers (T-2PFN).

5. Illustrative example

Using Type-2 Pentagonal Fuzzy Numbers (T2PFN), we attempt to get equivalent proverb from two different languages through English. We consider Tamil as source language, Hindi as target language and English as medium language. Now choosing a Tamil proverb we elucidate its core meaning in Tamil. The core meaning is expressed through context meaning words in English. Using the context meaning words in English, we select the proverb in English that is equivalent to Tamil proverb. In order to get equivalent the translation of the word from Tamil to English we assign a primary membership of Type-2 Pentagonal Fuzzy Numbers (T2PFN) with the range [0, 1]. The
scale of linguistic term of the similarity of the translation as follows:

Table 1: Fuzzy Linguistic Scale of the Similarity of the Translation

Linguistic scale	Pentagonal fuzzy value
Dissimilar	{(0,0,0,0,0), (0,0,0,0,.1), (0,0,0,.1,.2)}
Very less similar	{(0,0,.1,.2,.3), (0,.1,.2,.3,.4)}
Less similar	{(.1,.2,.3,.4,.5), (.2,.3,.4,.5,.6)}
Almost similar	{(.3,.4,.5,.6,.7)}
Similar	{(.4,.5,.6,.7,.8), (.5,.6,.7,.8,.9)}
Highly similar	{(.6,.7,.8,.9,1), (.7,.8,.9,1,1)}
Exactly same	{(.8,.9,1,1,1), (.9,1,1,1,1), (1,1,1,1,1)}

Next we examine the contexts meaning of the translation of the word for the proverb with membership value. This has been considering the secondary membership value of Type-2 Pentagonal Fuzzy Numbers (T2PFN) with the range [0, 1]. To bring out the accuracy of the context meaning of the translated words, we assign the following denomination for the linguistic terms:

Table 2: Fuzzy Linguistic Scale of the Context Meaning of the Translated Words

Linguistic scale	Pentagonal fuzzy value
Meaningless	{(0,0,0,0,0), (0,0,0,0,.1), (0,0,0,.1,.2)}
Somewhat meaningful	{(0,0,.1,.2,.3), (0,.1,.2,.3,.4)}
Slightly meaningful	{(.1,.2,.3,.4,.5), (.2,.3,.4,.5,.6)}
Meaningful	{(.3,.4,.5,.6,.7)}
Highly meaningful	{(.4,.5,.6,.7,.8), (.5,.6,.7,.8,.9)}
Very highly meaningful	{(.6,.7,.8,.9,1), (.7,.8,.9,1,1)}
Exactly meaningful	{(.8,.9,1,1,1), (.9,1,1,1,1), (1,1,1,1,1)}

We, consider the following proverb from Tamil language. "*பதறிய காரியம் சிதறும்*"[9]. It is read as "Pathariya Kaariyam Sitharum". The core meaning of this proverb is expressed through context meaning words as "*அவசரத்தில் செய்யும் எந்த வேலையும் பாழாய்போகும்*" which means any work done in a hurry will become a waste. We identify the words "*அவசரம்*" (hurry) and "*பாழாய்போகும்*" (wasted) as the key words from this expression of the proverb. Then we translate these words into English and find it's synonymous. We assign membership values as mentioned earlier. Membership values of the translated words on T2PFNs are given in table 3.
Next we do the max-min composition and projection between two clusters of the translated key words of Tamil proverb. i.e. $T\underset{\approx}{A}_i \circ T\underset{\approx}{B}_j$ where $i = 1,2,3...,n$ and $j = 1,2,...,m;\ m,n \in N$. (shown in Table 4.)
We compare all global projection as, $\max\left(T\underset{\approx}{A}_i \circ T\underset{\approx}{B}_j\right)$. The words "Haste" and "Waste" are highly meaningful in context wise and we obtain $T\underset{\approx}{A}_3 \circ T\underset{\approx}{B}_1$ as maximum global projection as (0.1)(0.7, 0.8, 0.9, 1, 1).
Then we can choose an equivalent proverb in English which involves the words "Haste" and "Waste". That proverb is "Haste makes waste"[9] and the core meaning of this proverb is "Doing something too quickly causes mistakes that result in time, effort, material, etc., being wasted". We check the similarity between the core meaning of English and Tamil proverbs.

From the core meaning of English proverb we identify the key words of this proverb through context meaning words along with some similar words. “Quickly”, “Hurry”, “haste”, “Urgency”, “Loss” and “Waste” are the key words of the English proverb. Then we assign the membership value of these words as Type-2 Pentagonal Fuzzy Numbers (T2PFNs) value (see Table 5). We do the max-min composition and projection to get the words which is highly meaningful in context wise. Highly meaningful words “Haste” and “Waste” are chosen along with some similar words and translated into Hindi. i.e. Quickly - जल्दी जल्दी (jaldee jaldee); Hurry – जल्दी कीजिये (jaldee keejiye); Haste – जल्दी (jaldee); Urgency – तात्कालिकता (taatkaalikata); Waste – बेकार (bekaar); Loss – नुकसान (nukasaan). (see Table 7)

We do the same procedure as done for Tamil and English proverbs; we get the words “जल्दी” (jaldee) and “बेकार” (bekaar) which is highly meaningful in this context as seen in table 8.

Then we choose an equivalent proverb in Hindi which involves the words “जल्दी”(jaldee) and “बेकार”(bekaar). That proverb is “जल्दी का काम अच्छा नहीं होता” [9] (any work done in a hurry does not yield good result). And the core meaning of the proverb is “Work done in haste does not give better result. We check the equivalence of the core meaning of the Hindi proverb with the core meaning of English and Tamil proverbs. Since, Meaning 1 (Tamil) = Meaning 2 (English) = Meaning 3 (Hindi). Thus we get Hindi equivalent proverb for its Tamil version.

Next, we use transitivity relation between Tamil proverb to English proverb; English proverb to Hindi proverb with the same membership value on T2PFNs. We get the same result as the result obtained by max-min composition. (Shown in Table 9.)

Hence we verify that, the key words “Haste” and “Waste” are involving in both Tamil and Hindi proverbs. This result by transitivity relation is obtained using max-min composition.

Table 3: Membership Values of the Translated Key Words of the Tamil Proverb on T2pfns

Trans-late words	Primary Membership value of T2PFNs.	Secondary Membership value of T2PFNs.
Hurry	$\underset{\sim}{TA}_1 = \left(\mu_{TA_1}(0.3), \mu_{TA_1}(0.4), \mu_{TA_1}(0.5), \mu_{TA_1}(0.6), \mu_{TA_1}(0.7)\right)$	(.3(.3,.4,.5,.6,.7), .4(.3,.4,.5,.6,.7), .5(.2,.3,.4,.5,.6), .6(.3,.4,.5,.6,.7), .7(.3,.4,.5,.6, .7))
Urgency	$\underset{\sim}{TA}_2 = \left(\mu_{TA_2}(0.3), \mu_{TA_2}(0.4), \mu_{TA_2}(0.5), \mu_{TA_2}(0.6), \mu_{TA_2}(0.7)\right)$	(.3(0,.1,.2,.3,.4), .4(0,.1,.2,.3,.4), .5(0,0,.1,.2,.3), .6(.2,.3,.4,.5,.6), .7(.1,.2,.3,.4,.5))
Haste	$\underset{\sim}{TA}_3 = \left(\mu_{TA_3}(0.2), \mu_{TA_3}(0.3), \mu_{TA_3}(0.4), \mu_{TA_3}(0.5), \mu_{TA_3}(0.6)\right)$	(.2(.7,.8,.9,1,1), .3(.7,.8,.9,1,1), .4(.6,.7,.8,.9,1), .5(.5,.6,.7,.8,.9), .6(.3,.4,.5,.6,.7))
Quick	$\underset{\sim}{TA}_4 = \left(\mu_{TA_4}(0.1), \mu_{TA_4}(0.2), \mu_{TA_4}(0.3), \mu_{TA_4}(0.4), \mu_{TA_4}(0.5)\right)$	(.1(.3,.4,.5,.6,.7), .2(.3,.4,.5,.6,.7), .3(.5,.6,.7,.8,.9), .4(.5,.6,.7,.8,.9), .5(.4,.5,.6,.7, .8))
Waste	$\underset{\sim}{TB}_1 = \left(\mu_{TB_1}(0.1), \mu_{TB_1}(0.2), \mu_{TB_1}(0.3), \mu_{TB_1}(0.4), \mu_{TB_1}(0.5)\right)$	(.1(.8,.9,1,1,1), .2(.8,.9,1,1,1), .3(.7,.8,.9,1,1), .4(.7,.8,.9,1,1), .5(.8,.9,1,1,1))
Useless	$\underset{\sim}{TB}_2 = \left(\mu_{TB_2}(0.3), \mu_{TB_2}(0.4), \mu_{TB_2}(0.5), \mu_{TB_2}(0.6), \mu_{TB_2}(0.7)\right)$	(.3(.4,.5,.6,.7,.8), .4(.4,.5,.6,.7,.8), .5(.5,.6,.7,.8,.9), .6(.3,.4,.5,.6,.7), .7(.5,.6,.7,.8, .9))
Loss	$\underset{\sim}{TB}_3 = \left(\mu_{TB_3}(0.2), \mu_{TB_3}(0.3), \mu_{TB_3}(0.4), \mu_{TB_3}(0.5), \mu_{TB_3}(0.6)\right)$	(.2(.3,.4,.5,.6,.7), .3(.3,.4,.5,.6,.7), .4(.3,.4,.5,.6,.7), .5(.4,.5,.6.,7,.8), .6(.4,.5,.6,.7, .8))
Damage	$\underset{\sim}{TB}_4 = \left(\mu_{TB_4}(0.1), \mu_{TB_4}(0.2), \mu_{TB_4}(0.3), \mu_{TB_4}(0.4), \mu_{TB_4}(0.5)\right)$	(.1(0,0,.1,.2,.3), .2(0,0,.1,.2,.3), .3(0,.1,.2,.3,.4), .4(0,.1,.2,.3,.4), .5(.1,.2,.3,.4,.5))

Table 4: Value of the Max-Min Composition and Projection of Tamil Proverb

$\underset{\sim}{TA}_i \circ \underset{\sim}{TB}_j$	$\underset{\sim}{TB}_1$ = Waste	$\underset{\sim}{TB}_2$ = Useless	$\underset{\sim}{TB}_3$ = Loss	$\underset{\sim}{TB}_4$ = Damage
$\underset{\sim}{TA}_1$ = Hurry	0.1(.3,.4,.5,.6,.7)	0.1(.3,.4,.5,.6,.7)	0.2(.3,.4,.5,.6,.7)	0.1(.1,.2,.3,.4,.5)
$\underset{\sim}{TA}_2$ = Urgency	0.1(.2,.3,.4,.5,.6)	0.3(.2,.3,.4,.5,.6)	0.2(.2,.3,.4,.5,.6)	0.1(.1,.2,.3,.4,.5)
$\underset{\sim}{TA}_3$ = Haste	0.1(.7,.8,.9,1,1)	0.2(.5,.6,.7,.8,.9)	0.2(.4,.5,.6,.7,.8)	0.1(.1,.2,.3,.4,.5)
$\underset{\sim}{TA}_4$ = Quick	0.1(.5,.6,.7,.8,.9)	0.1(.5,.6,.7,.8,.9)	0.1(.4,.5,.6,.7,.8)	0.1(.1,.2,.3,.4,.5)

Table 5: Membership Values of the Key Words of the English Proverb on T2PFNs

Translate words	Primary Membership value of T2PFNs.	Secondary Membership value of T2PFNs.
Quickly	$\underset{\sim}{EA}_1 = \left(\mu_{A_1}(0.8), \mu_{A_1}(0.9), \mu_{A_1}(1), \mu_{A_1}(1), \mu_{A_1}(1)\right)$	(.8(.8,.9,1,1,1),.9(.8,.9,1,1,1),1(.7,.8,.9,1,1),1(.6,.7,.8,,9,1),1(.7,.8,.9,1,1))
Hurry	$\underset{\sim}{EA}_2 = \left(\mu_{A_2}(0.7), \mu_{A_2}(0.8), \mu_{A_2}(0.9), \mu_{A_2}(1), \mu_{A_2}(1)\right)$	(.7(.8,.9,1,1,1),.8(.8,.9,1,1,1),.9(.7,.8,.9,1,1),1(.6,.7,.8,,9,1),1(.7,.8,.9,1,1))
Haste	$\underset{\sim}{EA}_3 = \left(\mu_{A_3}(0.8), \mu_{A_3}(0.9), \mu_{A_3}(1), \mu_{A_3}(1), \mu_{A_3}(1)\right)$	(.8(.8,.9,1,1,1),.9(.9,1,1,1,1),1(.8,.9,1,1,1),1(.8,.9,1,1,1),1(.8,.9,1,1,1))
Urgency	$\underset{\sim}{EA}_4 = \left(\mu_{A_4}(0.5), \mu_{A_4}(0.6), \mu_{A_4}(0.7), \mu_{A_4}(0.8), \mu_{A_4}(0.9)\right)$	(.5(.5,.6,.7,.8,.9),.6(.5,.6,.7,.8,.9),.7(.5,.6,.7,.8,.9),.8(.5,.6,.7,.8,.9),.9(.6,.7,.8,.9,1))
Waste	$\underset{\sim}{EB}_1 = \left(\mu_{B_1}(0.8), \mu_{B_1}(0.9), \mu_{B_1}(1), \mu_{B_1}(1), \mu_{B_1}(1)\right)$	(.8(.8,.9,1,1,1),.9(.9,1,1,1,1),1(.8,.9,1,1,1),1(.8,.9,1,1,1),1(.8,.9,1,1,1))

Loss	$\underset{\approx}{EB}_2 = \left(\mu_{\underset{\approx}{B}_2}(0.8), \mu_{\underset{\approx}{B}_2}(0.9), \mu_{\underset{\approx}{B}_2}(1), \mu_{\underset{\approx}{B}_2}(1), \mu_{\underset{\approx}{B}_2}(1)\right)$	(.8(.5,.6,.7,.8,.9),.9(.5,.6,.7,.8,.9),1(.5,.6,.7,.8,.9),1(.5,.6,.7,.8,.9),1(.5,.6,.7,.8,.9))

Table 6: Value of the Max-Min Composition and Projection of English Proverb

$\underset{\approx}{EA}_i \circ \underset{\approx}{EB}_j$	$\underset{\approx}{EB}_1$ = Waste	$\underset{\approx}{EB}_2$ = Loss
$\underset{\approx}{EA}_1$ = Quickly	0.8(0.8,0.9,1,1,1)	0.8(0.5,0.6,0.7,0.8,0.9)
$\underset{\approx}{EA}_2$ = Hurry	0.7(0.8,0.9,1,1,1)	0.7(0.5,0.6,0.7,0.8,0.9)
$\underset{\approx}{EA}_3$ = Haste	0.8(0.9,1 1,1,1)	0.8(0.5,0.6,0.7,0.8,0.9)
$\underset{\approx}{EA}_4$ = Urgency	0.5(0.6,0.7,0.8,0.9,1)	0.5(0.5,0.6,0.7,0.8,0.9)

Table 7: Membership Values of the Key Words of the Hindi Proverb on T2PFNs

Translate words	Primary Membership value of T2PFNs.	Secondary Membership value of T2PFNs.
Quickly, Hurry, Haste – जल्दी , जल्दी कीजिये	$\underset{\approx}{HA}_1 = \left(\mu_{\underset{\approx}{HA}_1}(0.6), \mu_{\underset{\approx}{HA}_1}(0.7), \mu_{\underset{\approx}{HA}_1}(0.8), \mu_{\underset{\approx}{HA}_1}(0.9), \mu_{\underset{\approx}{HA}_1}(1)\right)$	(.6(.7,.8,.9,1,1), .7(.7,.8,.9,1,1), .8(.8,.9,1,1,1), .9(.7,.8,.9, 1,1), 1(.6,.7,.8,.9,1))
Urgency - तात्कालिकता	$\underset{\approx}{HA}_2 = \left(\mu_{\underset{\approx}{HA}_2}(0.4), \mu_{\underset{\approx}{HA}_2}(0.5), \mu_{\underset{\approx}{HA}_2}(0.6), \mu_{\underset{\approx}{HA}_2}(0.7), \mu_{\underset{\approx}{HA}_2}(0.8)\right)$	(.4(.3,.4,.5,.6,.7), .5(.3,.4,.5,.6,.7), .6(.5,.6,.7,.8,.9), .7(.6,.7,.8,.9,1), .8(.6,.7,.8,.9,1)
Waste - बेकार	$\underset{\approx}{HB}_1 = \left(\mu_{\underset{\approx}{HB}_1}(0.7), \mu_{\underset{\approx}{HB}_1}(0.8), \mu_{\underset{\approx}{HB}_1}(0.9), \mu_{\underset{\approx}{HB}_1}(1), \mu_{\underset{\approx}{HB}_1}(1)\right)$	(.7(.7,.8,.9,1,1), .8(.6,.7,.8,.9,1), .9(.5,.6,.7,.8,.9), 1(.5,.6,.7,.8,.9), 1(.5,.6,.7,.8,.9))
Loss – नुकसान	$\underset{\approx}{HB}_2 = \left(\mu_{\underset{\approx}{HB}_2}(0.3), \mu_{\underset{\approx}{HB}_2}(0.4), \mu_{\underset{\approx}{HB}_2}(0.5), \mu_{\underset{\approx}{HB}_2}(0.6), \mu_{\underset{\approx}{HB}_2}(0.7)\right)$	(.3(.5,.6,.7,.8,.9), .4(.5,.6,.7,.8,.9), .5(.3,.4,.5,.6,.7), .6(.3,.4,.5,.6,.7), .7(.3,.4,.5,.6,.7)

Table 8: Value of the Max-Min Composition and Projection of Hindi Proverb

$\underset{\approx}{HA}_i \circ \underset{\approx}{HB}_j$	$\underset{\approx}{HB}_1$ = बेकार (Waste)	$\underset{\approx}{HB}_2$ = नुकसान (Loss)
$\underset{\approx}{HA}_1$ = जल्दी , जल्दी कीजिये (Quickly, Hurry, Haste)	0.6(0.7, 0.8, 0 .9, 1, 1)	0.3(0.5, 0.6, 0.7, 0.8, 0.9)
$\underset{\approx}{HA}_2$ = तात्कालिकता (Urgency)	0.4(0.6, 0.7, 0.8, 0.9, 1)	0.3(0.5, 0.6, 0.7, 0.8, 0.9)

Table 9: Transitivity Relation between Tamil and Hindi Proverb

$\underset{\approx}{TA}_i \circ \underset{\approx}{HA}_j$	$\underset{\approx}{HA}_1$ = Quickly, Hurry, Haste – जल्दी	$\underset{\approx}{HA}_2$ = Urgency - तात्कालिकता
$\underset{\approx}{TA}_1$ = Hurry	0.3(0.3, 0.4, 0.5, 0.6, 0.7)	0.3(0.3, 0.4, 0.5, 0.6, 0.7)
$\underset{\approx}{TA}_2$ = Urgency	0.3(0.2, 0.3, 0.4, 0.5, 0.6)	0.3(0.2, 0.3, 0.4, 0.5, 0.6)
$\underset{\approx}{TA}_3$ = Haste	0.2(0.7, 0.8, 0.9,1, 1)	0.2(0.5, 0.6, 0.7, 0.8, 0.9)
$\underset{\approx}{TA}_4$ = Quick	0.1(0.5, 0.6, 0.7, 0.8, 0.9)	0.1(0.5, 0.6, 0.7, 0.8, 0.9)
$\underset{\approx}{TB}_i \circ \underset{\approx}{HB}_j$	$\underset{\approx}{HB}_1$ = waste – बेकार (bekaar)	$\underset{\approx}{HB}_2$ = loss – नुकसान (nukasaan)
$\underset{\approx}{TB}_1$ = Waste	0.1(0.7, 0.8, 0.9,1,1)	0.1(0.5, 0.6, 0.7, 0.8, 0.9)
$\underset{\approx}{TB}_2$ = Useless	0.3(0.5, 0.6, 0.7, 0.8, 0.9)	0.3(0.5, 0.6, 0.7, 0.8, 0.9)
$\underset{\approx}{TB}_3$ = Loss	0.2(0.4, 0.5, 0.6, 0.7, 0.8)	0.2(0.4, 0.5, 0.6, 0.7, 0.8)
$\underset{\approx}{TB}_4$ = Damage	0.1(0.1, 0.2, 0.3, 0.4, 0.5)	0.1(0.1, 0.2, 0.3, 0.4, 0.5)

6. Conclusion

To analyze the complex structural representation and categorization of words, we have introduced Type-2 Pentagonal Fuzzy Number. Translating a proverb from one language to the other is more complex exercise. In order to resolve this complexity, we use Type-2 Pentagonal Fuzzy numbers which is a convenient tool that results in finding equivalent proverbs from two different Indian Languages (IL), namely Hindi and Tamil. Type-2 Pentagonal Fuzzy Numbers gives an equivalent proverb.

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