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Optimization A New Mathematical Model of Hybrid Flow Shop Work Order Problem By The Genetic Algorithm

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Abstract

In competitive business, one of the challenges of management in industrial units is reducing the prime cost. Hybrid flow shop is one of the common production environments which lead to a significant decrease in production costs if it has a good and appropriate scheduling in production. Hybrid flow-shop problems overcome one of the limitations of the classical flow-shop model by allowing parallel processors at each stage of task Processing. In this paper we study the hybrid flow shop work order problems. A brief enumeration of the essential constraints that characterize this kind of organization is given. Problem is minimizing the production and inventory cost in Hybrid flow-shop organization. To solve problem we used the genetic algorithm to obtain the minimum of production cost. An illustrative example explains in detail the feature of the proposed model.

Keywords: *hybrid flow shop ,mathematical model, production cost,inventory cost, genetic algorithm.*

1 Introduction

One of the managers' problems is reduce production costs in the management of manufacturing companies. Hybrid Flow Shops (HFS) is common manufacturing environments in which a set of n jobs are to be processed in a series of w stages. There are two important questions that should be considered in scheduling, including. First, when can we produce a product? and Second , How much is produce? The first question is in relation with production scheduling problems.Most research focus on scheduling problems. So far, many literatures are available for scheduling problems with limited buffers. Hall and Sriskan darajah N. G. Hall, C. Sriskan darajah [9] provided a survey for scheduling problems with blocking and no-wait in process. M. S. Salvador[10] first considered hybrid flow shop with no buffers between stages. He applied branch-and-bound techniques to minimize makespan. C. Rajendran, D. Chaudhuri[11]utilized branch-and-bound but they also focus only onpermutation schedules. S. A. Brah, J. L. Hunsucker[12] used branch-and-bound in the hybrid flow shop with an arbitrary number of stages and intermediate buffers.Moreover, they provide a means by which nonpermutation schedules or schedules with inserted idle time can be created.T. Sawik [15]presented a mixed integer programming formulation for scheduling flexibleflow line with limited buffers and also a FFS with limited intermediate buffers T. Sawik [13]and with no-process buffers.T. Sawik[14]. Torabi, B. Karimi [7]proposed two meta-heuristic algorithms, including the HGA and simulated annealing (SA), to solve a new 0–1mixed-nonlinear mathematical model of the economic lot-sizing and scheduling problem in flexible flow lines with unrelated parallel processors over a finite planning horizon. The objective determines a cyclic schedule by minimizing the sum of setup and inventory holding costs per unit time without any stock-out. Akrami et al. B. Akrami, B. Karimi, SM. Moattar-Hosseini[8]developed two heuristic approaches, including GA and an optimal enumeration method (OEM), to solve a new model of common cycle multi-product lot-sizing and scheduling problem in deterministic flexible flow shops with a finite planning horizon and limited intermediate buffers. The objective minimizes the sum of setup cost, inventory holding costs, and number of cycles. F. Riane, A. Artiba, S. Iassinovski [16] paid to the interaction between loading and scheduling problem in HFS environment. M.R. Garey, D.S. Johnson, R. Sethi [17] shows the optimal allocation of resources to activities over time is known to be combinatorially complex.

The HFS problem is, in most cases, NP-hard. For instance, HFS restricted to two processing stages, even in the case when one stage contains two machines and the other one a single machine, is NP-hard, after the results of J. N. D. Gupta [1]. Moreover, the special case where there is a single machine per stage, known as the flow shop, and the case where there is a single stage with several machines, known as the parallel machines environment, are also NP-hard, M. R. Garey and D. S. Johnson[2].

This study considers achieving optimal production volume in order to reduce production and inventory costs. Our main objective is to find a good allocation of jobs which generates a lower inventory level, a high plant efficiency and in which machines capacities are respected. The inventory level concerns in-process- stock and finished goods inventory. The efficiency of the plant means high machine utilization . This paper is organized as follows. In Section 2, we present the problem definition. In Section 3, we present the problem formulation. A solution procedure is introduced In Section 4. A to illustrate the propose approach, some examples are presented in Section 5.

2 Hybrid Flow Shop

A Hybrid Flow Shop (HFS) consists of series of production stages, each of which has several machines operating in parallel, that can be identical, uniform or unrelated. Some stages may have only one machine, but at least one stage must have multiple machines. The flow of jobs through the shop is unidirectional. Each job is processed by one machine in each stage and it must go through one or more stage (see fig.1)

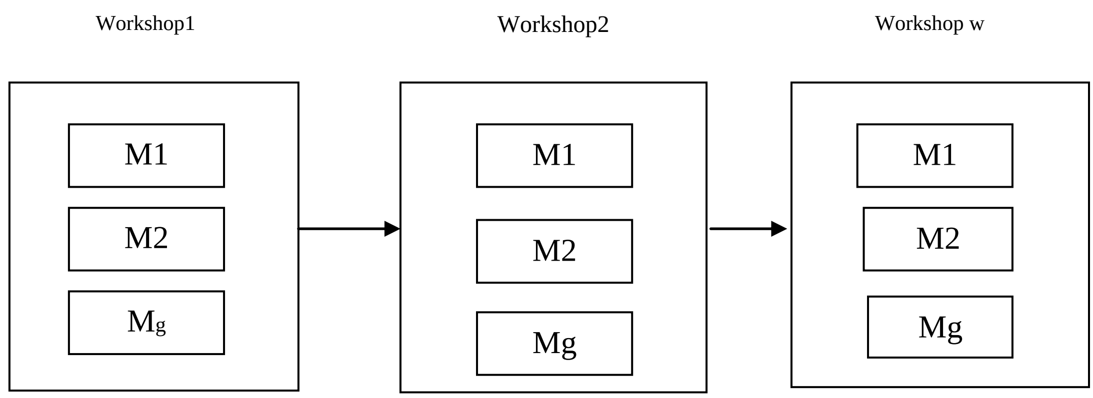

Fig.1.A structure of a hybrid flow shop organization

A job consists of several operations to be performed by none, one or more machines on each stage. The i-th operation of a products (jobs) $i\_\{i / i=1, 2,\ldots, n\}$ to be performed at the w-th workshop that Each workshop consists of gw centers of mg identical machines , requires units of time and can start only after the completion of the previous operation from the operation sequence of this job. Between the various stages of an hybrid flow shop environment there can be a definite amount of space, called buffer. This is a location in where the workflow can wait to be worked in the next stage. In the standard problem all jobs and machines are available at time zero, machines at a given stage are identical, any machine can process only one operation at a time and any job can be processed by only one machine at a time; setup times are negligible, preemption is not allowed, the capacity of buffers between stages is unlimited and problem data is deterministic and know in advance. A flow diagram of production planning and scheduling system is given in fig.2. In this paper, we presented a new mathematical model for the allocation jobs to each machine in each stage to optimize production and inventory costs in production planning process.

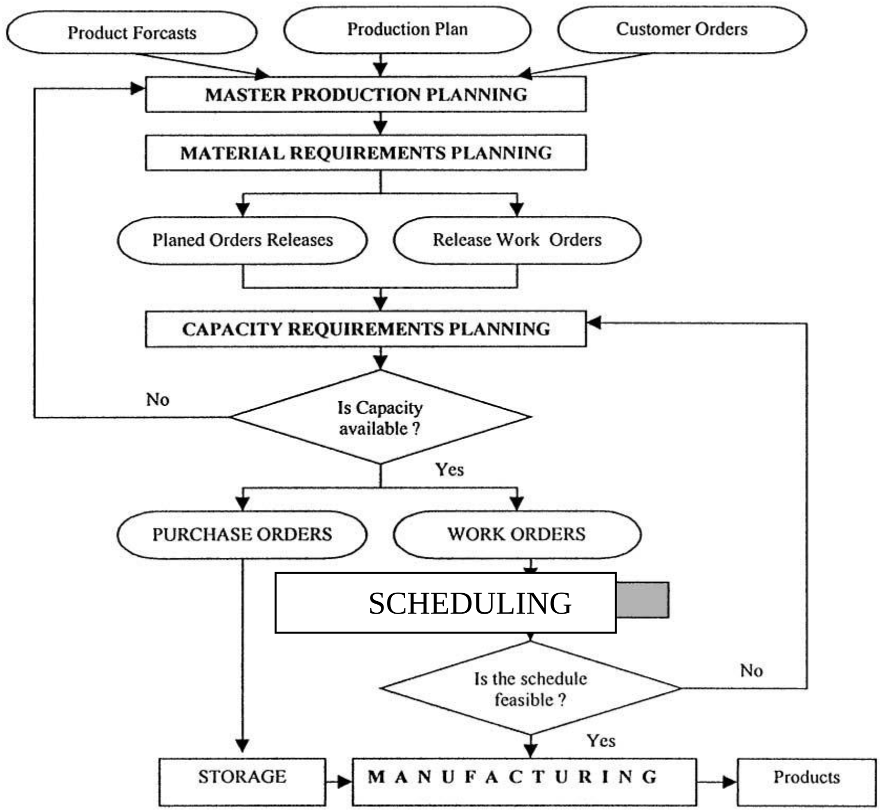

Fig.2. A flow diagram of production planning and scheduling system

3 Mathematical Model

The aims of this paper are minimize the:

- The total cost of production in t period.
- The total cost of inventory in the t period.
- The total cost of external production in the t period,

and model assumptions are:

- The available time for each workshop is determined.
- The cost of internal production of products in each workshop is constant and deterministic.
- The cost of external production of products i is constant and deterministic.

- Demand is constant and deterministic.
- Initial inventory level of products is determined.
- Available space for holding products for the period is specified.

Notations used for the problem formulation are as follow:

i :Product Type
j :workshop type
m :The number of machines in each workshop
t :The number of time periods.
X_{ijmt} : Amount of producing product i th in workshop j th with machine (m) in period t
Z_{it} :The quantity of external product i at period t
Y_{it} :The stock level of product i by the end of period t.
C_{ijmt} : Cost of producing product i th in workshop j th with machine (m) in period t.
CZ_{it} :The cost of external product i at period t.
CY_{it} :The inventory holding cost of product i
t_{ijmt} : Time of producing product i th in workshop j th with machine(m)in period t.
p_{jmt} :Time available for machine m in the workshop j
D_{it} : The demand of product i in period t.
fi : Available space for product i for stock .

The problem formulated by:

$$\min z = \sum_t \sum_j \sum_m \sum_i C_{ijmt} * X_{ijmt} + \sum_t \sum_i CZ_{it} * Z_{it} + \sum_t \sum_i CY_{it} * Y_{it} \quad (1)$$

$$Y_{i0} = Y_i \quad \forall i \quad (2)$$

$$\sum_i t_{ijmt} * X_{ijmt} \le P_{ijt} \quad \forall m, j, t \quad (3)$$

$$\sum_t \sum_i X_{ijmt} + Y_{i0} + Z_{it} + Y_{it-1} \ge D_{it} \quad \forall i, t \quad (4)$$

$$\sum_t \sum_i Y_{it} \le F \quad (5)$$

$$x_{ijmt} \ge 0 \,, Y_{it} \ge 0 \,, Z_{it} \ge 0$$

The objective function (1) minimizes the sum of internal production, external procurement, and inventory carrying costs. Equation (2) shows the initial inventory level at the time of beginning producing. Equation (3) shows the available time for each machine in each work station in any time period. Equation (4) shows minimal production level to meet the demands at any time period for each product. Equation (5) shows the maximum available space for keeping the inventory at end of each production period.

4 Genetic Algorithms for Problem Solving

In this paper, the genetic algorithm is used in optimization process. The concept of GA was developed by Holland and his colleagues in the 1960s and 1970s [18]. The ability of GA to simultaneously search different regions of a solution space makes it possible to find a diverse set of solutions for difficult problems with non-convex, discontinuous, and multi-modal solutions spaces.

4.1 Chromosome structure

In this paper, the chromosomes shows the quantity of job that allocated to each machine in each stage. At first, variables are encoded into binary threads. If variable u_j is in $[a_j, b_j]$, then number of genes are repeat shown as w_j and are calculated through equation (6).

$$2^{wj-1} \le (b_j - a_j) \times 10^{+k} \le 2^{wj} - 1 \tag{6}$$

That k is the accuracy of variable k.To convert a binary string to real numbers, we used equation 7.

$$u_j = a_j + \text{decimal (substring)} \times \frac{b_j - a_j}{2^{wj} - 1} \tag{7}$$

For example, let two variables u_1, u_2 regarding range [-3, 12.1]and [4.1 , 5.1]. If k=3 then:

$$For\ u_1 : 2^{17} \le (12.1 - (-3)) \times 10^{3} \le 2^{18} - 1 \rightarrow w_1 = 18$$

$$For\ u_2 : 2^{14} \le (5.8 - 4.1) \times 10^{3} \le 2^{15} - 1 \rightarrow w_2 = 15$$

As a result,the total chromosome length is 33 bits.

Genetic algorithm starts with a random initial population. The population is normally randomly initialized. The number of chromosomes inside the population is also need to be decided, because the number of the solutions determines the speed of the optimization and the accuracy of the solution found. If too many solutions are generated in the population, then longer duration of time is needed to find the fittest optimization. But, if the number of the solutions is too little or small, the genetic algorithm may face the problem of finding the fittest optimizations.

4.2 Fitness functions and selection mechanism

The selection mechanism is one of the main search components in evolutionary algorithms. Since the solution space is very big, this study used normalizing technique to select parents to produce next generation. Equation 8 and 9 shows standard deviation of each generation and standard fitness function of each chromosome in each generation, respectively.

$$\sigma_g = \left(\frac{\sum_{i=1}^{n}(\mathrm{fit}_{(i)} - m_g)^2}{n}\right)^{1/2} \tag{8}$$

$$z_i^g = \frac{\mathrm{Fit(i)} - m_g}{\sigma_g} \tag{9}$$

Which 6_g, mg are mean and standard deviation of fitness functions in generation *g* th and fit (i) is fitness function of chromosome *i*th in generation (g) th. After calculating the z_i^g ,if $z_i^g \le 0$,we selected chromosome i for next generation Because the problem goal is minimizing .(see Table.1).

Table.1. Mean and variance in generation *g* and *g+1*

generation *g+1*			generation *g*		
Normalized fitness	Fit	Chromosome	Normalized fitness	Fit	Chromosome
$z_1^{(g+1)}$	f_1^{g+1}	u_1^{g+1}	z_1^y	f_1^g	u_1
z_2^{g+1}	f_2^{g+1}	u_2^{g+1}	z_2^g	f_2^g	u_2
. . .	. . .	. . .	. . .	. . .	. . .
z_n^{g+1}	f_n^{g+1}	u_n^{g+1}	z_n^g	f_N^g	u_n^g
m_g ، σ_g		mean and variance	m_g , σ_g		mean and variance

4.3 Design the genetic operators

GA use two operators to generate new solutions from existing ones: crossover and mutation. The crossover operator is the most important operator of GA. In crossover, generally two chromosomes, called parents, are combined together to form new chromosomes, called offspring. In this study for produce offspring from parents for entering the next generation ,we used one-point crossover. For instance, in a intersection with two phases, two random chromosomes with feasible genes can be as follows:

Parent 1= [0,1,1,1,0,0,1,0,0,1,1,0,0,0]
Parent 2= [1,1,0,0,1,1,1,1,0,0,1,1,0,1]

The offspring produced from these parents are as follows:

ONE-cut-point crossover with random points (e1=9).

Offspring 1= [0,1,1,1,0,0,1,0,0,0,1,1,0,1]

Offspring 2= [1,1,0,0,1,1,1,1,0, 1,1,0,0,0]

The mutation operator used to be in order to create change, diversity and divergence in the population mutation operator in the classic mode,which means changing the values of two distinct genes(Mutations Replacement) together, or change the amount of the permissible scope of asingle gene(mutation changes). For instance, one point mutation with random points (e1=6).

Parent = [0,1,1,1,0,0,1,0,0,1,1,0,0,0]

Offspring= [0,1,1,1,0,1,1,0,0,1,1,0,0,0]

The main steps of the Solution procedure are as follows:

Step 1: Set g =1. Randomly generate N solutions to form the first population, p_g .

Step 2: Evaluate the fitness of solutions in p_g .

Step 3: Selecting chromosome from the initial population with equation 8 and 9.

Step 4: Crossover and mutation functions, reproducing the offspring and adding them to the initial population.

Step 5: g=g+1and repeating steps 2-4 until a termination condition has been reached.

5 Illustrative Example

We consider the two-workshop of hybrid flowshop organization .Each workshop consist two different centers of machines (see fig.3). The model is programmed in the Microsoft Excel 2007 software using the Visual Basic Application (VBA).The problem data given in tables. 2, 3 and 4 is entered in the application software.

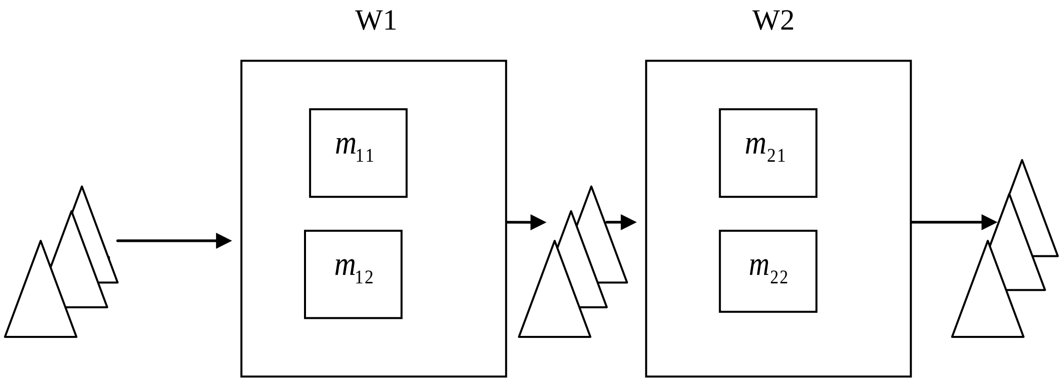

Fig.3.Description of the production process

Table.3.Internal production cost for each product

Product	1	2	3
m_{11}	300	800	150
$\mathbf{m}_{12}$	300	800	150
$\mathbf{m}_{21}$	800	400	500
$\mathbf{m}_{22}$	800	400	500

Table.2.The processing times in the workshop

Product	1	2	3	capacity (min)
m_{11}	10	15	30	3200
$\mathbf{m}_{12}$	6	20	25	3200
$\mathbf{m}_{21}$	25	40	15	3200
$\mathbf{m}_{22}$	30	10	20	3200

Table.4.External production cost for each product

Product	1	2	3
cost	2000	2000	2000

Table.7. The initial inventory level, demands, available space and holding cost for the jobs

Product	1	2	3
y_{i0}	30	30	30
CY_{it}	4000	4000	4000
fi	60	60	60
D_{it}	80	80	60

The previous tables summarize the necessary data to be used in optimization process. Table 2 represents the processing times in the workshop 1and 2.Table 3 represents internal production cost for each product of the workshop 1and 2.table 4 represents external production cost for each product and table 5 represents the initial inventory level , demands , available space and holding cost. We assumed the production capacity of each center of each workshop equal to 3200 minutes .Our main objective is to find a optimal work order generates a lower inventory level,with lower internal and external production cost.

5.1 Computing Results

These algorithms are coded and implemented in Excel 2007 by the VBA and are ran on a Pentium 4 PC with CPU 2.8 GHz and512 MB of RAM memory. Problem coded for both time periods t = 1(the feasible solution equal to 2^96) and t =2(the feasible solution equal to 2^196) . (see table 8,9 and 10)

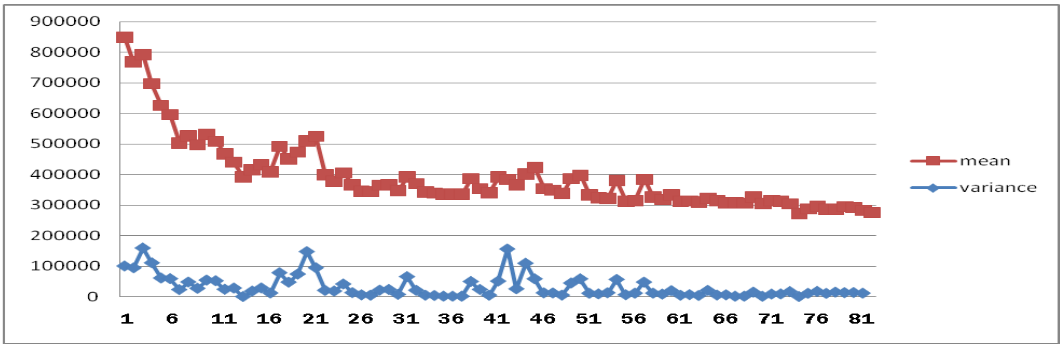

Fig.4. Mean and variance value with increasing the number of generations**.**

Reducing of fitness functions, variances in each repetition shows converging of the answers .due to the fact that reducing fitness functions variances show more similarities of chromosomes in each generation. Reduction of fitness functions mean in each repeat shows moving toward optimality.

Table.8.Sensitivity analysis for t=1

Generation	population	dimensions	objective function	Time(s)	Mean	Variance
400	100	18*10	399907	107	631475	75266
400	200	18*10	384308	205	526948	74599
400	300	18*10	331062	275	512208	71008
400	400	18*10	330103	390	510001	70521
400	600	18*10	290520	410	400098	70498

Table.9.Sensitivity analysis for t=2

Generation	population	dimensions	objective function	Time(s)	Mean	Variance
400	50	36*31	2153494	130	2608258	223452
400	200	36*31	1630049	240	1847430	102148
1000	400	36*31	1578420	375	1728064	122608
1000	800	36*31	1617845	415	1775159	91647
1500	900	36*31	1419179	540	1710210	81962

Table.10.Best solution with GA

t=1	x1111	x1211	x1112	x1212	x2111	x2211	x2112	x2212	x3111	x3211	x3112	x3212
	5	44			18	1			29	64		
	x1121	x1221	x1122	x1222	x2121	x2221	x2122	x2222	x3121	x3221	x3122	x3222
	76	37			36	54			12	8		
	y11	3	y12		y21	1	y22		y31	1	y32	
	z11	0	z12		z21	30	z22		z31	19	z32	
t=2	x1111	x1211	x1112	x1212	x2111	x2211	x2112	x2212	x3111	x3211	x3112	x3212
	90	45	50	120	24	20	39	10	12	20	31	43
	x1121	x1221	x1122	x1222	x2121	x2221	x2122	x2222	x3121	x3221	x3122	x3222
	6	51	128	18	56	60	60	89	11	13	34	23
	y11	61	y12	42	y21	10	y22	88	y31	37	y32	130
	z11	45	z12	17	z21	1	z22	14	z31	75	z32	36

6 Conclusions

In this paper, we proposed and optimization mathematical model of hybrid flow shop work order problem. . To solve the given problem, we applied the GA for solving the presented model. The high speed of the proposed algorithm and its quick convergence makes it desirable for large hybrid flow shop work order problem. Reduce the production costs of operational strategies and goals is the most productive institutions. For this purpose, we have optimized the production and inventory costs of each of the products produced in each period. However, as the problem is classified as NP-hard, it is very difficult to achieve optimal solutions using the integer programming(IP)model for larger problems. Therefore, we also introduce a genetic algorithm to efficiently find a good solution for the problem.

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