International Journal of Engineering & Technology, 7 (3.2) (2018) 667-671

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

The Definition of the Optimal Energy-Efficient form of the Building

Oleg Sergeychuk[1], Viacheslav Martynov[2]*, Dmytro Usenko[3]

[1]*Kyiv National Institute Of Building And Architecture, Ukraine*
[2]*Kyiv National Institute Of Building And Architecture, Ukraine*
[3]*Poltava National Technical Yuri Kondratyuk University, Ukraine*
**Corresponding Author E-Mail: Arx.Martynov@Gmail.Com*

Abstract

We consider the problem of finding a geometrical form of a body in a thermal radiation field, for which the thermal balance between the body and the surrounding air is minimal. The case of a point source of heat is investigated. To consider an analogous problem for buildings, one must know the value of incoming thermal energy to a unit square in relation to its orientation. We develop an application package in MATLAB that represents this relation in table form and takes into consideration the direct, diffuse, ground-reflected solar radiation and the thermal radiation of atmosphere.

Key words: *form, geometry model, optimization, solar radiation, thermal radiation.*

1. Introduction

We study how the form of a building implies on its energy-conservation attributes. Not by accident that houses of aborigines on territories with very hot or very cold climate have a semi-spherical form. Interest in energy-conservation buildings has increased in the past few years because of depletion of traditional sources of energy [16].
The problem of minimizing the thermal balance between a building and the surrounding air is considered in [14,15] for buildings having the form of parallelogram, triangular pyramid, or circular cylinder. To study this problem one must know the value of incoming thermal energy to a unit square in relation of its orientation. There are many mathematical models for this process. Some of them are primitive; they either drop essential physical factors (orientation, feculence of atmosphere, and altitude), or are based on particular data [3-6]. Others are very complicated and the speed of modern computers is insufficient for their solution [7].
It is important to construct a high-speed computer model of heat transmission from the Sun and atmosphere on an arbitrarily oriented plane. This problem is important for optimization of the geometrical form not only of buildings, but also of solar hot-water generators [10] and concentrators of solar energy [1]. An analogous problem arises in agriculture: to find the best time of planting on slopes [11].
Problems of geometrical modeling of transmission of direct solar radiation are the most extensively studied, mainly for the clear sky; investigators consider only the probability of solar days [8, 12, 17].
The paper is organized as follows. In Section 1 we consider the problem of optimizing the form of a geometrical body of a fixed volume that is heated by a light source located infinitely far away. In Section 2 we consider an analogous problem but for a heat source located at some fixed distance. In Sections 3 and 4 we construct models of receipt of atmosphere, ground-reflected, and solar thermal radiation.

2. Main Body

1. Body from optimization heated by an infinitely far located source

Let us consider a geometrical body Q of a fixed volume V that is radiated by a heat source P located infinitely far away. Let p be the value of thermal energy transmitted from P to a unit square that is perpendicular to its rays. Let q be the value of thermal energy radiated by a unit square on the surface of Q.
Let the form of this body be determined by the following condition: its thermal balance Δ is 0 or a negative number maximally close to 0.
Then the equation of thermal balance has the form:

$$\Delta = pS_{\perp} - qS, \qquad (1)$$

in which $S_{\perp}$ is the projection of Q to a plane that is perpendicular to beams of P and S is the surface area of Q. It can be proved that Q is a body of rotation around an axis parallel to thermal beams of P; the form of Q depends on p and q as illustrated in Fig. 1:

(i) If $p = 0$ then $\Delta < 0$ and Q is the sphere of radius

$$R = \sqrt[3]{\frac{3V}{4\pi}}\ ; \qquad (2)$$

(ii) if $0 < p < 2q$ then $\Delta < 0$ and Q is the body formed by two mutually symmetric spherical segments, each of height h and the common base of radius r, where

Copyright © 2018 Authors. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

$$h = \sqrt[3]{\frac{3}{2\pi} \cdot V \cdot \frac{2q - p}{4q + p}}\,, \qquad r = \sqrt{\frac{3V - \pi h^3}{3\pi h}}\,; \qquad (3)$$

(iii) if $p = 2q$ then $\Delta = 0$ and Q degenerates to the circle of infinite radius that is perpendicular to beams of the source P;

(iv) if $2q < p < 4q$ then $\Delta = 0$ and Q has the same form as in case (ii) with

$$h = \sqrt[3]{\frac{3}{\pi} \cdot V \cdot \frac{p - 2q}{4q + p}}\,, \qquad r = \sqrt{\frac{3V - \pi h^3}{3\pi h}}\,; \qquad (4)$$

(v) If $p = 4q$ then $\Delta = 0$ and Q is a sphere as in case (i);

(vi) If $p > 4q$ then $\Delta = 0$ and Q is the circular cylinder of radius r and height h with two semicircles of the same radius on its ends, where segments on tops.

$$h = \frac{p - 4q}{2q}\sqrt[3]{\frac{6Vq}{\pi(3p - 4q)}}\,, \quad r = \sqrt[3]{\frac{6Vq}{\pi(3p - 4q)}}\,. \qquad (5)$$

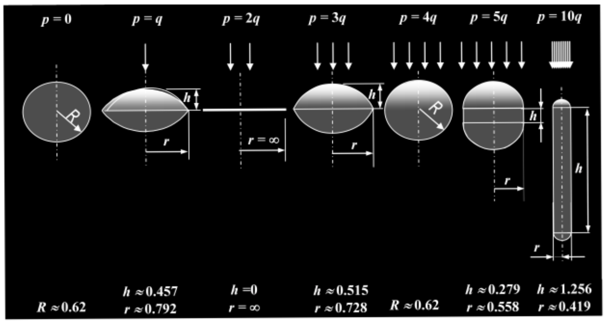

Fig.1. Body form change of a unit volume that has minimal thermal balance with the surrounding air, depending on the intensity of an external infinitely located heat source.

2. Body form optimization heated by a source located at a fixed distance.

Now let a source P be located at some finite distance H from the weight center O of a body Q and volume V. Let the source radiate heat stream p.

It is proved that Q is a rotational body inscribed in a cone Ω with axis of rotation PO and the top point P. Then the thermal balance equation is

$$\Delta = \frac{p}{2}(1 - \cos\gamma) - qS\,, \qquad (6)$$

in which γ is the angle between PO and a generating line of Ω. The body Q is the minimum area body inscribed in the cone Ω. It consists of the truncated cone with spherical segments on tops. We call it *maximally compact*. There are 3 values of γ at which Q changes its structure [13]:

- γ_1 that is defined from the equation

$$V = \frac{64 \cdot \pi \cdot H^3 \cdot \left(\cos^4\gamma + 2\cdot\sin\gamma - 2\cdot\sin\gamma\cdot\cos^2\gamma + 2 - 3\cdot\cos^2\gamma\right)}{3\cdot(\cos^2\gamma + 2\cdot\sin\gamma + 2)^3} \qquad (7)$$

- γ_2 defined by

$$\gamma_2 = \arcsin\left(\frac{1}{H}\cdot\sqrt[3]{\frac{3V}{4\pi}}\right), \qquad (8)$$

- γ_3 that is defined from the equation

$$V = \frac{2}{3}\cdot\pi\cdot H^3 \cdot \frac{2\cdot\sin\gamma - 2\cdot\sin\gamma\cdot\cos^2\gamma + 3\cdot\cos^2\gamma - \cos^6\gamma - 2}{\cos^6\gamma}\,. \qquad (9)$$

Fig. 2 illustrates the transformation of the form of maximally compact body depending on the angle γ.

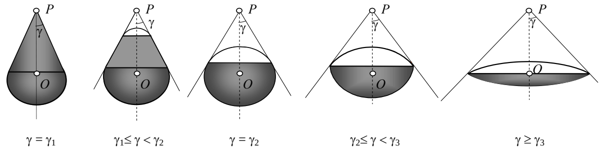

Fig.2. Maximum compact body form for different values of the angle γ.

It is impossible to find analytically the optimal value of angle γ from the equation (6) because of complicated transcendental dependence on γ of the surface area of Q and the γ itself from V and H. To find a computing solution, we have developed a package of M-files “Optima” in system MATLAB.

It realizes the following algorithm:

(i) find Δ as a function of γ for a maximum compact body Q with the weight center O, inscribed in the cone Ω (in which γ varies from γ_1 to 90 ° with step 1°);

(ii) find the corners γ_1, γ_2, … for which Δ is 0 or is maximally close to 0 (for negative values of Δ);

(iii) find the areas S_1, S_2, … of Q for γ equaling γ_1, $\gamma_{2,}$;

(iv) find $\gamma = \gamma_{opt}$ for which $S = \min(S_1, S_2, \ldots)$;

(v) find geometrical parameters that define the form of Q and the value of Δ..

An example of the graphic information obtained with the package “Optima” is given in Fig. 3.

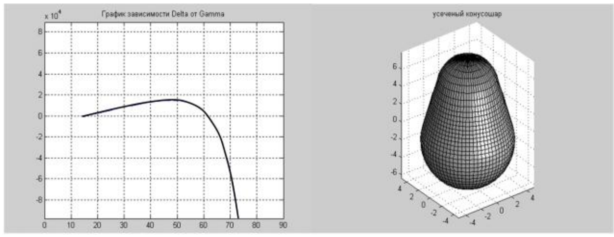

Fig.3. The “Optima” output for H=16 m, V=660 m^3, q = 10 watt/m^2, and p = 210000 watt.

3. Modeling of atmosphere receipt and ground-reflected thermal radiations

Studying thermal radiation for the clear sky, we accept the following assumptions:

1. The Earth is the sphere of radius 6371.21 km.
2. The height of atmosphere is 25 km. The atmosphere is isotropic and translucent body for thermal radiation.
3. The absorbing and radiating substances in atmosphere are water steam, carbonic acid, and ozone. Practically all water steam and more than 95% of carbonic acid are contained in the atmosphere layer at the height of 25 km. The distribution of water steam depending on height is given by the formula (see [7]):
4.

$$e = e_0 \cdot 10^{-\frac{h}{6} - \frac{h^2}{120}}\,, \qquad (10)$$

in which e and e_0 are the pressure of water steam at height of h km and on the ground level. The volume content of carbonic acid in atmosphere is 0.033% and does not depend on height. The relation of its density to the density of dry air is 1.529. Ozone is concentrated in the layer of thickness 3.4 mm and at height of 20 km from the ground. This assumption for optimizing the buildings form is correct since the height of buildings is much lesser than of the ozone layer.

4. The temperature t of atmosphere depends on the height h as follows:

- from the ground to 1 m (the layer of thermal roughness) by the formula

$$t = t_l + (t_0 - t_l)\sqrt{2h - h^2}\ , \tag{11}$$

in which t_l is the temperature of the ground surface and t_0 is the temperature of air at height of 2 m;

- from 1 m to 50 m (to the upper border of surface layer) the temperature is stable and is equal to t_0;
- from 50 m to 11 km (to the upper border of troposphere) the temperature changes linearly; the temperature at 11 km is 56.5 ° and the average gradient of temperature in troposphere is 6.5 °;

5. The absorption coefficients of radiating gases are determined by the formulas

$$k_\lambda(h) = k_{\lambda 0}\sqrt{\frac{p(h)}{p_0}}\cdot\sqrt[4]{\frac{T_0}{T(h)}}; \tag{12}$$

where $k_{\lambda 0}$ is the value of absorption coefficient at some fixed height in which the temperature is T_0, and pressure is p_0; h is the height on which k_λ is calculated; and λ is the wave length.

6. The atmosphere consists of 100 layers of different thickness. Within each layer physical properties of atmosphere are constant. The thickness of the layers changes from 2 cm near the ground to 700 m at height of 25 km.

7. The ground surface is a grey isotropic body of temperature t_z. This means that radiance of a surface is the same in all directions.

8. We take into consideration the radiation of gases of waveband 4-99 microns; by Planck's law, more than 99% of radiation of absolutely black body (in atmosphere conditions) are in this waveband.

9. This waveband is divided into 37 segments. Within each segment, the absorption coefficient is considered as constant.

Under these assumptions, the equation of heat transfer in atmosphere at clear sky takes the form

$$\begin{cases} G(h,\alpha) = \sum\limits_{\lambda=1}^{37}\int\limits_h^{25000}\sum\limits_{i=1}^{3} k_{\lambda i}(\xi)\cdot\rho_i(\xi)\cdot k_\alpha(\xi)\cdot E_\lambda(\xi)\cdot e^{-\int\limits_h^{\xi}\sum\limits_{i=1}^{3} k_{\lambda_i}(\zeta)\rho_i(\zeta)d\zeta} d\xi; \\ U(0,\alpha) = \sum\limits_\lambda \left(\delta\cdot E_\lambda(0) + 2\cdot(1-\delta)\int\limits_0^{\pi/2} G_\lambda(0,\alpha)\cos\alpha\sin\alpha d\alpha\right); \\ U(h,\alpha) = \sum\limits_\lambda \left(U_\lambda(0,\alpha) e^{-\int\limits_0^{h}\sum\limits_{i=1}^{3} k_{\lambda i}(\zeta)\rho_i(\zeta)d\zeta} + \int\limits_0^h \sum\limits_{i=1}^{3} k_{\lambda_i}(\xi)\rho_i(\xi)k_\alpha(\xi)E_\lambda(\xi)e^{-\int\limits_\xi^{h}\sum\limits_{i=1}^{3} k_{\lambda_i}(\zeta)\rho_i(\zeta)d\zeta} d\xi \right), \end{cases} \tag{13}$$

in which h is the height of the design point over ground; α is the inclination of a beam; G and U are the intensity of integrated radiation from the upper and the bottom hemispheres of the space; G_λ and U_λ are the intensity of monochrome radiation within a segment λ; $k_{\lambda i}$ is the mass coefficient of absorption of the i-th gas; k_α is a coefficient that reflects the curvature of atmosphere and refraction; ρ_i is the density of i-th gas; E_λ is Planck's function on segment λ for the temperature at height h; δ is the absorption coefficient of the ground.

For the cloudy sky we assume that each cloud is an absolutely black body. For the height of clouds in several hundreds of meters this assumption is correct since the thickness of the bottom part of clouds, where thermal radiation is absorbed only partially, is only several tens of meters.

In the case of overcast, equations similar to (13) were obtained under additional boundary conditions:

$$G_\lambda(H_{cl},\alpha) = E_\lambda(H_{cl});\quad 0 \le h \le H_{cl}, \tag{14}$$

in which H_{cl} is the height of the bottom surface of clouds.

For incomplete nebulosity, the value of intensity of an irradiation of a point B_α in a direction α is calculated by the formula

$$B_\alpha = B_{0\alpha}(1 - n_\alpha) + B_{10\alpha} n_\alpha, \tag{15}$$

in which $B_{0\alpha}$ and $B_{10\alpha}$ are the intensity of irradiation of the design point for the clear and for the completely cloudy sky; n_α is the number of clouds in the direction α, which admits to take into account the projective increase of the number of clouds near horizon because of their height.

On the base of the obtained formulas, we develop the package of M-files "Long-wavelength radiation", which calculates a two-dimensional file of values $B(H,\alpha)$ of the stream of thermal radiation at height H in the direction α. The input parameters are the following: t, f, and p are temperature, relative humidity, and pressure of atmosphere in two meters over the ground; t_l is the ground temperature; H_{max} is the maximal height of the design point over the ground; H_{cl} is the height of the bottom surface of clouds; n is the number of clouds. An example of the use of this package is given on Fig. 4.

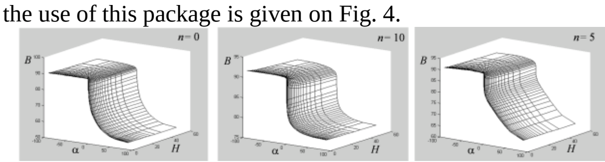

Fig. 4. Calculations results of by the package "Long-wavelength radiation" for t = –10°, t_l = –5°, f = 70%, p = 1000 gPa, δ = 0.95, H_{max}= 50 m, H_{cl} = 2000 m.

4. Modeling of solar radiation receipt

Under the clear sky the direct solar radiation I_0 arriving to any plane is calculated by the well-known formula

$$I_0 = Ip^m\ , \tag{16}$$

where $I = f(\varphi, N, D, Hr, \alpha, \beta)$ is the intensity of direct solar radiation on any plane on the top border of atmosphere; φ is the geographic latitude, N is the month of year, D is the day of month; Hr is the time of day; α and β are the angle of slope and the azimuth of normal to the design plane; p is the atmosphere transparency; m is the optical weight of atmosphere in the normal direction.

Geometrical bases for calculation of I are given in [9]. Finding the optical weight m of atmosphere, we take into account its curvature and refraction. The atmosphere transparency p is calculated by the formula, which was obtained by P.M.Tverskiy method [4]:

$$p = \left(0.906\, m_1^{0.018}\right)^{T_{m_1}} \left(\frac{m}{m_1}\right)^{a_m}\ , \tag{17}$$

in which 0.906 is the transparency of ideal atmosphere for an integrated stream at m=1; m_1 is the weight of atmosphere for which the feculence atmosphere factor T_{m1} is determined; 0.018 and a_m are the values, for the ideal and the real atmosphere, of some parameter defined in [4].

For a cloudy sky the value of direct solar radiation is determined by a formula similar to (15) and $I_{10} = 0$.

Radiance distribution modeling of diffused radiation is based on the assumption that the light and the energy streams of diffused radiation are proportional. This assumption is valid because the maximum of energy of diffused solar radiation is located in visible light being the electromagnetic waves of 0.425-0.450 microns, [4,7]. Then the radiance of clear sky is defined by R.Kitler's formula:

$$L_0(z,\gamma) = L_z \cdot \frac{\left(1-e^{0.32\sec z}\right)\cdot\left(a+be^{-3\gamma}+d\cos^2\gamma\right)}{0.274\left(a+be^{-3z_0}+d\cos^2 z_0\right)}, \qquad (18)$$

and the radiance for overcast by the formula

$$L_{10} = L_z \cdot \left[a_1 + (1-a_1)\sin\alpha\right], \qquad (19)$$

[2]. Here L_z is the brightness of the sky in zenith; z_0 and z are the zenith distances in the sky of the Sun and of the flow point; γ is the flow point angular distance of the to the Sun; a, b, d are factors that depend on the atmosphere transparency; a_1 is a factor depending on ground albedo.

We propose to determine the radiance of the cloudless sky as follows.

It is known that for the clear sky the ratio k_1 between the stream of diffused radiation falling on a horizontal plane i_h, and the intensity of direct solar radiation $I_\perp$ depends only on the atmosphere transparency. If the optical weight of atmosphere is m = 1.5, then basing on experimental data the authors of [4,7] obtain

Factor $T_{1.5}$ of atmospheric turbidity	4.26	3.62	2.92	2.41	1
The coefficient k_1	0.22	0.17	0.12	0.09	0.05

(20)

Using (20), we can determine the function k_1 (T_{m1},m_1,m), in which T_{m1}, m_1, m are as in (17). Then

$$L_z = \frac{I_\perp \cdot k_1(\grave{O}_{m_1}, m_1, m)}{\int_0^{2\pi} d\gamma \int_0^{\pi/2} \frac{\left(1-e^{0.32\sec z}\right)\cdot\left(a+be^{-3\gamma}+d\cos^2\gamma\right)}{0.274\left(a+be^{-3z_0}+d\cos^2 z_0\right)} dz} \qquad (21)$$

Determining L_z in the case of overcast, we use the following additional assumptions:

1. Clouds are located in a single layer.
2. The solar radiation to the top border of clouds arrives through the clear sky.
3. The feculence factor T_{m1} depends on the elevation over sea level as illustrated on Fig. 5. This dependence is implied from the experimental data [4].
4. The function P of solar radiation transmission through clouds depends on the height of the Sun and is given in Fig. 6. To determine it, we used [7] containing data on streams of diffused radiation for clouds of different types and for different heights of the Sun.
5. All radiation that has passed through clouds is diffused and reaches the design point.

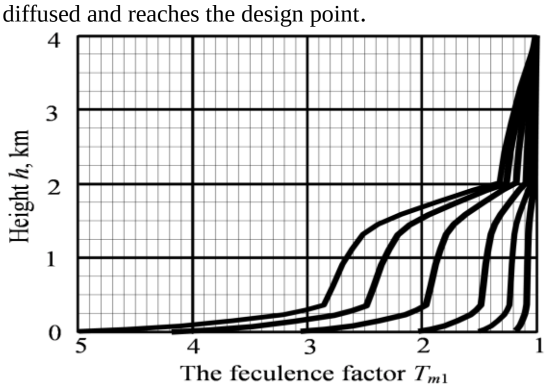

Fig.5. Feculence factor change in dependence to height

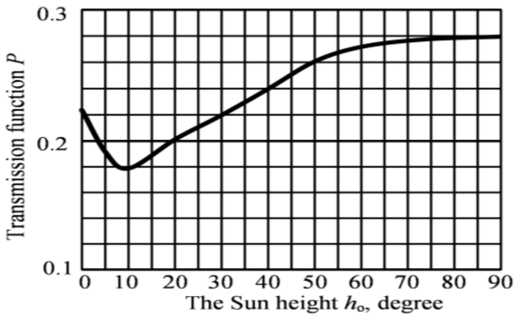

Fig.6. The function P of solar radiation transmission trough clouds

Under these assumptions,

$$L_z = \frac{\left(I_{cl} + 3_{cl}\right)P}{2\pi \int_0^{\pi/2} \left(a_1 + (1-a_1)\sin\alpha\right)\sin\alpha\cos\alpha d\alpha}, \qquad (22)$$

in which I_{cl} and i_{cl} are the streams of direct and diffused radiation on the top border of clouds.

Distribution of radiance of diffused radiation for any cloudiness is defined under the formula which is similar to (15). Total solar radiation is reflected isotropically from the ground in the form of diffused radiation.

The considered dependences of solar radiation receipt are realized in the form of a package of M-files.

3. Conclusion

The considered examples of optimizing the geometrical body form heated by a point source (possibly, positioned infinitely far away) illustrate the importance of a body form in energy saving problems. To solve optimizing problems, we developed a MATLAB package that in table form finds the value of incoming thermal energy to a unit square relating to its orientation and takes into consideration the direct, diffuse, ground-reflected solar radiation, and the atmosphere thermal radiation. The results obtained with this package coincide very closely with the experimental data.

References

[1] Dvoretsky A.T., 2001: Geometrical Simulation of the Reflected Power Flows in Solar Thermal Technology (in Russian). Doctor of Science dissertation. KNUBA. Kyiv: p. 325.

[2] Ivanchenko V. T., 2002: *Determination of Light Exposure of Housings by Natural Light* (in Russian). AVOK-Press, Moscow: p.80.

[3] Harkness E. L., Mehta M. L., 1978: *Solar Radiation Control in Buildings*. Applied Science Publishers Ltd, London: p.176.

[4] Kondrat'ev K. Ya., 1954: *Radiant Energy of the Sun* (in Russian). Gidrometeoizdat, Leningrad: pp. 560.

[5] Kruglova A. I, 1970. *Climate and Protecting Design* (in Russian). Stroyizdat, Moscow: p.167.

[6] Markus T. A., Morris E. N., 1980: Buildings, Climate and Energy. Pitman Publishing, S.Lo, Canada: p.540.

[7] Matveev L. T, 2000: *Course of General Meteorology. Physics of Atmosphere* (in Russian). GIMIZ, St.-Petersburg: p.778.

[8] Pidgorny O. L., 2004: Geometry of solar irradiation and problems of architectural designing and solar engineering (in Ukrainian). In: "Geometrical and computer modeling: energy saving, ecology, design. Proceedings of the 1-st Scientific-Practical Conference. KNUTD. Kyiv: pp. 9-13.

[9] Podgorny A. L., 1981: *To the problem of automation of insolation calculations* (in Russian). In: "The Applied Geometry and Engineering Graphics", V. 31. Budivel'nik. Kyiv: pp.12-15.

[10] Podgorny A. L., Martynov V. L, 1992: *Determination of the optimum angle of slope of a solar collector plane for a given*

azimuth (in Russian). In: "The Applied Geometry and Engineering Graphics", V. 53. Budivel'nik. Kyiv: pp. 8-12.

[11] Podgorny A. L., 1998: To the question of geometrical modeling of priority of temperature maturing of soils (in Russion). In: "Papers of the Taurian State Agrotechnical Academy. - Issue 4. The Applied Geometry and Engineering Graphics. V. 2. TGATA. Melitopol: pp. 22-24.

[12] Pugachov E.V., 2006: *Insolation and an irradiation of a ramp with regards to probability of solar light* (in Ukrainian). In: "Geometrical and computer modeling: energy saving, ecology, design. Proceedings of the 3-rd Scientific-Practical Conference. KNUTD. Kyiv: pp. 133-137.

[13] Sergeychuk O.V., 2005: *Bodies of the minimum area inscribed in a cone* (in Russian). In: "Modern problems of geometrical modeling. Proceedings of the Ukrainian-Russian Scientific-Practical Conference". Kharkov: p. 245-250.

[14] Tabunshcikov Yu. A., 1992: Mathematical Models of Thermal Conditions in Buildings. Taylor & Francis Ltd., Hardback: p. 240.

[15] Tabunschikov, Yu. , Brodatch, M., 2002. *Mathematical Modeling and Optimization of Thermal Efficiency of Buildings* (in Russian). AVOK-Press, Moscow: p. 194.

[16] Tabunshcikov Yu. A., Brodach M. M., Shinkin N. V., 2003: *Energy-efficient Buildings* (in Russian). AVOK-Press, Moscow: pp. 200.

[17] Twarowski M., 1970: *The Sun in Architecture* (in Poland). Arcady. Warsaw: pp. 296.