International Journal of Engineering & Technology, 7 (3.2) (2018) 334-338

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Strength Design of Compressed Reinforced Concrete Elements by Deformation Method Based on Extreme Criterion

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Abstract

Investigations of the normal sections strength of compressed elements using a deformation method were performed. The shape of the section, the class of concrete and the percentage of reinforcement were taken into account when performing analytical calculations. Analytical, numerical, optimization techniques etc. were used for solving of this task. The results of the analytical calculations and the data of the experimental studies were compared.

***Keywords**: bearing capacity; deformation model; strength; the extreme criterion; ultimate strain.*

1. Introduction

The tasks [1-3] pays particular attention to the upgrading of such methods [10, 11] for a more complete description of mathematical models with allowance for strength characteristics materials, cross-section shapes, etc. This technique takes into account the complete diagrams of the work of reinforcement and concrete. It is realized as an optimization problem. The deformation technique proposed by the authors makes it possible to calculate the characteristics of materials of the normal section in the ultimate state.

The maximum $2\left(\varepsilon_{c1};f_{ck,prism}\right)$ of the function (Fig. 1, c) of the diagram of the normal section compressed zone entails a maximum $4\left(\varepsilon_{cu1};N_{max}\right)$ of the function (Fig. 1, d) (axial force-deformation of the most compressed concrete fiber), that is, to the existence of an extreme criterion [1-3].

$$N = f\left(\varepsilon_{cm}\right)\Big|_{\varepsilon_{cm}=\varepsilon_{cu1}} = max. \qquad (1)$$

Experimental researches by other authors confirm this [2, 4-9].

The deformation technique proposed by the authors, unlike the existing ones, which allows us to analytically determine the ultimate parameters of concrete and reinforcement. In existing methods deforation ε_{cu1}, is determined experimentally. Their values are given in the tabular form of documents [11], depending on the strength class of the compresed concrete. This does not allow taking into account the section shape, the reinforcement percentage and other factors.

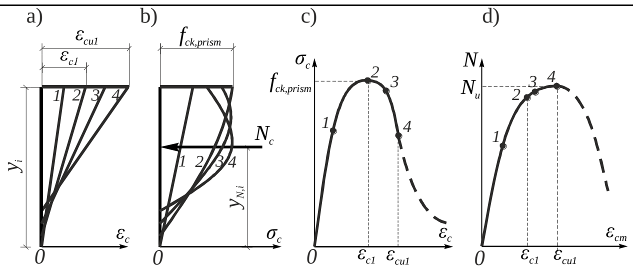

Fig. 1: Preultimate (1,2,3) and ultimate (4) development of deformations (a) and tensions (b) in the compressed area of concrete of RCE. Correspondence of states 1, 2, 3, 4 in the diagram of compressing concrete (c) and the curve «the efforts in the section is the deformation in compressed verge of concrete» Dotted lines I, II, III characterize pseudoplastic, plastic, and plastic bodies which are consolidated accordingly

2. Main Body

The researches were limited by the task of calculating the strength of a normal section. The functions of stresses and deformations and other characteristics of compressed reinforced concrete element sections were determined according to expressions (2) - (16). The design scheme is shown in Fig. 2.

1 Physical equations for concrete (a) and reinforcement (b):

a) we apply the estimation approved by EN-2 [11]

$$\sigma_c = f_{ck,prism}\left(K\eta-\eta^2\right)/\left[1+\left(K-2\right)\eta\right], \qquad (2)$$

in which is the strain degree η and is the parameter of the strain-strength (mechanical) property of concrete K determined by the formulas

$$\eta=\varepsilon_c/\varepsilon_{c1}, \quad K=1{,}1E_{cm}\varepsilon_{c1}/f_{ck,prism}, \qquad (3)$$

where E_{cm} –is the initial concrete elasticity module, is found by the formula $E_{cm} = 1{,}1 \cdot 10^4 f_{ck,prism}^{0,3}$ [15]; $f_{ck,prism}, \varepsilon_{c1}$ – are the stress and strain in the point of curve maximum $\sigma_c - \varepsilon_c$ (Fig. 1,c). The strain ε_{c1} is determined by ECB-FIP as

$$\varepsilon_{c1} = 70 \cdot 10^{-5} f_{ck,prism}^{0,31}; \tag{4}$$

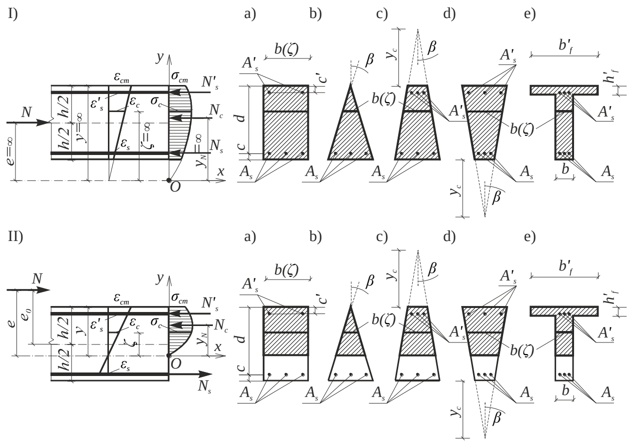

Fig. 2: The design schemes of normal cross sections compressed element

b) The dependences between stresses and strains in the reinforcement with physical σ_y (Fig.3a) and conventional $\sigma_{0,2}$ (Fig.3, b) yield strength were determined according to equations (5) - (8).

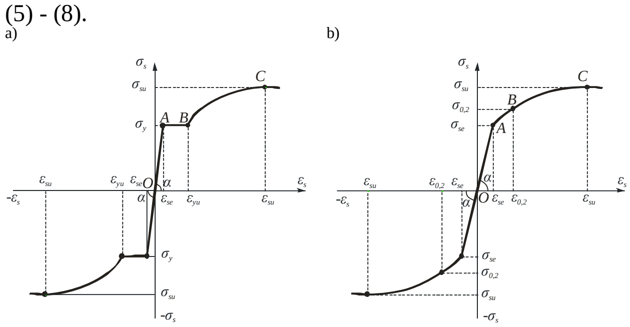

Fig. 3: Complete diagrams of the reinforcement work

$$\sigma_s = \sigma_{su} / \left(1 - \varepsilon_{yu} / \varepsilon_{su}\right)^2 - \left[\left(1 - \sigma_y / \sigma_{su}\right)\left(2 - \varepsilon_s / \varepsilon_{su}\right) \times \right. \\ \left. \times \varepsilon_s / \varepsilon_{su} + + \sigma_y / \sigma_{su} + \left(\varepsilon_{yu} / \varepsilon_{su}\right)^2 - 2\varepsilon_{yu} / \varepsilon_{su}\right], \tag{5}$$

where ε_{yu} – is the deformation of reinforcement (point *B*, fig. 3,a), $\sigma_{su}, \varepsilon_{su}$ – is the strength limit (stress) and strain in the maximum point of diagram *C* $\sigma_s - \varepsilon_s$ (fig. 3,a).

The diagram shown in figure 3, b is described by the expressions (6) - (8).

$$\begin{cases} 0 \le \varepsilon_s \le \varepsilon_{se}, \quad \sigma_s = E_s \varepsilon_s, \\ \varepsilon_{se} \le \varepsilon_s \le \varepsilon_{0,2}, \sigma_s = -\alpha \varepsilon_s^2 + \beta \varepsilon_s + \gamma, \\ \varepsilon_{0,2} \le \varepsilon_s \le \varepsilon_{su}, \sigma_s = -a \varepsilon_s^2 + b \varepsilon_s + c, \end{cases} \tag{6}$$

where

$$\begin{cases} \alpha = p - q, \ \beta = 2p\varepsilon_{0,2} - q\left(\varepsilon_{se} + \varepsilon_{0,2}\right), \\ \gamma = \sigma_{0,2} - p\varepsilon_{0,2}^2 + q\varepsilon_{se}\varepsilon_{0,2}, \\ p = \left(\sigma_{0,2} - \sigma_{se}\right) / \left(\varepsilon_{0,2} - \varepsilon_{se}\right)^2, \quad q = \sigma'_{0,2} / \left(\varepsilon_{0,2} - \varepsilon_{se}\right), \\ \sigma'_{0,2} = -2a\varepsilon_{0,2} + b, \end{cases} \tag{7}$$

$$a = \left(\sigma_{su} - \sigma_{0,2}\right) / \left(\varepsilon_{su} - \varepsilon_{0,2}\right)^2, \ b = 2a\varepsilon_{su}, \ c = \sigma_{su} - a\varepsilon_{su}^2, \tag{8}$$

where the following parameters are used: the modulus of elasticity E_s, proportionality limit σ_{se}, conventional yield point $\sigma_{0,2}$, strength limit σ_{su} and the strains corresponding to these limits – $\varepsilon_{se}, \varepsilon_{0,2}, \varepsilon_{su}$ [2].

2 Geometrical equations are based on the hypothesis of plane sections and this allows to express through the ε_{cm} (Fig. 2)

$$\varepsilon_c = \varepsilon_{cm} \cdot \zeta / y; \tag{9}$$

$$\varepsilon_s = \varepsilon_{cm} \cdot (d / y - 1); \tag{10}$$

$$\varepsilon'_s = \varepsilon_{cm} \cdot (1 - c' / y), \tag{11}$$

where y, c', d – geometric parameters shown in fig. 2.

Provided that the relative deformation of the most compressed concrete fibre in the normal section is described by the expression $\alpha = \varepsilon_{cm} / \varepsilon_{c1}$, then the value η will be equal $\eta = \alpha\zeta / y$.

3 Equations of statics

$$\sum X = 0; \quad N - \sigma_s \cdot A_s - N_c - \sigma'_s \cdot A'_s = 0; \tag{12}$$

$$\sum M_O = 0; \quad N \cdot e - N_c \cdot y_N - \sigma'_s \cdot A'_s \cdot (y - c') - \\ - \sigma_s \cdot A_s \cdot (d - y) = 0, \tag{13}$$

where N_c – is the concrete resultant of the compressed part

$$N_c = \int_0^y \sigma_c b(\zeta) d\zeta = N_c(\alpha, y); \tag{14}$$

e –is the eccentricity application of the axial force N (Fig. 2)

$$e = e_0 - h/2 + y = e(y); \tag{15}$$

e_0 –is the eccentricity of the force N application relatively to the center of the normal cross section, $e_0 = 0$;

y_N – shown in fig. 2 and is calculated by the formula

$$y_N = \left(\int_0^y \sigma_c \cdot \zeta \cdot b(\zeta) d\zeta\right) / N_c = y_N(\alpha, y). \tag{16}$$

To determine the unknown parameters N, α, y, we use expressions (2) - (16). As a result, we obtain a system of equations (17, 18)

$$\begin{cases} N - \sigma_s(\alpha, y) \cdot A_s - N_c(\alpha, y) - \sigma'_s(\alpha, y) \cdot A'_s = 0; & \textbf{(17)} \\ N \cdot e(y) - N_c(\alpha, y) \cdot y_N(\alpha, y) - \sigma'_s(\alpha, y) \cdot A'_s(y - c') - \\ - \sigma_s(\alpha, y) \cdot A_s(d - y) = 0. & \textbf{(18)} \end{cases}$$

To notice the unspecified N, α, y the equations (17), (18) and the extra state in the normal cross section shape of extreme strength criterion (1). The solution of equations (17), (18) with the goal function (1) taken into account is realized as an optimization problem of nonlinear mathematical programming [13]. The algorithm of this methodology for calculating such elements is implemented in the program CRC-12 for PC (Fig. 4).

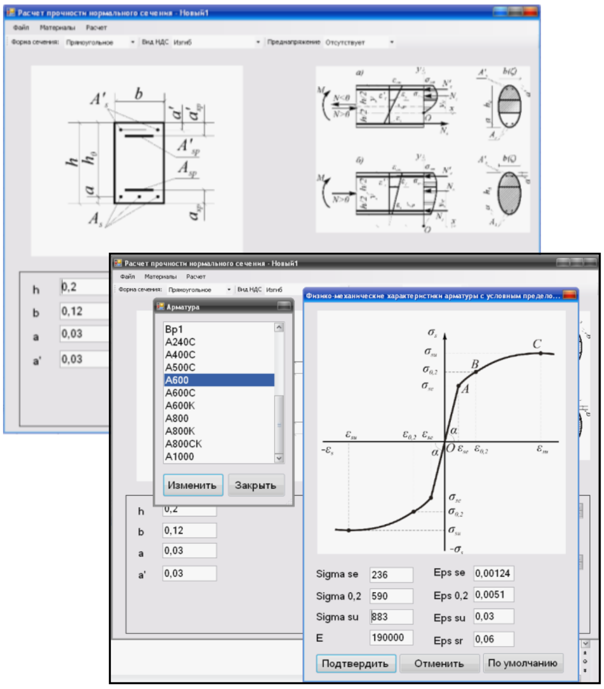

Fig. 4: Software program CRC – 12

The results of calculations of such elements on the basis of the above method were compared with the data of the experimental researches. Statistical processing of the results comparison was carried out. The initial parameters of the reinforcement were taken from the experimental data [14] (Fig. 3).

The initial data for research were taken: different shapes normal cross section (Fig. 2); $K = 1.2...3.5$ ($f_{cd} = 120MPa...7.5MPa$); the reinforcement percentage $\rho_f = 0.41...5.03\,\%$; the reinforcement class – *A400C*; $b = 0{,}25\ m$, $h = 0{,}3\ m$, $tg\beta = 0.217$ (Fig. 2, a-d); $h = 0{,}3\ m$, $b = 0{,}12\ m$, $h'_f = 0{,}06\ m$, $b'_f = 0{,}25\ m$ (Fig. 2, e). The calculations result of the normal section ultimate strength parameters according to the above technique are shown in Figures 5-9.

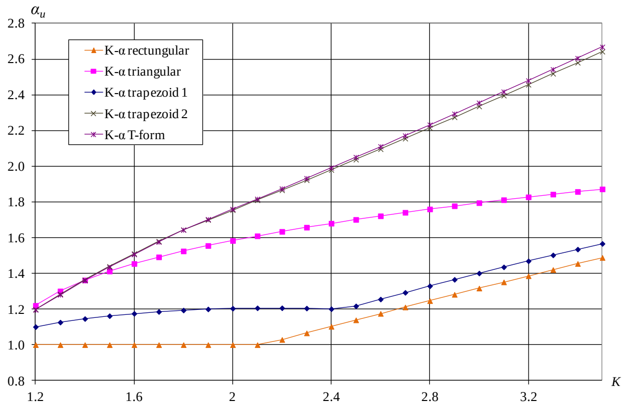

Fig. 5: Dependence of ultimate relative strain $\alpha_u = \varepsilon_{cu1} / \varepsilon_{c1}$ of the compressed concrete part from normal cross section shape and K under central compression with symmetric reinforcement (2Ø16 A400C in the compressed and tensile parts of section)

As can be seen from Fig. 5, the parameter K essentially depends on the ε_{cu}, on cross section shape and on percentage of reinforcement ρ_f [3]. The ultimate deformation ε_{cu} significantly changes its values when the stresses in the reinforcement exceed the σ_y (Figure 6).

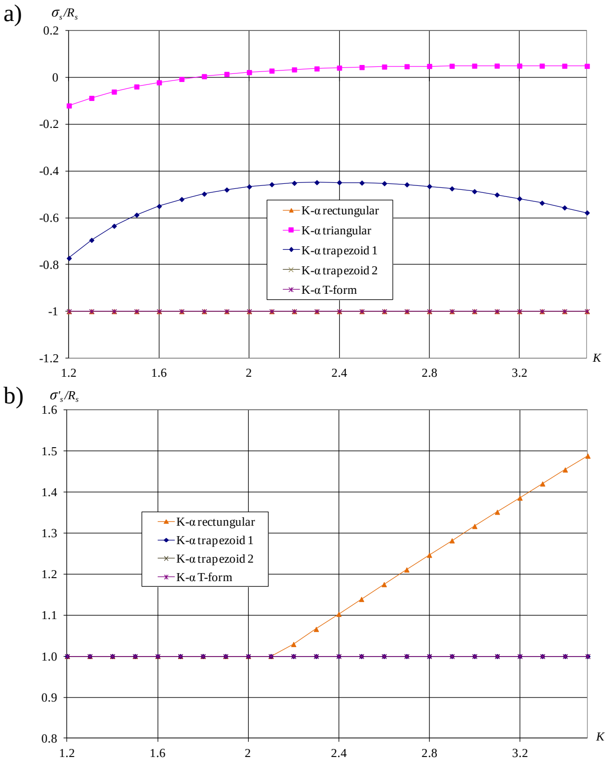

Fig. 6: The influence of parameter K and normal cross section shape of the centrally compressed element on relative stresses in reinforcement of compressed and tension parts

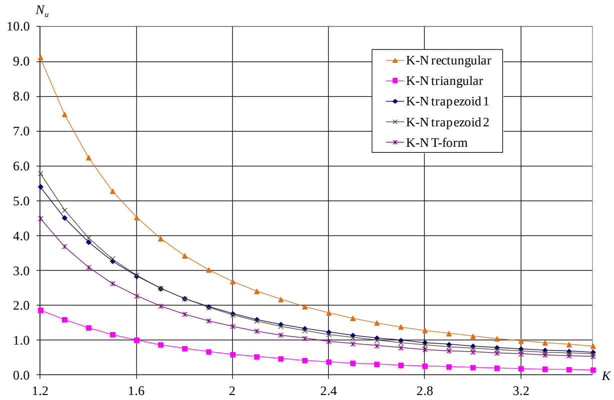

Fig. 7: Dependence of the normal section strength of the centrally compressed element (N_u) on the parameter K for symmetric reinforcement (2Ø16 A400C in the compressed and tensile parts of section)

The dependence of N_u on the normal section shapes and the parameter K for symmetric reinforcement is shown in Fig. 7. The analysis of the graphs shows that N_u essentially depends on the section shape.

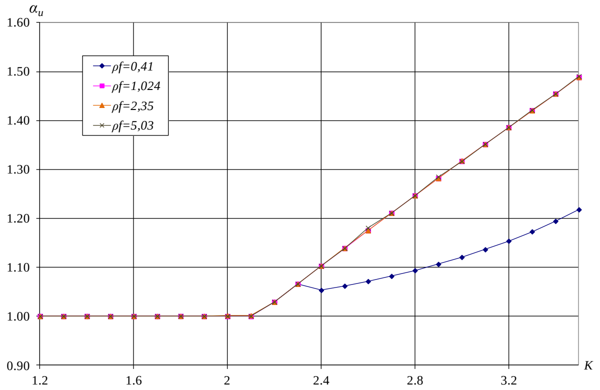

Fig. 8: Dependence of ultimate strain α_u of reinforcement percentage ρ_f, of parameter K of centrally compressed element

Analysis of the data on the graphs (Fig. 8) shows the dependence of the $\alpha_u - K$ parameters on different reinforcement percentages in the $\rho_f = 0{,}41...5{,}03\%$. As a consequence of this graph, it should be noted that when the class of concrete and the reinforcement percentage change, the ultimate characteristics of the section parameters and its strength change significantly.

Figure 9 shows the dependence curves of the strain ε_{cu} from concrete strength class $f_{ck,prism}$, obtained according to DM of ESC for different stress-strain states RCE and with different number of reinforcement in stretched and compressed zones of normal cross section (curves 1-6) and curves 7 and 8 which correspond to the recommendations of Eurocode 2 [11] and SRIBC [18]. Calculations according to DM with ESC are executed for RCE rectangular reinforcement of class A400C without preliminary tension.

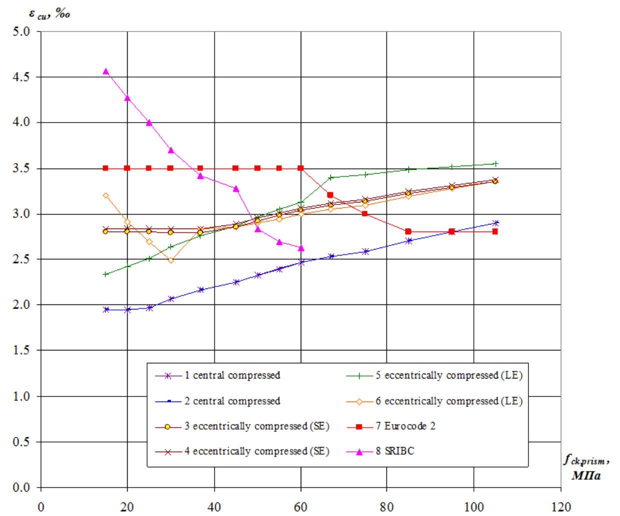

Fig. 9 Curves $f_{ck,prism} - \varepsilon_{cu}$ according to DM with ESC for $\rho_f = \rho_f^{'}$ (in %): 1, 3, 5 – 0,88; 2, 4, 6 – 2,7

Curves $f_{ck,prism} - \varepsilon_{cu}$ according DM with ESC (fig. 9) form a beam that narrows from $\varepsilon_{cu} \approx 2...4$ ‰ for the low strength to $\varepsilon_{cu} \approx 2{,}9...3{,}7$ ‰ for the high concrete strength. It makes for the logical relative position of curves. In the middle of the beam there are curves with increased ρ_f, during the eccentrically compression of small (SE) and large (LE) eccentricities. Lower curves for the central compressed RCE almost repeated strain curves of concrete under axial compression ε_{c1} and therefore do not cause doubts in their reality. The upper curves are approximately at the constant level to $\varepsilon_{cu} \approx 3{,}7$ ‰ close to the adopted in Eurocode 2 constant value $\varepsilon_{cu} = 3{,}5$ ‰ for concretes of low and medium strength. But according to DM with ESC the level of $\varepsilon_{cu} \approx 3{,}7$ ‰ is retained for high strength concrete up to *B* = *105 MPa*, for which in Eurocode 2 $\varepsilon_{cu} \approx 2{,}8$ ‰. However, in tests of beams with cylindrical strength of concrete $f_{cm} = 97{,}9...108{,}3\ MPa$, reinforcement of physical yield point $\sigma_{sy} = 430{,}9\ MPa$ and parameter $\xi = y/h = 0{,}282$ the value $\varepsilon_{cu} = 3{,}21...3{,}74$ ‰ was obtained [19], that means that the experimental data support the calculation ε_{cu} according to DM with ESC better than Eurocode 2.

Reduction of the ultimate strain for the high-strength concrete in Eurocode 2 is caused by the increased fragility and taken in order to ensure reliability of RCE.

Figure 9 shows that the curve $f_{ck,prism} - \varepsilon_{cu}$ of Eurocode 2 is located in the «corridor» limited with curves DM with ESC though for high-strength concrete curves it gradually decreases to the curves under central compression $f_{ck,prism} - \varepsilon_{cu}$. SRIBC curve, which has a big slope and crosses the indicated corridor in the area of low and medium strength concrete, behaves absolutely differently.

The technique given above was applied for normal cross section of centrally compressed RCE strength calculation. The experimental data of different authors [16, 17] were also used. The design results have being compared with the experimental data of the normal cross section strength of the reinforced concrete elements. The results of the strength calculation of compressed elements according to the above technique were compared with the experimental data (33 samples). Statistical analysis of the sampling of the correlation ratios shows their good convergence. The arithmetic mean deviation is *0,978*. The mean square deviation is *0,083*. The variation coefficient is – *8,5%*.

3. Conclusion

1. The implementation of the algorithm of this technique for calculating the strength of a normal section is released by the authors in the software package for PC and allows to analyze the full complex of parameters in the limiting stage, to analyze the actual work of the concrete and reinforcement.
2. The proposed method for calculating the strength of compressed concrete elements of normal cross section makes it possible to take into account influence of strength concrete, the reinforcement amount, the section shapes, etc. on the ε_{cu1}. It is evident from experiments [16, 17].
3. Accounting the real diagrams of reinforcement renders the essential effect on the characteristics of the normal cross section and on its strength in ultimate state. Over-reinforcing of the normal cross section reinforced concrete element significantly increases the ultimate strain of most compressed fibre of the concrete. Decreasing of concrete strength having constant reinforcement results in the same phenomenon.
4. Ultimate strain ε_{cu} depends not only on the parameters E_{cm}, $f_{ck,prism}$ of concrete, but also on the character of RCE concrete complete compression diagram the number of reinforcement A_S, $A_S^{'}$, and its location, section shape, the character of the complete diagram of reinforced steel and other factors that are taken into account only in DM with ESC. Therefore ε_{cu}, is not a criterion value, which determines the state of destruction only of concrete, and is one of the parameters of the ultimate state of

the normal cross section RCE and it cannot be constant, as it is accepted in Eurocode-2 [11] and DBN V.2.6-98: 2009 [18].

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