International Journal of Engineering & Technology, 7 (2) (2018) 935-938

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET
doi: 10.14419/ijet.v7i2.12720
Research paper

Divided square difference cordial labeling of some special graphs

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Abstract

In this article, we have introduced the concepts of divided square difference cordial labeling behavior of some special graphs called Jewel graph,$C_{n-2}+K_2$, Wheel graph, Helm graph, Flower graph,$P_n+\overline{K_m}$, $\overline{K_m}\cup P_n+2K_1$ and Bistar.

Keywords: Bistar; $C_{n-2}+K_2$; Flower Graph; Helm Graph; Jewel Graph; $\overline{K_m}\cup P_n+2K_1$;$P_n+\overline{K_m}$; Wheel Graph.

1. Introduction

For basic notation and terminology in graph theory we refer to Bondy and Murty [2], F. Harary [7] and Rosen Kenneth.H [10] while for number theory we refer Burton [4].Most graph labeling methods were introduced by Rosa [9] in 1967. A dynamic survey on different graph labeling along with an extensive bibliography was found in Gallian [6]. The concept of cordial labeling was introduced by Cahit [3]. Dhavaseelan et.al [5] introduced the concept of even sum cordial labeling graphs.The concept of divisor cordial labeling was introduced by P. Lawrence Rozario Raj and R. Valli [8]. Also further results on divisor cordial labeling was given by S.K.Vaidya and N.H.Shah[11]. Alfred Leo et.al [1] introduced the concept of divided square difference cordial labeling graphs.
In this paper, the concepts of divided square difference cordial labeling behavior of jewel graph,$C_{n-2}+K_2$, Wheel graph, Helm graph, Flower graph,$P_n+\overline{K_m}$, $\overline{K_m}\cup P_n+2K_1$ and Bistar are introduced

2. Preliminaries

Definition 2.1:*[6]The Graph labeling is an assignment of numbers to the edges or vertices or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges), then the labeling is called a vertex (edge) labeling.*

Definition 2.2:*[6]A mapping $f: V(G)\to\{0,1\}$ is called binary vertex labeling of G and $f(V)$ is called the label of the vertex v of G under f.*

The concept of cordial labeling was introduced by Cahit [3].

Definition 2.3:*[3]A binary vertex labeling f of a graph G is called a Cordial labeling if $|v_f(0)-v_f(1)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$. A graph G is cordial if it admits cordial labeling.*

Definition 2.4:*[1]Let $G=(V,E)$ be a simple graph and $f: V\to \{1,2,3,\dots|V|\}$ be bijection. For each edge uv, assign the label 1 if $\left|\frac{(f(u))^2-(f(v))^2}{f(u)-f(v)}\right|$ is odd and the label 0 otherwise. f is called divided square difference cordial labeling if $|e_f(0)-e_f(1)|\le 1$, where $e_f(1)$ and$e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively.*

A graph G is called divided square difference cordial if it admits divided square difference cordial labeling.

Definition 2.5:*[10]A Wheel graph $W_n$is a graph formed by connecting a single universal vertex to all vertices of a cycle. A Wheel graph with n vertices can also be defined as the 1-skeleton of an n-1 gonal pyramid.*

Definition 2.6:*[5]The Helm graph H_n is the graph obtained from a wheel graph W_n by adjoining a pendent edge at each node of the cycle.*

Definition 2.7:*[5]The Flower graph Fl_n is the graph obtained from the Helm graph $H_n$by joining each pendent vertex to apex of the Helm H_n.*

Definition 2.8:*[5]Bistar$B_{m,m}$is the graph obtained by joining the apex vertices of two copies of star $K_{1,n}$.*

Proposition 2.9 [1]
1) Any path P_n is a divided square difference cordial graph.
2) Any cycle C_n is a divided square difference cordial graphexcept $n = 6, 6 + d, 6 + 2d, \dots$ whered $= 4$.
3) The Star graph $K_{1,n}$ is a divided square difference cordial graph.

3. Main result

Proposition 3.1
The Jewel graph is a divided square difference cordial graph.
Proof
Let G be a Jewel graph.
The jewel graph can be constructed by taking
$$V(G)=\{u,v,x,y,u_i,1\le i\le n\}\text{ and}$$

$$E(G) = \{ux, xy, uv, vy, xu_i, vu_i, 1 \le i \le n\}.$$

In this graph, $|V(G)| = n + 4$ and $|E(G)| = 2n + 4$.
Now we can label the graph by defining a map $f: V(G) \to \{1,2,\dots,n+4\}$ and assign the label values as $f(u) = 1, f(x) = 2, f(y) = 3, f(v) = p$ where p is the largest prime number and $p \le n + 4$. Also we can label the vertices $u_1, u_2, \dots, u_n$ with labels $4,5,6 \dots, n+4$ other than p.
Then we get $e_f(0) = e_f(1)$.
Thus $|e_f(0) - e_f(1)| \le 1$.
Hence G is a divided square difference cordial graph.

Example 3.2

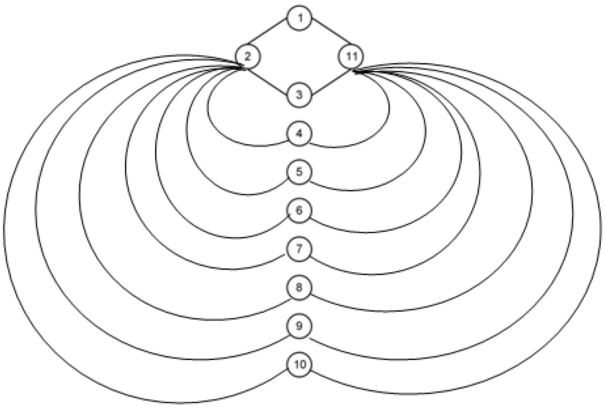

Fig. 1: Jewel Graph When n = 7.

Proposition 3.3
The graph $C_{n-2} + K_2$ is a divided square difference cordial graph except for $n - 2 = 6, 6 + d, 6 + 2d, \dots$ where $d = 4$.

Proof
Let G be a graph $C_{n-2} + K_2$.
Let $v_1, v_2, \dots, v_{n-2}$ are the vertices of C_{n-2} and v_{n-1}, v_n are the vertices of K_2. Construct the graph $C_{n-2} + K_2$.
Define a map $f: V(G) \to \{1,2,\dots,n\}$ as follows.
First we can label the cycle C_{n-2} by Proposition 2.9.
Then label K_2 by taking $f(v_{n-1}) = n - 1, f(v_n) = n$.
Then we get $|e_f(0) - e_f(1)| \le 1$.
Hence G is a divided square difference cordial graph.

Example 3.4

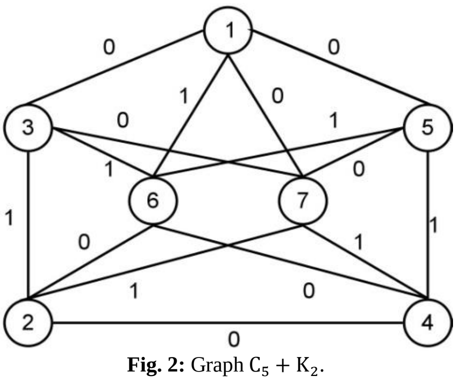

Fig. 2: Graph $C_5 + K_2$.

Proposition 3.5
The Wheel graph W_n is a divided square difference cordial graph except for $n = 6, 6 + d, 6 + 2d, \dots$ where $d = 4$.

Proof
Let G be a Wheel graph W_n. Let $u, v_1, v_2, \dots, v_n$ are the vertices of W_n. Here u is the apex vertex. In this graph, $|V(G)| = n + 1$ and $|E(G)| = 2n$.
Now, define a map $f: V(G) \to \{1,2,\dots,n+1\}$ as follows. First we can label the cycle C_n by Proposition 2.9. Then we can label the apex vertex $f(u) = n + 1$.
Then we get $|e_f(0) - e_f(1)| \le 1$.
Hence G is a divided square difference cordial graph.

Example 3.6

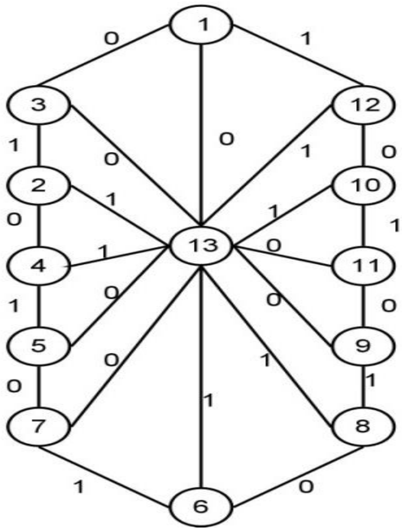

Fig. 3: Wheel Graph W_{12}.

Proposition 3.7
The Helm graph H_n is a divided square difference cordial graph except for $n = 6, 6 + d, 6 + 2d, \dots$ where $d = 4$.

Proof
Let G be a Helm graph H_n. Let $x, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n$ are the vertices of H_n. Here x is the apex vertex, $v_1, v_2, \dots, v_n$ are the vertices of the cycle C_n and $u_1, u_2, \dots, u_n$ are the pendent vertices. In this graph, $|V(G)| = 2n + 1$ and $|E(G)| = 3n$.
Now, define a map $f: V(G) \to \{1,2,\dots,2n+1\}$ as follows. First we can label the Wheel W_n by proposition 3.5. Then we can label the pendent vertices $u_1, u_2, \dots, u_n$ as $f(u_k) = n + k + 1, 1 \le k \le n$.
Then we get

$$e_f(0) - e_f(1) = \begin{cases} 0, \text{if n is even} \\ 1, \text{if n is odd} \end{cases}$$

Thus, $|e_f(0) - e_f(1)| \le 1$.
Hence G is a divided square difference cordial graph.

Example 3.8

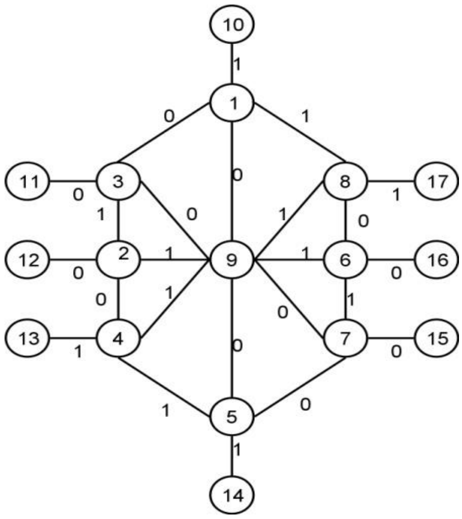

Fig. 4: Helm Graph H_8.

Proposition 3.9
The flower graph Fl_n is a divided square difference cordial graph.

Proof
Let G be a flower graph Fl_n. Let $x, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n$ are the vertices of Fl_n. Here x is the apex vertex, $v_1, v_2, \dots, v_n$ are the vertices of the cycle C_n and $u_1, u_2, \dots, u_n$ are the pendent vertices. In this graph, $|V(G)| = 2n + 1$ and $|E(G)| = 4n$. Now, define a map $f: V(G) \to \{1,2,\dots,2n+1\}$ as follows. First we can label the

Helm H_n by proposition 3.7. Then join the pendent vertices to the apex vertex to get the Fl_n graph.
Then we get $|e_f(0) - e_f(1)| \leq 1$.
Hence G is a divided square difference cordial graph.

Example 3.10

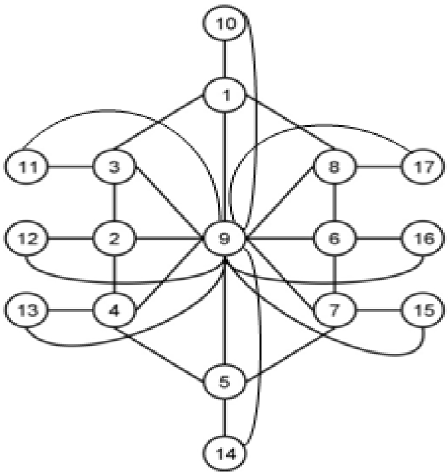

Fig. 5: Flower Graph Fl_8.

Proposition 3.11

The graph $P_n + \overline{K_m}$ is a divided square difference cordial graph.

Proof

Let G be a $P_n + \overline{K_m}$ graph. Let $v_1, v_2, \dots, v_m$ are the vertices of $\overline{K_m}$ and $u_1, u_2, \dots, u_n$ are the vertices of P_n.
Then $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_m$ are the vertices of $P_n + \overline{K_m}$.
Let the edge set be

$$E(G) = \{u_i u_{i+1}, u_1 v_j, u_n v_j, 1 \leq i \leq n-1, 1 \leq j \leq m\}$$

In this graph, $|V(G)| = m + n$ and $|E(G)| = 2m + n - 1$.
Now, define a map $f: V(G) \to \{1,2,\dots,m+n\}$ as follows.
We can label the path by Proposition 2.9 and $\overline{K_m}$ by $f(v_j) = n + j, 1 \leq j \leq m$.
Then we get $|e_f(0) - e_f(1)| \leq 1$.
Hence G is a divided square difference cordial graph.

Example 3.12

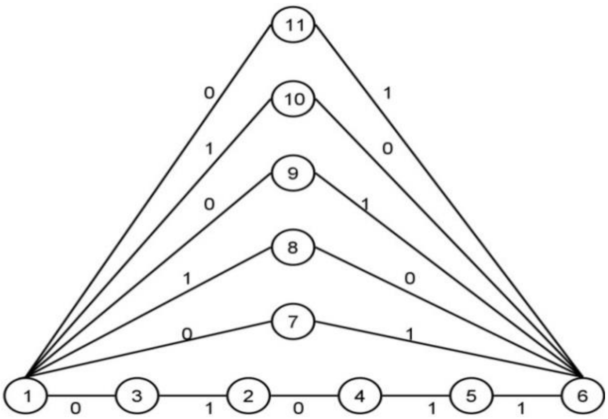

Fig. 6: Graph $P_6 + \overline{K_5}$.

Proposition 3.13

The graph $(\overline{K_m} \cup P_n) + 2K_1$ is a divided square difference cordial graph.

Proof

Let G be a $(\overline{K_m} \cup P_n) + 2K_1$ graph.
Let x, y are the vertices of $2K_1$, $v_1, v_2, \dots, v_m$ are the vertices of $\overline{K_m}$ and $u_1, u_2, \dots, u_n$ are the vertices of P_n.
Then $x, y, u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_m$ are the vertices of $(\overline{K_m} \cup P_n) + 2K_1$.
In this graph, $|V(G)| = m + n + 2$ and $|E(G)| = 2m + 3n - 1$.
Now, define a map $f: V(G) \to \{1,2,\dots,m+n+2\}$. We can construct the path P_n by Proposition 2.9, label $\overline{K_m}$ by $f(v_j) = n + j, 1 \leq j \leq m$ and $2K_1$ by $f(x) = m + n + 1$, $f(y) = m + n + 2$.
Then we get $|e_f(0) - e_f(1)| \leq 1$.
Hence G is a divided square difference cordial graph.

Example 3.14

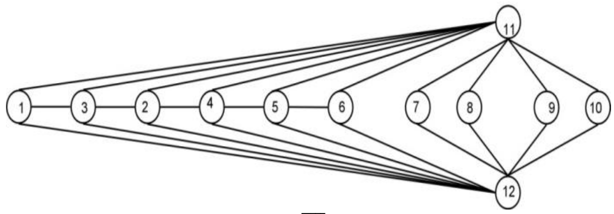

Fig. 7: Graph$(\overline{K_4} \cup P_6) + 2K_1$.

Proposition 3.15

Bistar $B_{m,m}$ is a divided square difference cordial graph.

Proof

Let G be a $B_{m,m}$ graph.
Let $u_1, u_2, \dots, u_m$ and $v_1, v_2, \dots, v_m$ are the vertices of each copy of $K_{1,m}$ with the apex vertex x and y.
In this graph, $|V(G)| = 2m + 2$ and $|E(G)| = 2m + 1$.
Let the edge set be $E(G) = \{xy, yv_i, xu_i, 1 \leq i \leq m\}$.
Now, define a map $f: V(G) \to \{1,2,\dots,2m+2\}$ as follows.
We can label the vertices by taking,

$$f(x) = 1, f(y) = m + 2, f(u_i) = i, 2 \leq i \leq m + 1$$

$$f(v_i) = m + 2 + i, 1 \leq i \leq m$$

Then we get $|e_f(0) - e_f(1)| \leq 1$.
Hence G is a divided square difference cordial graph.

Example 3.16

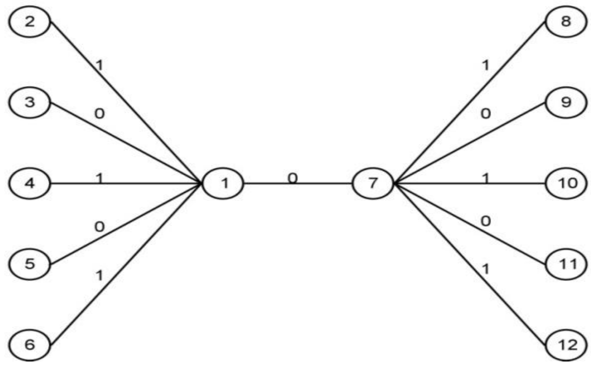

Fig. 8: Bistar $B_{5,5}$.

4. Conclusion

In this paper, the concepts of divided square difference cordial labeling behaviour of Jewel graph, $C_{n-2} + K_2$, Wheel graph, Helm graph, Flower graph, $P_n + \overline{K_m}$, $\overline{K_m} \cup P_n + 2K_1$, Bistar $B_{m,m}$ were discussed. This work can be extended to other type of graphs such as neutrosophic graphs, fuzzy graphs, intuitionistic fuzzy graphs.

Acknowledgement

The authors are highly thankful to the anonymous referees for constructive suggestions and comments. Also thankful to Dr.R.Dhavaseelan for his valuable feedback and comments.

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