International Journal of Engineering & Technology, 7 (2.7) (2018) 142-145

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research Paper

Tracking of pendulum by particle smoother

Mohan Ram Vemuri [1]*, Hima Bindu Bade [2], Koteswara Rao.S[3], V. Gopi Tilak[4]

[1,2]*B. Tech Student, Koneru Lakshmaiah Education Foundation, India.*
[3]*Professor, Koneru Lakshmaiah Education Foundation, India.*
[4]*M.Tech student, Koneru Lakshmaiah Education Foundation, India.*
**Email: vemurimohanram@gmail.com*

Abstract

The series of tracking algorithms accelerated from linear state to non-linear state estimations like the Particle filter. Due to its vibrant computation, tracking signal gets diverged at peaks. Smoothing makes perfect estimation possible, even at that minute portions by modifying its trace based on all the prior measurement values. So, a Particle smoother is used which uses Monte Carlo approximations for smoothing in a non-linear system. Different types of Particle Smoothers can be implemented by using various algorithms. Here, a Backward Simulation Particle smoother is used which is relatively less degenerate than other smoothing algorithms.

Keywords: *Backward Simulation Particle Smoother, Gaussian Approximations, Monte Carlo approximations, Particle filter, Particle Smoother.*

1. Introduction

The importance of Particle filter came into existence in early 2000 after overcoming the problems that arise from the primitive model. Degeneracy is one which makes the particle filter concepts limited to theoretical assumptions [1]. When degeneracy occurs, the zero-weighted particles stops further processing of tracking because the current value depends upon its previous state values. This implicates no signal can be tracked further. The solution to this problem is found as resampling. Resampling is the technique by which the diverging (less probable) particles are replaced by the high weighted particles. Even though many such techniques are available, most of the basic resampling techniques add extra variance [2]. So, one should be cautious while selecting a resampling technique. We have selected an unbiased resampling technique called Stratified resampling and implemented it to particle filter [3]. This technique will be explained in later units of this paper.
The RMSE obtained with particle filter is better than many methods like EKF and UKF, but to make it still better smoothing is used. Smoother smooths the signal trace and gives a minute error. But smoothing performs relatively more computations than filtering in a result we were obtained with the best estimation. Smoothing is certainly different from filtering by means of its computational selection of values. Generally, in filters the calculations are based on the prior values and with the current value of measurement we estimate the next(future) state equation. Whereas in the Smoothing routine, when we are obtained with plenty of output values, hence correct estimation is possible.

2. Particle filter

Particle filter usually derived from the name Sequential Monte Carlo which deals with the terrified nonlinear estimation problems. When dynamic systems are considered, the sensor data obtained will have lots of perturbations, such irreverent signals can be traced out of nonlinear noise by splitting into more particles for correct estimation, hence called as Particle filter.
The probability estimation that starts with the Bayesian equations can be categorized into continuous and discrete signals. The continuous signal equations were solved by Gaussian approximation and the discrete signals with Monte Carlo approximation. These filtering methods are modeled to solve the problems like estimating the internal states of a dynamic system and to eliminate the random variance obtained from sensor data. One of the posterior distribution models that work well with highly non-linear data is Particle filter. The tool used to plough a land cannot suit to dig soil in the flower pot. Similarly, PF cannot work for linear data, the problem arises is sample impoverishment [4,5]. So, particle filters were designed and can be implemented only to nonlinear and partially obtained data from the sensors. Here we implemented Sequential Importance Resampling (SIR) technique and the algorithm is as follows
Consider S samples with the probability of their existence at that instance as

$$v_0^{(i)} \sim p(v_0)\ ;\ i=1,2,\ldots,S$$

$$w_0^{(i)} = \frac{1}{S}\ ;\ i=1,2,..,S \quad (1)$$

Where

$v_0^{(i)}$ - Initial estimated value at i[th] sample.

$w_0^{(i)}$ -Initial weight of i[th] sample.

For each time step d=1,……,T perform the below steps
1)Get the samples $v_0^{(i)}$ from the importance distribution

$$v_0^{(i)} \sim \pi\left(v_{d-1}^{(i)}, u_{1:S}\right) \quad i=1,\ldots.N. \quad (2)$$

2)Calculate new weights by

$$w_d^{(i)} \ \alpha \ w_{d-1}^{(i)} \frac{p\left(u_d \middle| v_d^{(i)}\right) p\left(v_d^{(i)} \middle| v_{d-1}^{(i)}\right)}{\pi\left(v_d^{(i)} \middle| v_{d-1}^{(i)}, u_{1:d}\right)} \quad (3)$$

Where

$w_d^{(i)}$- weight of i[th] particle at d[th] time step.
$u_d^{(i)}$ - measurement value of i[th] particle at d[th] time step.
$v_d^{(i)}$ - estimation value of i[th] particle at d[th] time step.

And then normalize their sum to unity.

3)If the obtained number of particles is less, then perform resampling.

The filtering distribution of SIR resampling technique is approximated by

$$p(v_d|u_{1:d}) \sim \sum_{i=1}^{N} w_d^{(i)} \delta\left(v_d - v_d^{(i)}\right) \quad (4)$$

The optimal variance can be obtained by the importance distribution when it is approximated as

$$\pi\left(v_d^{(i)} \middle| v_{d-1}^{(i)}, u_{1:d}\right) = p(v_d|v_{d-1}, u_d) \quad (5)$$

The importance distribution can be made easy when we approximate (5) as $p(v_d|v_{d-1})$ usually called as Bootstrap filter.

2.1 Stratified resampling:

The idea involved in this technique is to subdivide the whole particles into multiple sets(stratum), called as Strata [6]. Hence it is called as Stratified resampling.
The random value generated is based on the uniform distribution with range (0,1].This set is again partitioned into disjoint intervals as (0,1/S] ,…, (1-1/S,1] and the random variable is obtained by the equation

$$R_d^{(i)} \sim (\tfrac{i-1}{S}, \tfrac{i}{S}] \text{, i =1,2,…S} \quad (6)$$

Where $R_d^{(i)}$- uniform random number generated.
Number of random numbers used is equal to the number of samples (S). So, the order of computational complexity is O(S).
The bounded condition that should be satisfied while getting i[th] selection is

$$B_d^{(k-1)} < R_d^{(i)} \le B_d^{(k)} \quad (7)$$

Where

$$B_d^{(k)} = \sum_{i=1}^{k} w_d^{(i)}$$

Since the bounded values are given by the cumulative sum of the normalized weights. Therefore, the probability of selecting $u_d^{(i)}$is same as that of $R_d^{(i)}$. In this technique the particle gets replicated to the minimum limit of $\max([Nw_d^{(k)}] - 1{,}0)$ and maximum limit of $[Nw_d^{(k)}] + 2$.The difference between random numbers of present and previous states are given by

$$\Delta R = R_d^{(i)} - R_d^{(i-1)} \quad (8)$$

When ΔR=0 then particles get resampled twice.

When ΔR > 2/N the particles that weights between 1/N and ΔR can be discarded.

3. Backward simulation particle smoother

This smoother uses less degenerate particles unlike SIR particle smoother. The SIR filter simply stores the full histories of the particles whereas this smoother reuse the filtering results. The Backward Simulation Particle Smoother algorithm starts from the last step to the first which implies the backward simulation of the individual trajectories.
The Backward Simulation Algorithm contains the following steps:

1. Initially, the particles weighted set must be given as the input to determine the distributions of the filtering and this set is represented as

$$\boldsymbol{w_d^i, v_d^i: i = 1, 2, \ldots.S; d = 1, \ldots\ldots, T} \quad (9)$$

2. For d=T-1,…,0 (simulation from last step to first step**)**

   i. Compute new weights using the equation

$$\boldsymbol{w_{d|d+1}^{(i)} \propto w_d^{(i)} p\left(v_{d+1}^* \middle| v_d^{(i)}\right)} \quad (10)$$

   ii. Again choose $\boldsymbol{v_d^* = v_d^{(i)}}$ with probability $\boldsymbol{w_{d|d+1}^{(i)}}$

Here, the new weights are computed by assuming a trajectory simulated form a smoothing distribution which is given as
For trajectory $v_{d+1:T}^*$

$$\boldsymbol{p(v_d|v_{d+1}^*, u_{1:T}) = \frac{p(v_{d+1}^*|v_d)\, p(v_d|u_{1:d})}{p(v_{d+1}^*|u_{1:d})}}$$
$$= Z\, p(v_{d+1}^*|v_d)\, p(v_d|u_{1:d}) \quad (11)$$

Where, Z is a normalisation constant.
Substitute the above (11) equation in equation (4), we get

$$\boldsymbol{p(v_d|v_{d+1}^*, u_{1:T}) = Z \sum_i v_d^{(i)}\, p(v_{d+1}^*|v_d) \delta\left(v_d - v_d^{(i)}\right)} \quad (12)$$

Now, By getting a sample $v_d^{(i)}$from the above distribution having the probability $\alpha\, w_d^{(i)}\, p(v_{d+1}^*|v_d)$.
By repeating this algorithm S times which implies $v_{0:T}^{*(j)}\ j = 1, \ldots, N$ and the smoothing distribution is approximated as

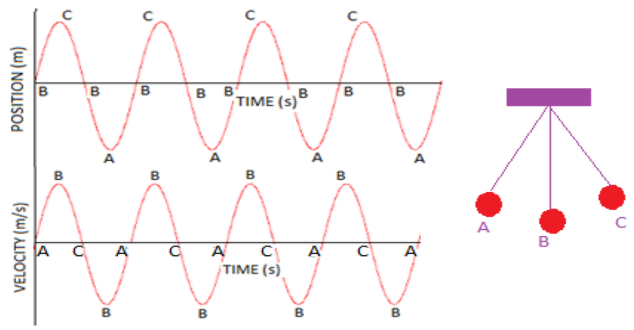
$$p(v_{0:T}|u_{1:T}) \approx \frac{1}{N} \sum_j \delta\left(v_{0:T} - v^*{}_{0:T}^{(j)}\right) \quad (13)$$

Where N – number of iterations the smoothing is repeated

The complexity of the smoothing distribution is O (S T N). If the complexity is quadratic in number of particles, then the memory required is O (N T S^2). This particle implementation iterates recursively through the filtered posterior estimates and, without changing the support of the distribution, modifies the particle weights. This smoother modifies the particle weights without disturbing the distribution support and uses filtered posterior estimates to iterate recursively.

4. Pendulum basics

Let us consider a perfect ideal pendulum by neglecting all types of losses. The pendulum bob is naturally at the centre when the pendulum is in rest, that position is called as Equilibrium position(B). When the pendulum sets into motion, the displacement which is to the right of equilibrium position was considered as positive displacement and left side is negative displacement.



The accelerating positive peak is obtained when the bob is moving from B-C, and the decelerating positive position will be obtained when the bob moves from C-B (see position vs time plot above). Similarly, it follows negative peak in B-A and A-B path

The velocity of bob changes continuously, it will be positive for right ward motion(A-C) and negative in reverse direction(C-A). As the distance from equilibrium position(B) increases velocity goes on decreasing and it will be maximum at B.

The differential equation of this pendulum model is given by

$$\frac{d^2a}{dt^2} = -g\ sin(a) + w(t) \tag{14}$$

Where

a-angle that is made by equilibrium axis to the displacement of pendulum rod

g-Gravitational acceleration

w(t)-random noise

When the above equations are represented in terms of state space model, then

$$\frac{d}{dt}\binom{v1}{v2} = \binom{v2}{-g\sin(v1)} + \binom{0}{1}w(t) \tag{15}$$

Where $v1 = a$ $\quad v2 = \frac{da}{dt}$

When the view point of measurement is horizontal, then there is no offence in considering the above equations as non-linear. Now the measurement equation is

$$u_d = \sin(v1(d)) + r_d \tag{16}$$

To model expressions to discretize the above equations are

$$v_d = F(m_{d-1}, q_{d-1})$$

$$u_d = H(u_d, v_d) \tag{17}$$

Where $q_{d-1} \sim N$ and $r_d \sim N(0, R)$ are noise vectors to be considered.u_d is the measurement vector and v_d is the estimation vector

By discretizing the above continuous non-linear equation, we get

$$\binom{v_{1,d}}{v_{2,d}} = \binom{v1(d-1) + v2(d-1) * \Delta T}{v2(d-1) - g * sin(x1(k-1)) * \Delta T} + q_{d-1} \tag{18}$$

Jacobian matrices for f and h are in the form

$$Q = q^c(\frac{\Delta T^3}{3}\ \frac{\Delta T^2}{2};\frac{\Delta T^2}{2}\ \Delta T) \tag{19}$$

Where q^c is the spectral density of continuous time process noise

Now the weights of pendulum is computed as

$$W \sim e^{\frac{-1}{2R(u(d)-sin(v(d,i)))^2}} \tag{20}$$

Where d-State ; i-1,2,…S ; R=variance

Normalize the weight by

$$w_i = \frac{w_i}{\sum_{j=0}^{S} w_j} \tag{21}$$

Then apply resampling step mentioned in section 2 of this paper.

5. Results

The below results depict the simulation outcomes of 10,000 samples obtained from pendulum data. True and measured values were obtained by simulating the pendulum wave equations mentioned in section-4. The steps considered was 500.The smoother runs for 100 times. The obtained results were plotted below.

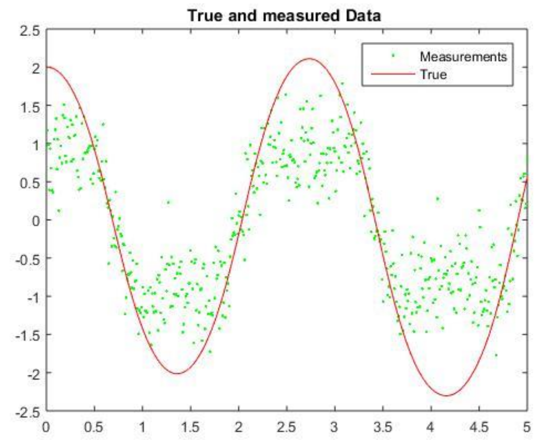


Fig. 1: Simulation result of pendulum wave equation with true and measured values. Red line represents true signal (to be tracked) and the green dots represents measured data points. The horizontal axis is considered as time axis and the vertical axis is referred as pendulum angle.

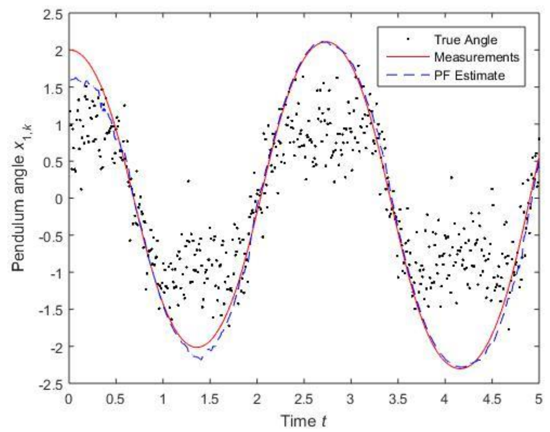


Fig. 2: Tracking based on particle filter estimation. The dots (.) indicates the true angle whereas the red line represents the measurement values and the blue lines states the estimated values of pendulum signal. Pendulum angle is taken along vertical axis and the horizontal axis is time axis in seconds.

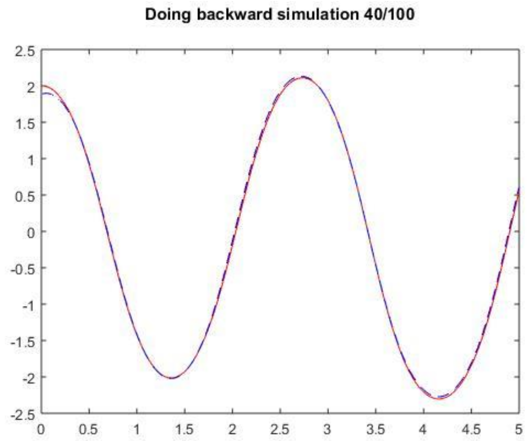


Fig. 3: Applying smoothing on filtered data for 40 times gives the smoothed better curve. The red line shows the true signal of pendulum and the blue dotted line indicates the dynamic trace that estimates the true signal.

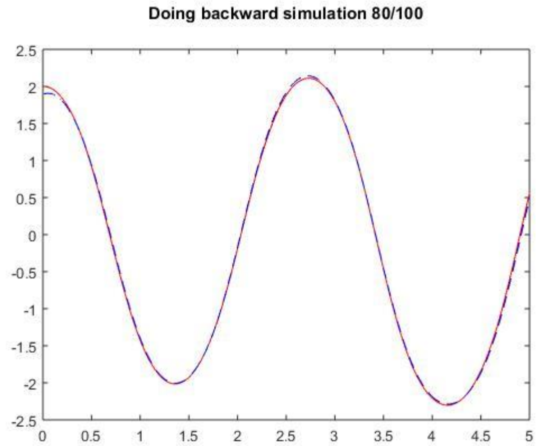


Fig. 4: Applying smoothing on filtered data for 80 times gives the smoothed better curve. The red line shows the true signal of pendulum and the blue dotted line indicates the dynamic trace that estimates the true signal.

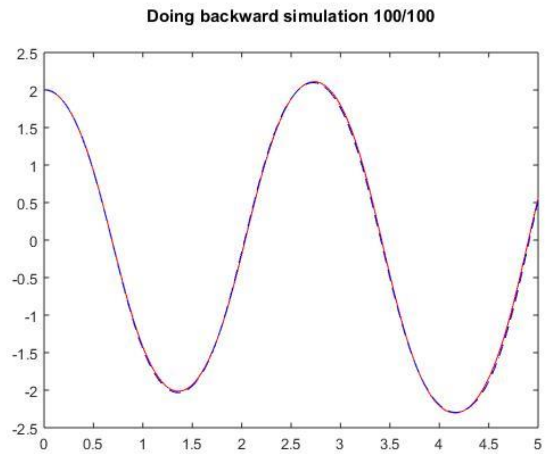


Fig. 5: Smoothing result of pendulum signal. By successive smoothing for 100 times the original signal is tracked correctly. Original signal is denoted by red line and the blue stripes represents the tracking signal. Horizontal axis is a time reference and the vertical axis as pendulum angle.

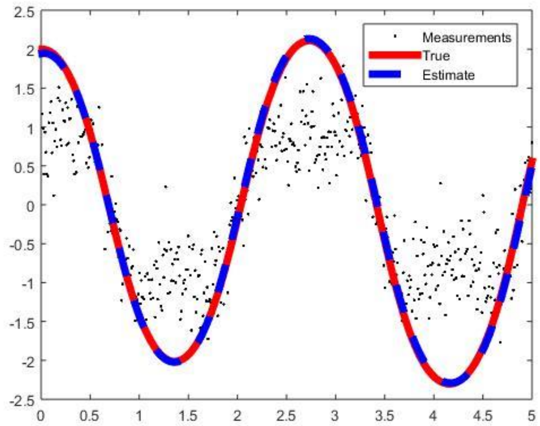


Fig. 6: Tracking signal simulation result. The red line indicates the target signal and the broken blue lines indicates the tracking signal.The time information is given by the horizontal axis and the pendulum angle is given by the vertical axis.

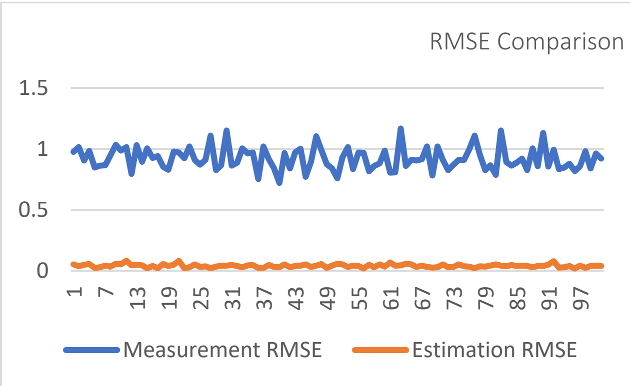


Fig. 7: Monte Carlo simulation for RMSE. Blue colored line indicates Measurement RMSE and the orange colored line indicates pendulum angle estimate.

6. Conclusion

The desired optimal estimation of position can be measured in terms of root mean square error(RMSE) of obtained values. The RMSE of measurement values is 0.919669 and the RMSE of the particle filter is 0.136697 whereas the RMSE value of smoother is 0.039995. From the obtained results, it is clear that the RMSE value obtained from the plot clearly elucidates that the Particle smoother gives the smoothed version of tracking. Hence the Particle smoother gives an optimal estimation of position than a filter.

References

[1] Simo Sarkka, *Bayesian filtering and smoothing (2013),* Vol.3, Cambridge University Press.

[2] Godsill, S. J., Doucet, A., and West, M. 2004. “Monte Carlo smoothing for nonlinear time series”. Journal of the American Statistical Association, 99(465), 156–168

[3] Bindu B, Mohanram V, Koteswararao S,” Tracking of pendulum using particle filter with stratified resampling”, *International Journal of Pure and Applied Mathematics,* Volume 117 No. 18 2017, 299-304

[4] Dan Simon, *Optimal State Estimation Kalman, H∞, and Nonlinear Approaches*, John Wiley and sons Inc., publishers

[5] Douc, R., Garivier, A., Moulines, E., and Olsson, J. 2011. “Sequential Monte Carlo smoothing for general state space hidden Markov models”. Annals of Applied Probability, 21(6), 2109–2145

[6] Tiancheng Li, Miodrag Bolic, Petar M. Djuric, “Resampling Methods For Particle Filtering”,IEEE Signal Processing Magazine, May 2015/1053-5888/15©2015IEEE