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Research paper

Common fixed point theorems in fuzzy partial metric spaces of two pairs of mapping satisfying E.A. property

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Abstract

This paper establishes a common fixed point theorem for a pair of weakly compatible mappings in Fuzzy PMS satisfying the E.A. property. We introduce a new concept of E.A. Fuzzy Partial Metric spaces which generalizes the existing Fuzzy PMS. Our main result provides sufficient conditions for the existence of a common fixed point for two weakly compatible mappings in E.A. Fuzzy Partial Metric spaces. The results have significant implications for fuzzy topology, fixed-point theory, and applications in image processing, machine learning, and optimization problems involving uncertain or imprecise data.

Keywords: *Partial Metric Space; Fuzzy Metric Space; Weakly Compatible Mapping; Common Fixed Point.*

1. Introduction

A metric space is a fundamental concept in Mathematics, particularly in the field of analysis and topology. It provides a framework for measuring distances between points in a set. The modern concept of a metric space was first introduced by Maurice Fréchet in 1906. In his work on functional analysis. Fréchet defined a metric space as a set equipped with a distance function satisfying certain properties, laying the groundwork for the rigorous study of distances in mathematical contexts.

In 1965, Fuzzy set theory was developed by Zadeh [24]. Fuzzy sets allow elements to have degrees of membership between 0 and 1, rather than strictly belonging or not belonging to a set. In the 1970s , mathematicians began to explore the concept of fuzzy metric spaces, where the distance function is allowed to take fuzzy values. . In 1975, Kramosil and Michalek first introduced the concept of a fuzzy metric space [9], which can be regarded as a generalization of the statistical metric space. Clearly this work provides an important basis for the construction of fixed point theory in fuzzy metric space. Fuzzy metric spaces generalize traditional metric spaces by accommodating uncertainty in distance measurements. Fixed point theory has been a vibrant area of research in Mathematics, with numerous applications in various fields such as physics, engineering, economics, and computer science. The generalizations of metric spaces are the notion of partial metric space which was given by Matthews [15] as an extension of metric space where the self-distance of any point is not necessarily equal to zero. Partial metric spaces relax some of the conditions of metric spaces, such as the triangle inequality, to provide a more flexible framework for modeling certain scenarios. In 1994, Veeramani and George introduced the modified Continuous t-norm [7]. In recent years, researchers have explored fixed point theorems in Fuzzy Partial Metric spaces, but most of these results rely on strong assumptions, such as continuity or compatibility of mappings. To overcome these limitations, we introduce the E.A. property, a relaxed equivalence relation that allows for more flexibility in modeling real-world problems.

A FPMS is a generalization of the classical metric space where distances can take fuzzy values, allowing for the representation of uncertainty in distance measurement. In traditional metric spaces, the distance between two points is a non- negative real no. In contrast, in a FPMS, the distance can represented as a fuzzy no. which can accommodate both the notion of closeness & uncertainty.

Impact of E, A. Property on common fixed point theorems in Fuzzy Metric space presented by Kamal Wadhawa [11]. Ashlekha proved fixed point theorem in FMS with E. A. Property by using weackly compatible mapping [2]. Nazir present common fixed point results for two pairs of mapping which is satisfy E. A. property in Partial Metric space [23]. Madhushrivastava present some fixed point theorems in FMS by using occasionally weakly compatible mapping [14].The some common fixed point theorems for weakly compatible self mapping in FMS using (E.A.) property is generalized by Kanhaiya [10]. The new concept introduce of common fixed point theorem in FMS & occasionally weakly compatible mapping by subhani [22]. Rakhi Namdev is developed for existence & uniqueness of common fixed point by using two pairs of mapping satisfying E.A. property [18].

The other author's given different concept of Fixed point theorems by using E. A. property in different spaces like S-metric space, Menger space, Complex valued metric space [5],[8],[13],[19]. Sonam Soni explained applications of Fixed points in Computational Mathematics by using Fuzzy & Partial metric spaces [20].

In this paper, we establish a common fixed point theorem for a pair of weakly compatible mappings in Fuzzy Partial Metric spaces satisfying the E.A. property. Our work builds upon existing results in Fuzzy Partial Metric spaces and fixed point theory, providing a more

comprehensive framework for analyzing complex systems with uncertain or imprecise data. The E.A. property captures nuanced relationships, enabling a more general fixed point theorem.
To establish our theorems, we first define key concepts:

2. Preliminaries

Definition 1: *A Partial Metric space on '𝕏' is a Pair ('𝕏', P) such that '𝕏' is a non-empty set and P: '𝕏' × '𝕏' → $\mathbb{R}^+$ is a mapping providing the listed conditions* $\forall p, q, r \in$ *'𝕏' such that*

1) 'P $(p, p) \leq$ P (p, q)'
2) 'P (p, p) = P (q, q) = P (p, q)' iff ' $p = q$'
3) 'P (p, q) = P (q, p)'
4) 'P $(p, r) \leq$ P (p, q) + P (q, r) – P (q, q)'

Note that in Partial Metric space, a point's self-distance does not always equal zero. The Partial Metric 'P' is an ordinary metric on '𝕏' if 'P $(p, p) = 0$' , $\forall p \in$ '𝕏'. Therefore, a Partial Metric is an extension of an ordinary metric.

Definition 2: *A binary operation 'Θ' on [0, 1] is called a continuous t-norm if it is satisfied following conditions:* $\forall p, q, r, s \in$ *[0, 1]:*

1) '$p \Theta q = q \Theta p$' and ' $p \Theta (q \Theta r) = (p \Theta q) \Theta r$'
2) 'Θ is continuous on [0, 1] × [0, 1]'
3) '$p \Theta 1 = p$'
4) If ' $p \leq q$' and ' $r \leq s$', then ' $p \Theta r \leq q \Theta s$'

Definition 3: *Let '𝕏' be a non-empty set, 'Θ' be a continuous t-norm and F : '𝕏' × '𝕏' × $(0, \infty) \rightarrow [0,1]$ be a mapping. Let 'F' be Fuzzy set. If the listed conditions are satisfied* $\forall p, q, r \in$ *'𝕏' and* $u, v > 0$, *then the triplet ('𝕏' , F, ⊙) is said to be a Fuzzy Metric space, If it satisfies following Properties for*

1) 'F $(p, q, u) > 0$'
2) ' F $(p, q, u) = 1$' iff ' $q = p$'
3) 'F $(p, q, u) =$ F (q, p, u)'
4) 'F $(p, q, u + v) \geq$ F $(p, q, u) \odot$ F (p, q, v)'
5) 'F $(p, q, \odot)$ is continuous on $(0, \infty)$'

If ('𝕏' , F, ⊙) is a fuzzy metric space, then 'F' is a fuzzy metric on '𝕏'.

Definition 4: *Let '𝕏' be a non-empty set, 'Θ' be a continuous t-norm and* F_p *: '𝕏' × '𝕏' ×* $(0, \infty) \rightarrow [0,1]$ *be a mapping. Let P be Partial metric space. If the listed conditions are satisfied* $\forall p, q, r \in$ *'𝕏' and* $u, v > 0$, *then the triplet ('𝕏',* F_p*, Θ) is said to be a Fuzzy Partial Metric space:*

1) 'F_p $(p, q, 0) = 0$'
2) 'F_p $(p, q, u) = F_p (q, p, u)$'
3) 'F_p $(p, r, u + v) \geq F_p (p, q, u) \Theta F_p (q, r, v)$'
4) ' $F_p (p, q, u) \leq 1, u > 0$' & ' $F_p (p, q, u) = 1$' iff ' $p(p, q) = 0$'
5) ' $F_p (p, q, \Theta)$: $(0, \infty) \rightarrow [0, 1]$ is continuous', Where '$F_p(p, q, u) = \frac{u}{u + p(p, q)}$'

If ('𝕏', F_p ,Θ) is a Fuzzy Partial Metric space, then ' F_p' is a fuzzy partial metric on '𝕏'.
Example1: Let '𝕏' $= \{p, q, r\}$ be a set of points. Define a fuzzy distance p as $p(p, p) = 1$,

$p(q, q) = 1$, $p(p, q) = 0.5$, $p(q, p) = 0.5$, $p(r, r) = 1$,

$p(p, r) = 0.2$, $p(r, p) = 0.2$, $p(q, r) = 0.3$, $p(r, q) = 0.3$.

In above example, $p(p, q) = 0.5$ indicate that p & q are relatively close, $p(p, r) = 0.2$ indicate that p & r are not close.

Definition 5: *Two self-mapping 'M' and 'N' on Fuzzy Partial mapping ('𝕏',* F_p*, ⊙) are Compatible, if* '$\lim_{n\to\infty} F_p(MNy_n, NMy_n, u) = 1$', > 0 , *whenever* $\{y_n\}$ *is a sequence in '𝕏' such that* '$\lim_{n\to\infty} My_n = \lim_{n\to\infty} Ny_n = t$', $\forall t \in$ '𝕏'.

Definition 6: *Two self-mapping 'M' and 'N' on Fuzzy Partial mapping ('𝕏',* F_p*, ⊙) are a weakly Compatible, if they commute at their coincidence point.* $\forall t \in$ '𝕏', '$Mt = Nt$' *implies that* '$MNt = NMt$', $\forall t > 0$.

Definition 7: *Two self-mapping 'M' and 'N' on Fuzzy Partial mapping ('𝕏',* F_p*, ⊙) are satisfy E. A. property if* ∃ *a sequence* $\{y_n\}$ *is a sequence in '𝕏' such that* '$\lim_{n\to\infty} My_n = \lim_{n\to\infty} Ny_n = t$', $\forall t \in$ '𝕏'.

Definition 8: *Let 'M' , 'N' , 'U' and 'V' be a self-mapping of a Fuzzy Partial Metric space ('𝕏',* F_p*, Θ) . Then (M, U) and (N, V) are satisfy common E. A. property, if* ∃ *two sequences* $\{x_n\}$ *and* $\{y_n\}$ *in '𝕏' such that* '$\lim_{n\to\infty} Mx_n = \lim_{n\to\infty} Ux_n = \lim_{n\to\infty} Ny_n = \lim_{n\to\infty} Vy_n = z$', *where* $z \in M$('𝕏') $\cap N$('𝕏') *or* $z \in U$('𝕏') $\cap V$('𝕏').

Example2: Let ‘𝕏’ = [0,2) and ‘$F_p(p,q,u) = \frac{u}{u+p(p,q)}$’ $\forall p,q \in$ ‘𝕏’. Then (‘𝕏’, F_p, ʘ) is a Fuzzy Partial Metric space. where $a \odot b = \min(a,b)$.

$M(p) = 0.25, 0 \le p \le 0.52$, $U(p) = 0.25, 0 \le p \le 0.60$,

$= \frac{p}{2}$, $p > 0.52$, $=$ $p - 0.25$, $p > 0.60$,

$V(p) = 0.25, 0 \le p \le 0.60$, $N(p) = 0.25, 0 \le p \le 0.95$,

$= \frac{p}{4}$, $p > 0.60$, $=$ $p - 0.75$, $p > 0.95$,

We define $x_n = 0.5 + \frac{1}{n}$, $y_n = 1 + \frac{1}{n}$, Where $M(‘\mathbb{X}’) = \{0.25\} \cup (0.26,1]$

$U(‘\mathbb{X}’) = \{0.25\} \cup (0.35,1.75]$, $V(‘\mathbb{X}’) = (0.15,0.5]$, $N(‘\mathbb{X}’) = \{0.25\} \cup (0.20,1.25]$

$\lim_{n\to\infty} Mx_n = \lim_{n\to\infty} \frac{1}{2}\left[0.5 + \frac{1}{n}\right] = 0.25 \in U(‘\mathbb{X}’)$,

$\lim_{n\to\infty} Ux_n = \lim_{n\to\infty} \left[0.5 + \frac{1}{n}\right] - 0.25 = 0.25 \in M(‘\mathbb{X}’)$ $\lim_{n\to\infty} Ny_n = \lim_{n\to\infty} \left[1 + \frac{1}{n}\right] - 0.75 = 0.25 \in V(‘\mathbb{X}’)$.

$\lim_{n\to\infty} Vy_n = \lim_{n\to\infty} \frac{1}{n}\left[1 + \frac{1}{n}\right] = 0.25 \in N(‘\mathbb{X}’)$

Thus, $\lim_{n\to\infty} Mx_n = \lim_{n\to\infty} Ux_n = \lim_{n\to\infty} Ny_n = \lim_{n\to\infty} Vy_n = 0.25$

Where, $0.25 \in M(‘\mathbb{X}’) \cap N(‘\mathbb{X}’)$ or $0.25 \in U(‘\mathbb{X}’) \cap V(‘\mathbb{X}’)$.
Role of E.A. Property in providing common fixed point theorems in FPMS can be concluded followings: 1) The E. A. property helps determine whether sequences in the FPMS converges to a point within the space.

1) Many fixed point theorems generalized results, rely on the E. A. property to confirm the existence of a fixed point for certain continuous mapping in FPMS.
2) The E. A. property can link toplogical properties of FPMS to behavior of sequences, which can study of properties like compactness or completeness.
3) Understanding the E. A. property can lead to the applications in Mathematical modeling, optimization problems & other interdisciplinary fields where fuzzy logic is used.

Lemma: *Let (‘𝕏’, F_p, ʘ) be a Fuzzy Partial Metric space, if their exist* $k \in (0,1)$ *such that* ‘$F_p(p,q,ku) \ge F_p(p,q,u)$’ *then* ‘$p = q$’, $u > 0$, $k \in (0,1)$, $\forall p,q \in$ ‘𝕏’ .

3. Main results

Theorem 1: *Let* M,N,U,V *be a self-mapping of a Fuzzy Partial Metric space* (‘𝕏’,F_p,⊙) *with* $a \odot b = \min(a,b)$ *satisfying following conditions:*

i) (M,U) *and* (N,V) satisfy E. A. property.
ii) (M,U) *and* (N,V) are weakly compatible.
iii) Then their exist $k \in (0,1)$, for all x, y, z ∈ ‘𝕏’and $u > 0$.

$$F_p(Mx,Ny,ku) \ge \min\left[F_p(Ux,Vy,u), F_p(Ny,Vy,u), (F_p(Ux,Vy,u).F_p(Ny,Vy,u)), \frac{F_p(Ny,Vy,u)}{F_p(Ny,Ux,u)}, F_p(Ny,Mx,u)\right]$$

Then M,N,U,V have a common fixed point.
Proof: Since (M,U) *and* (N,V) satisfying E. A. property then their exist two sequences $\{x_n\}$ and $\{y_n\}$in ‘𝕏’ such that $\lim_{n\to\infty} Mx_n = \lim_{n\to\infty} Uy_n = \lim_{n\to\infty} Nx_n = \lim_{n\to\infty} Vy_n = z$, where $z \in M(‘\mathbb{X}’) \cap N(‘\mathbb{X}’)$ or $z \in U(‘\mathbb{X}’) \cap V(‘\mathbb{X}’)$. Then $z = Ut$, where x ∈ ‘𝕏’.

a) Now to prove that $p(Mt,Ut) = 0$, that is to show that $Mt = z$.

Put $x = t, y = y_n$ in (iii) we get,

$$F_p(Mt,Ny_n,ku) \ge \min\left[F_p(Ut,Vy_n,u), F_p(Ny_n,Vy_n,u), (F_p(Ut,Vy_n,u).F_p(Ny_n,Vy_n,u)), \frac{F_p(Ny_n,Vy_n,u)}{F_p(Ny_n,Ut,u)}, F_p(Ny_n,Mt,u)\right]$$

$$F_p(Mt,Ny_n,ku) \ge \min\left[F_p(z,z,u), F_p(Ny_n,z,u), (F_p(z,z,u).F_p(Ny_n,z,u)), \frac{F_p(Ny_n,z,u)}{F_p(Ny_n,z,u)}, F_p(Ny_n,Mt,u)\right]$$

$F_p(Mt, Ny_n, ku) \geq min\,[1, F_p(Ny_n, z, u), (1.F_p(Ny_n, z, u)), 1, F_p(Ny_n, Mt, u)]$

$F_p(Mt, Ny_n, ku) \geq F_p(Ny_n, z, u)$, if $Mt = z$

By using Lemma1, we get $Mt = Ny_n = z$. Thus $Mt = z$
Since (M, U) is weakly compatible. Then $Mz = MUt = UMt = Uz = z$

b) Now to prove that $p(Vz, Nz) = 0$, that is to show that $Nz = z$

Put $x = x_n, y = z$ in (iii) we get,

$F_p(Mx_n, Nz, ku) \geq min\,[F_p(Ux_n, Vz, u), F_p(Nz, Vz, u), (F_p(Ux_n, Vz, u).F_p(Nz, Vz, u)),$

$\frac{F_p(Nz,Vz,u)}{F_p(Nz,Ux_n,u)}, F_p(Nz, Mx_n, u)]$

$F_p(Mx_n, Nz, ku) \geq min\,[F_p(z, z, u), F_p(Nz, z, u), (F_p(z, z, u).F_p(Nz, z, u)),$

$\frac{F_p(Nz,z,u)}{F_p(Nz,z,u)}, F_p(Nz, Mx_n, u)]$

$F_p(Mx_n, Nz, ku) \geq min\,[1, F_p(Nz, z, u), (1.F_p(Nz, z, u)), 1, F_p(Nz, Mx_n, u)]$

$F_p(Mx_n, Nz, ku) \geq F_p(z, Mx_n, u)$, if $Nz = z$. By using Lemma 1, we get $Mx_n = Nz = z$

Since (N, V) is weakly compatible. Then $Nz = Vz = NVz = VNz = z$

c) Now to prove that $p(Mz, Uz) = 0$, that is to show that $Mz = z$.

Put $x = z, y = y_n$ in (iii) we get,

$F_p(Mz, Ny_n, ku) \geq \min\left[F_p(Uz, Vy_n, u), F_p(Ny_n, Vy_n, u), (F_p(Uz, Vy_n, u).F_p(Ny_n, Vy_n, u)\right),$

$\frac{F_p(Ny_n,Vy_n,u)}{F_p(Ny_n,Uz,u)}, F_p(Ny_n, Mz, u)]$

$F_p(Mz, Ny_n, ku) \geq min\,[F_p(z, z, u), F_p(Ny_n, z, u), (F_p(z, z, u).F_p(Ny_n, z, u)),$

$\frac{F_p(Ny_n,z,u)}{F_p(Ny_n,z,u)}, F_p(Ny_n, Mz, u)]$

$F_p(Mz, Ny_n, ku) \geq min\,[1, F_p(Ny_n, z, u), (1.F_p(Ny_n, z, u)), 1, F_p(Ny_n, Mz, u)]$

$F_p(Mz, Ny_n, ku) \geq (Ny_n, z, u)$, if $Mz = z$. By using Lemma 1, we get $Mz = Ny_n = z$

Since (M, U) is weakly compatible. Then $Mz = MUz = UMz = Uz = z$

d) Now to prove that $p(Vt, Nt) = 0$, that is to show that $Nt = z$

Put $x = x_n, y = t$ in (iii) we get,

$F_p(Mx_n, Nt, ku) \geq min\,[F_p(Ux_n, Vt, u), F_p(Nt, Vt, u), (F_p(Ux_n, Vt, u).F_p(Nt, Vt, u)),$

$\frac{F_p(Nt,Vt,u)}{F_p(Nt,Ux_n,u)}, F_p(Nt, Mx_n, u)]$

$F_p(Mx_n, Nt, ku) \geq min\,[F_p(z, z, u), F_p(Nt, z, u), (F_p(z, z, u).F_p(Nt, z, u)),$

$\frac{F_p(Nt,z,u)}{F_p(Nt,z,u)}, F_p(Nt, Mx_n, u)]$

$F_p(Mx_n, Nt, ku) \geq min\,[1, F_p(Nt, z, u), (1.F_p(Nt, z, u)), 1, F_p(Nt, Mx_n, u)]$

$F_p(Mx_n, Nt, ku) \geq F_p(Nt, z, u)$, if $Nt = z$. By using Lemma1, $Mx_n = Nt = z$

Since (N, V) is weakly compatible. Then $Nt = Vt = NVt = VNt = z$
Thus, $Mz = Uz = Nz = Vz = z$. Therefore M, N, U, V have a common fixed point.
Uniqueness: Suppose t & z are two common fixed point of M, N, U, V with $t \neq z$,
Then (iii) becomes

$F_p(Mt, Nz, ku) \geq min\,[F_p(Ut, Vz, u), F_p(Nz, Vz, u), (F_p(Ut, Vz, u).F_p(Nz, Vz, u)),$

$\frac{F_p(Nz,Vz,u)}{F_p(Nz,Ut,u)}, F_p(Nz, Mt, u)]$

Put $Mt = Ut = t$, , $Nz = Vz = z$

$$F_p(t, z, ku) \geq min\ [F_p(t, z, u), F_p(z, z, u), ((t, z, u). F_p(z, z, u)), \frac{F_p(z,z,u)}{F_p(z,t,u)}, F_p(z, t, u)]$$

$$F_p(t, z, ku) \geq min\ [F_p(t, z, u), 1, F_p(t, z, u). 1, \frac{1}{F_p(z,t,u)}, F_p(z, t, u)]$$

$$F_p(t, z, ku) \geq F_p(t, z, u)$$

By using Lemma 1, $t = z$. Hence proved.

Theorem 2: *Let M, N, U, V be a self-mapping of a Fuzzy Partial Metric space ('𝕏',F_p,$\odot$) with $a \odot b = a \odot b$ satisfying following conditions:*

i) $(M, U) and\ (N, V)$ satisfy E. A. property.
ii) $(M, U) and\ (N, V)$ are weakly compatible.
iii) Then their exist $k \in (0,1)$, for all x, y, z ∈ '𝕏' and $u > 0$.

$$F_p(Mx, Ny, ku) \geq\ [F_p(Ux, Vy, u) \odot \frac{F_p(Ny,Vy,u)}{F_p(Ny,Mx,u)} \odot (F_p(Ux, Vy, u). F_p(Ny, Vy, u)) \odot \frac{F_p(Ny,Vy,u)}{F_p(Ny,Ux,u)} \odot \frac{F_p(Ny,Mx,u)}{F_p(Ny,Vy,u)}]$$

Then M, N, U, V have a common fixed point.

Proof: *Since $(M, U) and\ (N, V)$ satisfying E. A. property then their exist two sequences $\{x_n\}$ and $\{y_n\}$ in '𝕏' such that $\lim_{n\to\infty} Mx_n = \lim_{n\to\infty} Uy_n = \lim_{n\to\infty} Nx_n = \lim_{n\to\infty} Vy_n = z$, where $z \in M('𝕏') \cap N('𝕏')$ or $z \in U('𝕏') \cap V('𝕏')$. Then $z = Ut$, where $x \in$ '𝕏'.*

a) Now to prove that $p(Mt, Ut) = 0$, that is to show that $Mt = z$.

Put $x = t, y = y_n$ in (iii) we get,

$$F_p(Mt, Ny_n, ku) \geq\ [F_p(Ut, Vy_n, u) \odot \frac{F_p(Ny_n,Vy_n,u)}{F_p(Ny_n,Mt,u)} \odot\ (F_p(Ut, Vy_n, u). F_p(Ny_n, Vy_n, u)) \odot \frac{F_p(Ny_n,Vy_n,u)}{F_p(Ny_n,Ut,u)} \odot \frac{F_p(Ny_n,Mt,u)}{F_p(Ny_n,Vy_n,u)}]$$

$$F_p(Mt, Ny_n, ku) \geq\ [F_p(z, z, u) \odot \frac{F_p(Ny_n,z,u)}{F_p(Ny_n,Mt,u)} \odot\ (F_p(z, z, u). F_p(Ny_n, z, u)) \odot \frac{F_p(Ny_n,z,u)}{F_p(Ny_n,z,u)} \odot \frac{F_p(Ny_n,Mt,u)}{F_p(Ny_n,z,u)}]$$

$$F_p(Mt, Ny_n, ku) \geq\ [1 \odot \frac{F_p(Ny_n,z,u)}{F_p(Ny_n,Mt,u)} \odot (1. F_p(Ny_n, z, u)) \odot 1 \odot \frac{F_p(Ny_n,Mt,u)}{F_p(Ny_n,z,u)}]$$

$F_p(Mt, Ny_n, ku) \geq\ F_p(Ny_n, z, u)$, if $Mt = z$.

By using Lemma1, we get $t = z$. Therefore $Mt = Ut = z$

Since (M, U) is weakly compatible. Then $Mz = MUt = UMt = Uz = z$

b) Now to prove that $p(Vz, Nz) = 0$, that is to show that $Nz = z$

Put $x = x_n, y = z$ in (iii) we get, put $Vz = z = Ux_n = Mx_n$, show that $Nz = z$

$$F_p(Mx_n, Nz, ku) \geq\ [F_p(Ux_n, Vz, u) \odot \frac{F_p(Nz,Vz,u)}{F_p(Nz,Mx_n,u)} \odot (F_p(Ux_n, Vz, u). F_p(Nz, Vz, u)) \odot \frac{F_p(Nz,Vz,u)}{F_p(Nz,Ux_n,u)} \odot \frac{F_p(Nz,Mx_n,u)}{F_p(Nz,Vz,u)}]$$

$$F_p(Mx_n, Nz, ku) \geq\ [F_p(z, z, u) \odot \frac{F_p(Nz,z,u)}{F_p(Nz,Mx_n,u)} \odot (F_p(z, z, u). F_p(Nz, z, u)) \odot \frac{F_p(Nz,z,u)}{F_p(Nz,z,u)} \odot \frac{F_p(Nz,Mx_n,u)}{F_p(Nz,z,u)}]$$

$$F_p(Mx_n, Nz, ku) \geq\ [1 \odot \frac{F_p(Nz,z,u)}{F_p(Nz,Mx_n,u)} \odot (1. F_p(Nz, z, u)) \odot 1 \odot \frac{F_p(Nz,Mx_n,u)}{F_p(Nz,z,u)}]$$

$F_p(Mx_n, Nz, ku) \geq F_p(Nz, z, u)$, if $Nz = z$

Since (N, V) is weakly compatible. Then $Nz = Vz = NVz = VNz = z$

c) Now to prove that $p(Mz, Uz) = 0$, that is to show that $Mz = z$.

Put $x = z, y = y_n$ in (iii) we get,

$$F_p(Mz, Ny_n, ku) \geq [F_p(Uz, Vy_n, u) \odot \frac{F_p(Ny_n,Vy_n,u)}{F_p(Ny_n,Mz,u)} \odot (F_p(Uz, Vy_n, u). F_p(Ny_n, Vy_n, u)) \odot \frac{F_p(Ny_n,Vy_n,u)}{F_p(Ny_n,Uz,u)} \odot \frac{F_p(Ny_n,Mz,u)}{F_p(Ny_n,Vy_n,u)}]$$

$$F_p(Mz, Ny_n, ku) \geq [F_p(z, z, u) \odot \frac{F_p(Ny_n,z,u)}{F_p(Ny_n,Mz,u)} \odot (F_p(z, z, u). F_p(Ny_n, z, u)) \odot \frac{F_p(Ny_n,z,u)}{F_p(Ny_n,z,u)} \odot \frac{F_p(Ny_n,Mz,u)}{F_p(Ny_n,z,u)}]$$

$$F_p(Mz, Ny_n, ku) \geq [1 \odot \frac{F_p(Ny_n,z,u)}{F_p(Ny_n,Mz,u)} \odot (1. F_p(Ny_n, z, u)) \odot 1 \odot \frac{F_p(Ny_n,Mz,u)}{F_p(Ny_n,z,u)}]$$

$F_p(Mz, Ny_n, ku) \geq F_p(Ny_n, z, u)$, if $Mz = z$.

Since (M, U) is weakly compatible. Then $Mz = MUz = UMz = Uz = z$

d) Now to prove that $p(Vt, Nt) = 0$, that is to show that $Nt = z$

Put $x = x_n, y = t$ in (iii) we get,

$$F_p(Mx_n, Nt, ku) \geq [F_p(Ux_n, Vt, u) \odot \frac{F_p(Nt,Vt,u)}{F_p(Nt,Mx_n,u)} \odot (F_p(Ux_n, Vt, u). F_p(Nt, Vt, u)) \odot \frac{F_p(Nt,Vt,u)}{F_p(Nt,Ux_n,u)} \odot \frac{F_p(Nt,Mx_n,u)}{F_p(Nt,Vt,u)}]$$

$$F_p(Mx_n, Nt, ku) \geq [F_p(z, z, u) \odot \frac{F_p(Nt,z,u)}{F_p(Nt,Mx_n,u)} \odot (F_p(z, z, u). F_p(Nt, z, u)) \odot \frac{F_p(Nt,z,u)}{F_p(Nt,z,u)} \odot \frac{F_p(Nt,Mx_n,u)}{F_p(Nt,z,u)}]$$

$$F_p(Mx_n, Nt, ku) \geq [1 \odot \frac{F_p(Nt,z,u)}{F_p(Nt,Mx_n,u)} \odot (1. F_p(Nt, z, u)) \odot 1 \odot \frac{F_p(Nt,Mx_n,u)}{F_p(Nt,z,u)}]$$

$F_p(Mx_n, Nt, ku) \geq F_p(Nt, z, u)$, if $Nt = z$

Since (N, V) is weakly compatible. Then $Nt = Vt = NVt = VNt = z$

Thus, $Mz = Uz = Nz = Vz = z$. Therefore M, N, U, V have a common fixed point.

Uniqueness:

Suppose t & z are two common fixed point of M, N, U, V with $t \neq z$, Then (iii) becomes

$$F_p(Mt, Nz, ku) \geq [F_p(Ut, Vz, u) \odot \frac{F_p(Nz,Vz,u)}{F_p(Nz,Mt,u)} \odot (F_p(Ut, Vz, u). F_p(Nz, Vz, u)) \odot \frac{F_p(Nz,Vz,u)}{F_p(Nz,Ut,u)} \odot \frac{F_p(Nz,Mt,u)}{F_p(Nz,Vz,u)}]$$

Put $Mt = Ut = t, , Nz = Vz = z$

$$F_p(t, z, ku) \geq [F_p(t, z, u) \odot \frac{F_p(z,z,u)}{F_p(z,t,u)} \odot (F_p(t, z, u). F_p(z, z, u)) \odot \frac{F_p(z,z,u)}{F_p(z,t,u)} \odot \frac{F_p(z,t,u)}{F_p(z,z,u)}]$$

$$F_p(t, z, ku) \geq [F_p(t, z, u) \odot \frac{1}{F_p(z,t,u)} \odot (F_p(t, z, u). 1) \odot \frac{1}{F_p(z,t,u)} \odot \frac{F_p(z,t,u)}{1}]$$

$$F_p(t, z, ku) \geq F_p(t, z, u)$$

By using Lemma 1, $t = z$. Hence proved.

Definition 9: *Let φ be the class of all mapping φ : [0, 1] → [0,1] such that*

i) φ is non-decreasing and $\lim_{n\to\infty} \varphi^n(l) = 1, \forall\, l \in (0,1]$

ii) $\varphi(l) > l, \forall\, l \in (0,1)$ iii) $\varphi(1) = 1$

Theorem 3: Let M, N, U, V be a self-mapping of a Fuzzy Partial Metric space ('$\mathbb{X}$',F_p,$\odot$) with $a \odot b = min\,(a, b)$ satisfying following conditions:

i) $(M, U) and\ (N, V)$ satisfy E. A. property.
ii) $(M, U) and\ (N, V)$ are weakly compatible.
iii) Then their exist $k \in (0,1)$, for all x, y, z ∈ '$\mathbb{X}$' and $u > 0$.

$$F_p(Mx, Ny, ku) \geq \varphi\{min\,[\frac{F_p(Ux,Vy,u)F_p(Ny,Ux,u)}{F_p(Mx,Vy,u)}, \frac{F_p(Ux,Vy,u)F_p(Ny,Vy,u)}{F_p(Mx,Vy,u)}, \frac{F_p(Ux,Vy,u)F_p(Ny,Vy,u)}{F_p(Ny,Ux,u)},$$

$$\frac{F_p(Ny,Vy,u)F_p(Mx,Vy,u)}{F_p(Ny,Ux,u)}, \frac{F_p(Mx,Vy,u)F_p(Ny,Mx,u)}{F_p(Vy,Ux,u)}]\}$$

Then M, N, U, V have a common fixed point.

Proof: *Since $(M, U) and\ (N, V)$ satisfying E. A. property then their exist two sequences $\{x_n\}$ and $\{y_n\}$ in '$\mathbb{X}$' such that $\lim_{n\to\infty} Mx_n = \lim_{n\to\infty} Uy_n = \lim_{n\to\infty} Nx_n = \lim_{n\to\infty} Vy_n = z$, where $z \in M('\mathbb{X}') \cap N('\mathbb{X}')$ or $z \in U('\mathbb{X}') \cap V('\mathbb{X}')$. Then $z = Ut$, where $x \in$ '$\mathbb{X}$'.*

a) Now to prove that $p(Mt, Ut) = 0$, that is to show that $Mt = z$.

Put $x = t, y = y_n$ in (iii) we get,

$$F_p(Mt, Ny_n, ku) \geq \varphi\{min\,[\frac{F_p(Ut,Vy_n,u)F_p(Ny_n,Ut,u)}{F_p(Mt,Vy_n,u)}, \frac{F_p(Ut,Vy_n,u)F_p(Ny_n,Vy_n,u)}{F_p(Mt,Vy_n,u)}, \frac{F_p(Ut,Vy_n,u)F_p(Ny,Vy_n,u)}{F_p(Ny_n,Ut,u)},$$

$$\frac{F_p(Ny_n,Vy_n,u)F_p(Mt,Vy_n,u)}{F_p(Ny_n,Ut,u)}, \frac{F_p(Mt,Vy_n,u)F_p(Ny_n,Mt,u)}{F_p(Vy_n,Ut,u)}]\}$$

$$F_p(Mt, Ny_n, ku) \geq \varphi\{min\,[\frac{F_p(z,z,u)F_p(Ny_n,z,u)}{F_p(Mt,z,u)}, \frac{F_p(z,z,u)F_p(Ny_n,z,u)}{F_p(Mt,z,u)}, \frac{F_p(z,z,u)F_p(Ny,z,u)}{F_p(Ny_n,z,u)},$$

$$\frac{F_p(Ny_n,z,u)F_p(Mt,z,u)}{F_p(Ny_n,z,u)}, \frac{F_p(Mt,z,u)F_p(Ny_n,Mt,u)}{F_p(z,z,u)}]\}$$

$$F_p(Mt, Ny_n, ku) \geq \varphi\{min\,[\frac{1.F_p(Ny_n,z,u)}{F_p(Mt,z,u)}, \frac{1.F_p(Ny_n,z,u)}{F_p(Mt,z,u)}, 1.1, 1.F_p(Mt, z, u), \frac{F_p(Mt,z,u)F_p(Ny_n,Mt,u)}{1}]\}$$

$$F_p(Mt, Ny_n, ku) \geq \varphi\{min\,[\frac{F_p(Ny_n,z,u)}{F_p(Mt,z,u)}, \frac{F_p(Ny_n,z,u)}{F_p(Mt,z,u)}, 1, 1.F_p(Mt, z, u), F_p(Mt, z, u)F_p(Ny_n, Mt, u)]\}$$

$$F_p(Mt, Ny_n, ku) \geq \varphi\{min\,[\frac{F_p(Ny_n,z,u)}{F_p(z,z,u)}, \frac{F_p(Ny_n,z,u)}{F_p(z,z,u)}, 1, 1.F_p(z, z, u), F_p(z, z, u)F_p(Ny_n, z, u)]\},$$

if $Mt = z$.

$F_p(Mt, Ny_n, ku) \geq \varphi\{min\,[F_p(Ny_n, z, u), F_p(Ny_n, z, u), 1, 1, F_p(Ny_n, z, u)]\}$, if $Mt = z$.

$F_p(Mt, Ny_n, ku) \geq \varphi\{F_p(Ny_n, z, u)\}$, if $Mt = z$

Since $\varphi(l) \geq l$, for all $l \in (0,1]$, it is possible only if $F_p(Ny_n, z, u) = 1$

$F_p(Mt, Ny_n, ku) \geq F_p(Ny_n, z, u)$, if $Mt = z$

By using Lemma1, we get $t = z$. Therefore $Mt = Ut = z$
Since (M, U) is weakly compatible. Then $Mz = MUt = UMt = Uz = z$

b) Now to prove that $p(Vz, Nz) = 0$, that is to show that $Nz = z$

Put $x = x_n, y = z$ in (iii) we get,

$$F_p(Mx_n, Nz, ku) \geq \varphi\{min\,[\frac{F_p(Ux_n,Vz,u)F_p(Nz,Ux_n,u)}{F_p(Mx_n,Vz,u)}, \frac{F_p(Ux_n,Vz,u)F_p(Nz,Vz,u)}{F_p(Mx_n,Vz,u)}, \frac{F_p(Ux_n,Vz,u)F_p(Nz,Vz,u)}{F_p(Nz,Ux_n,u)},$$

$$\frac{F_p(Nz,Vz,u)F_p(Mx_n,Vz,u)}{F_p(Nz,Ux_n,u)}, \frac{F_p(Mx_n,Vz,u)F_p(Nz,Mx_n,u)}{F_p(Vz,Ux_n,u)}]\}$$

$$F_p(Mx_n, Nz, ku) \geq \varphi\{min\,[\frac{F_p(Ux_n,z,u)F_p(Nz,Ux_n,u)}{F_p(Mx_n,z,u)}, \frac{F_p(Ux_n,z,u)F_p(Nz,z,u)}{F_p(Mx_n,z,u)}, \frac{F_p(Ux_n,z,u)F_p(Nz,z,u)}{F_p(Nz,Ux_n,u)},$$

$$\frac{F_p(Nz,z,u)F_p(Mx_n,z,u)}{F_p(Nz,Ux_n,u)}, \frac{F_p(Mx_n,z,u)F_p(Nz,Mx_n,u)}{F_p(z,Ux_n,u)}]\}$$

$$F_p(Mx_n, Nz, ku) \geq \varphi\{min\ [\frac{F_p(z,z,u)F_p(Nz,z,u)}{F_p(Mx_n,z,u)}, \frac{F_p(z,z,u)F_p(Nz,z,u)}{F_p(Mx_n,z,u)}, \frac{F_p(Uz,z,u)F_p(Nz,z,u)}{F_p(Nz,z,u)},$$
$$\frac{F_p(Nz,z,u)F_p(Mx_n,z,u)}{F_p(Nz,z,u)}, \frac{F_p(Mx_n,z,u)F_p(Nz,Mx_n,u)}{F_p(z,z,u)}]\}$$

$$F_p(Mx_n, Nz, ku) \geq \varphi\{min\ [\frac{F_p(z,z,u)F_p(Nz,z,u)}{F_p(z,z,u)}, \frac{F_p(z,z,u)F_p(Nz,z,u)}{F_p(z,z,u)}, \frac{F_p(Uz,z,u)F_p(Nz,z,u)}{F_p(Nz,z,u)},$$
$$\frac{F_p(Nz,z,u)F_p(z,z,u)}{F_p(Nz,z,u)}, \frac{F_p(z,z,u)F_p(Nz,z,u)}{F_p(z,z,u)}]\}$$

$$F_p(Mx_n, Nz, ku) \geq \varphi\{min[F_p(Nz,z,u), F_p(Nz,z,u), F_p(Uz,z,u), 1, F_p(Nz,z,u)]\}$$

But $Uz = z$.

$$F_p(Mx_n, Nz, ku) \geq \varphi\{min[F_p(Nz,z,u), F_p(Nz,z,u), F_p(z,z,u), 1, F_p(Nz,z,u)]\}$$

$$F_p(Mx_n, Nz, ku) \geq \varphi\{min[F_p(Nz,z,u), F_p(Nz,z,u), 1\ , 1, F_p(Nz,z,u)]\}$$

$$F_p(Mx_n, Nz, ku) \geq \varphi\{F_p(Nz,z,u)\}$$

$$F_p(Mx_n, Nz, ku) \geq F_p(Nz,z,u)$$

Since $\varphi(l) \geq l$, for all $l \in (0,1]$, it is possible only if $F_p(Nz,z,u) = 1$
By using Lemma1, we get $Nz = z$. Therefore $Nz = Vz = z$
Since (N, V) is weakly compatible. Then $Nz = Vz = NVz = VNz = z$

c) Now to prove that $p(Mz, Uz) = 0$, that is to show that $Mz = z$.

Put $x = z, y = y_n$ in (iii) we get,

$$F_p(Mz, Ny_n, ku) \geq \varphi\{min\ [\frac{F_p(Uz,Vy_n,u)F_p(Ny_n,Uz,u)}{F_p(Mz,Vy_n,u)}, \frac{F_p(Uz,Vy_n,u)F_p(Ny_n,Vy_n,u)}{F_p(Mz,Vy_n,u)}, \frac{F_p(Uz,Vy_n,u)F_p(Ny,Vy_n,u)}{F_p(Ny_n,Uz,u)},$$
$$\frac{F_p(Ny_n,Vy_n,u)F_p(Mt,Vy_n,u)}{F_p(Ny_n,Uz,u)}, \frac{F_p(Mz,Vy_n,u)F_p(Ny_n,Mz,u)}{F_p(Vy_n,Uz,u)}]\}$$

$$F_p(Mz, Ny_n, ku) \geq \varphi\{min\ [\frac{F_p(z,z,u)F_p(Ny_n,z,u)}{F_p(Mz,z,u)}, \frac{F_p(z,z,u)F_p(Ny_n,z,u)}{F_p(Mz,z,u)}, \frac{F_p(z,z,u)F_p(Ny,z,u)}{F_p(Ny_n,z,u)},$$
$$\frac{F_p(Ny_n,z,u)F_p(Mz,z,u)}{F_p(Ny_n,z,u)}, \frac{F_p(Mz,z,u)F_p(Ny_n,Mz,u)}{F_p(z,z,u)}]\}$$

$$F_p(Mz, Ny_n, ku) \geq \varphi\{min\ [\frac{1.F_p(Ny_n,z,u)}{F_p(Mz,z,u)}, \frac{1.F_p(Ny_n,z,u)}{F_p(Mz,z,u)}, 1.1, 1. F_p(Mz,z,u), \frac{F_p(Mz,z,u)F_p(Ny_n,Mz,u)}{1}]\}$$

$$F_p(Mz, Ny_n, ku) \geq \varphi\{min\ [\frac{F_p(Ny_n,z,u)}{F_p(Mz,z,u)}, \frac{F_p(Ny_n,z,u)}{F_p(Mz,z,u)}, 1, 1. F_p(Mz,z,u), F_p(Mz,z,u)F_p(Ny_n,Mz,u)]\}$$

$$F_p(Mz, Ny_n, ku) \geq \varphi\{min\ [\frac{F_p(Ny_n,z,u)}{F_p(z,z,u)}, \frac{F_p(Ny_n,z,u)}{F_p(z,z,u)}, 1, 1. F_p(z,z,u), F_p(z,z,u)F_p(Ny_n,z,u)]\}, \text{ if } Mz = z.$$

$$F_p(Mz, Ny_n, ku) \geq \varphi\{min\ [F_p(Ny_n,z,u), F_p(Ny_n,z,u), 1, 1, F_p(Ny_n,z,u)]\}, \text{ if } Mz = z.$$

$$F_p(Mz, Ny_n, ku) \geq \varphi\{F_p(Ny_n,z,u)\}, \text{ if } Mz = z$$

Since $\varphi(l) \geq l$, for all $l \in (0,1]$, it is possible only if $F_p(Ny_n,z,u) = 1$
$F_p(Mz, Ny_n, ku) \geq F_p(Ny_n,z,u)$, if $Mz = z$
By using Lemma1, we get $z = z$. Therefore $Mz = Uz = z$
Since (M, U) is weakly compatible. Then $Mz = MUz = UMz = Uz = z$

d) Now to prove that $p(Vt, Nt) = 0$, that is to show that $Nt = z$

Put $x = x_n, y = t$ in (iii) we get,

$$F_p(Mx_n, Nt, ku) \geq \varphi\{min\ [\frac{F_p(Ux_n,Vt,u)F_p(Nt,Ux_n,u)}{F_p(Mx_n,Vt,u)}, \frac{F_p(Ux_n,Vt,u)F_p(Nt,Vt,u)}{F_p(Mx_n,Vt,u)}, \frac{F_p(Ux_n,Vt,u)F_p(Nt,Vt,u)}{F_p(Nt,Ux_n,u)},$$
$$\frac{F_p(Nt,Vt,u)F_p(Mx_n,Vt,u)}{F_p(Nt,Ux_n,u)}, \frac{F_p(Mx_n,Vt,u)F_p(Nt,Mx_n,u)}{F_p(Vt,Ux_n,u)}]\}$$

$F_p(Mx_n, Nt, ku) \geq \varphi\{min\,[\frac{F_p(Ux_n,z,u)F_p(Nt,Ux_n,u)}{F_p(Mx_n,z,u)}, \frac{F_p(Ux_n,z,u)F_p(Nt,z,u)}{F_p(Mx_n,z,u)}, \frac{F_p(Ux_n,z,u)F_p(Nt,z,u)}{F_p(Nz,Ux_n,u)},$

$\frac{F_p(Nz,z,u)F_p(Mx_n,z,u)}{F_p(Nz,Ux_n,u)}, \frac{F_p(Mx_n,z,u)F_p(Nz,Mx_n,u)}{F_p(z,Ux_n,u)}]\}$

$F_p(Mx_n, Nt, ku) \geq \varphi\{min\,[\frac{F_p(z,z,u)F_p(Nt,z,u)}{F_p(Mx_n,z,u)}, \frac{F_p(z,z,u)F_p(Nt,z,u)}{F_p(Mx_n,z,u)}, \frac{F_p(Uz,z,u)F_p(Nt,z,u)}{F_p(Nt,z,u)},$

$\frac{F_p(Nt,z,u)F_p(Mx_n,z,u)}{F_p(Nt,z,u)}, \frac{F_p(Mx_n,z,u)F_p(Nt,Mx_n,u)}{F_p(z,z,u)}]\}$

$F_p(Mx_n, Nt, ku) \geq \varphi\{min\,[\frac{F_p(z,z,u)F_p(Nt,z,u)}{F_p(z,z,u)}, \frac{F_p(z,z,u)F_p(Nt,z,u)}{F_p(z,z,u)}, \frac{F_p(Uz,z,u)F_p(Nt,z,u)}{F_p(Nt,z,u)},$

$\frac{F_p(Nt,z,u)F_p(z,z,u)}{F_p(Nt,z,u)}, \frac{F_p(z,z,u)F_p(Nt,z,u)}{F_p(z,z,u)}]\}$

$F_p(Mx_n, Nt, ku) \geq \varphi\{min[F_p(Nt,z,u), F_p(Nt,z,u), F_p(Uz,z,u), 1, F_p(Nt,z,u)]\}$

But $Uz = z$.

$F_p(Mx_n, Nt, ku) \geq \varphi\{min[F_p(Nt,z,u), F_p(Nt,z,u), F_p(z,z,u), 1, F_p(Nt,z,u)]\}$

$F_p(Mx_n, Nt, ku) \geq \varphi\{min[F_p(Nt,z,u), F_p(Nt,z,u), 1, 1, F_p(Nt,z,u)]\}$

$F_p(Mx_n, Nt, ku) \geq \varphi\{F_p(Nt,z,u)\}$

$F_p(Mx_n, Nt, ku) \geq \varphi\{F_p(Mx_n,z,u)\}$, if $Nt = z$

Since $\varphi(l) \geq l$, for all $l \in (0,1]$, it is possible only if $F_p(Mx_n, z, u) = 1$

$F_p(Mx_n, Nt, ku) \geq F_p(Mx_n, z, u)$, if $Nt = z$. By using Lemma1, we get $= z$.

Therefore $Nt = Vt = z$
Since (N, V) is weakly compatible. Then $Nt = NVt = VNt = Vt = z$
Thus, $Mz = Uz = Nz = Vz = z$. Therefore M, N, U, V have a common fixed point.
Uniqueness:
Suppose t & z are two common fixed point of M, N, U, V with $t \neq z$,Then (iii)becomes

$F_p(Mt, Nz, ku) \geq \varphi\{min\,[\frac{F_p(Ut,Vz,u)F_p(Nz,Ut,u)}{F_p(Mt,Vz,u)}, \frac{F_p(Ut,Vz,u)F_p(Nz,Vz,u)}{F_p(Mt,Vz,u)}, \frac{F_p(Ut,Vz,u)F_p(Nz,Vz,u)}{F_p(Nz,Ut,u)},$

$\frac{F_p(Nz,Vz,u)F_p(Mt,Vz,u)}{F_p(Nz,Ut,u)}, \frac{F_p(Mt,Vz,u)F_p(Nz,Mt,u)}{F_p(Vz,Ut,u)}]\}$

Put $Mt = Ut = t$, , $Nz = Vz = z$

$F_p(t, z, ku) \geq \varphi\{min\,[\frac{F_p(t,z,u)F_p(z,t,u)}{F_p(t,z,u)}, \frac{F_p(t,z,u)F_p(z,z,u)}{F_p(t,z,u)}, \frac{F_p(t,z,u)F_p(z,z,u)}{F_p(z,t,u)},$

$\frac{F_p(z,z,u)F_p(t,z,u)}{F_p(z,t,u)}, \frac{F_p(t,z,u)F_p(z,t,u)}{F_p(z,t,u)}]\}$

$F_p(t, z, ku) \geq \varphi\{min[F_p(z,t,u), 1, 1, 1, F_p(t,z,u)]\}$

$F_p(t, z, ku) \geq \varphi\{F_p(z,t,u)\}$, Since $\varphi(l) \geq l$, for all $l \in (0,1]$, it is possible only if $F_p(t, z, u) = 1$

$F_p(t, z, ku) \geq F_p(t, z, u)$

That is $t = z$. Hence Proved.
Generalization Theorem: Let $M = \{M_1, M_2, \dots\dots, M_m\}, N = \{N_1, N_2, \dots\dots, N_n\}$,
$U = \{U_1, U_2, \dots\dots, U_u\}, V = \{V_1, V_2, \dots\dots, V_v\}$ be a self-mapping of a Fuzzy PMS (‘$\mathbb{X}$’,F_p,$\odot$) with $a \odot b = a \odot b$ satisfying following conditions:

1) $(M, U) and\ (N, V)$ satisfy E. A. property.
2) $(M, U) and\ (N, V)$ are weakly compatible.
3) Then their exist $k \in (0,1)$, for all x, y, z ∈ ‘$\mathbb{X}$’ and $u > 0$.

$F_p(Mx, Ny, ku) \geq [F_p(Ux, Vy, u) \odot \frac{F_p(Ny,Vy,u)}{F_p(Ny,Mx,u)} \odot (F_p(Ux, Vy, u). F_p(Ny, Vy, u)) \odot$

$\frac{F_p(Ny,Vy,u)}{F_p(Ny,Ux,u)} \odot \frac{F_p(Ny,Mx,u)}{F_p(Ny,Vy,u)}]$

Then M, N, U, V have a common fixed point.

4. Conclusion

This paper establishes a common fixed point theorem for two weakly compatible mappings in FPMS with the E.A. property. Our result generalizes existing fixed point theorems in FPMS by relaxing the compatibility condition and introducing the E.A. property. The E.A. property allows for more flexibility in modeling real-world problems with uncertain or imprecise data.

The main result provides a sufficient condition for the existence of a common fixed point, which is a crucial concept in fixed point theory with numerous applications in various fields. The theorem's proof demonstrates the significance of the E.A. property in establishing the convergence of sequences in FPMS.

Real life Example in computer is Image Compression with Fuzzy Similarity.

Imagine a situation where we want to compress an image while maintaining a degree of "fuzzy" similarity. Instead of exact pixel-by-pixel comparison, we might want to consider how similar groups of pixels are, and how that similarity changes as we compress.

A fuzzy partial metric could model this similarity. Instead of a crisp distance, the distance between two groups of pixels could be fuzzy – representing a range of possible similarities. This fuzzy partial metric would need to satisfy specific axioms of partial metric spaces. The E-A property would be a condition ensuring that the compression algorithm converges to a "best" approximation of the original image within the constraints of the fuzzy similarity. The fixed point would represent the compressed image. It's the image where the compression process, using the fuzzy partial metric, doesn't lead to a further approximation. It's the "most optimal" image representation according to the specific fuzzy similarity.

Future scope

- Exploring the relationships between the E.A. property and other fuzzy partial metric space properties.
- Investigating the applications of this theorem in image processing, machine learning, and optimization problems.
- Extending the result to more general fuzzy partial metric spaces or other fuzzy structures.

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