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Research paper

Strategic supply chain optimization for eco-friendly products: impact of preservation investment, carbon tax, marketing strategy and pricing on environmental sustainability

Sachin Kumar Verma [1], Anupama Sharma [2], Amit Kumar [3], Vipin Kumar [2] *, Sanjay Kumar Sharma [4]

[1] *Department of Mathematics, Dewan Institute of Management Studies, Meerut*
[2] *Department of Mathematics, B.K. Birla Institute of Engineering and Technology, Pilani Raj. India*
[3] *Department of Mathematics, Shaheed Mangal Pandey Government Girls P.G. College, Meerut, India*
[4] *Department of Applied Science and humanities Sobhasaria Group of Institutes, Sikar, (Raj) 332001, India.*
**Corresponding author E-mail: drvkmaths@gmail.com*

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Abstract

The presents research paper of Economic order quantity includes the impact of multiple parameter demand which heighted the impact of these parameter to achieve the optimum result. This research paper uses the advertisement, selling price, and green product demand. Additionally, as the impact of deterioration results in reducing profit, the concept of preservation is included in this paper. Also, the partial backlog shortage situation has been part of this paper. After observing the past few years trend, it has come to notice that the concept of sustainability of product and carbon tax in supply chain optimization attracts the focus of management of any organization to gain expected results. Numerical examples and sensitivity analysis reveal how firms can gain optimum while mentioning environmental sustainability.

Keywords: *Economic Order Quantity; Sustainable Supply Chain Optimization; Carbon Tax; Green Product Demand; Preservation Technology Investments; Advertisement Frequency; Partially Backlogged Shortages.*

1. Introduction

1.1. Advertisement

Saha & Chakrabarti (2018) developed the concept where payment related situation highlighted under the effect of advertisement cost in supply chain management of inventory. Shaikh et al. (2020) explore the concept where demand of product based on selling price of items of non-instantaneous deteriorating product under the effect of trade credit. Shaikh et al. (2019) discuss the deterioration rate which is follow the pattern of Weibull distributed. Also impact of trade credit and frequency of advertisement also highlighted. Additionally, San-José et al. (2021) discovered the optimal order policy which integrate the demand dependent on price, time and frequency of advertisement. Shah & Vaghela (2017) integrate inflation with time and advertisement dependent demand in his model. The aim of this model is to optimizing the objective function which leads to the desired result. Rathore (2019) used concept of reliability in assumption in developing the model. This paper studied the impact of advertisement dependent demand and reliability which can lead to desired result in the favour of growth of organization. Khan (2020) investigated the effects of advanced payment on inventory model under the impact of advertisement and selling price dependent demand rate.

1.2. Demand

Poswal et al. (2022) used the inventory items which have degradation rate satisfied by Weibull distribution under the environment of fuzzy optimization, this research paper helps the management in taking decision about the situation of quadratic demand linked with the times-based holding cost. Verma et al. (2019) highlighted the concept of demand pattern which is flexible during the time, as the time change inventory start follow the different demand, by doing this researcher try to resolve the issue related to optimization of such kind of objective function with the aim of best ordering polices. Additionally, Verma et al. (2018) integrate the price of selling linked demand with holding cost and developed the model of items with is deteriorated in nature. Also, impact of exponent time function demand explores the model and suggest optimal result which help origination to deal competitive environment of current robust market. Trade credit environment and their impact is highlighted in the model developed by Verma et al. (2021). The aim of this research article is to integrate the stock dependent demand, partial backlogging to achieve the optimal ordering policy for deteriorating items. On the other hand, Singh & Goel (2024b) balanced the hybrid demand and situation of trade credit in post payment help in management to deal with real life rigours competitive environment. Also, Selling Price Demand Rate discussed and developed by the Verma et al. (2018). Researcher proposed the assumption in which such kind of situation faced by the management by balancing th effect of Holding Cost with Partial Backlogging and Constant Deteriorating rate. Rizwanullah et al. (2021) highlighted and offer suggestion to deal with taught market situation in handling the situation of supply-chain two-warehouse, additionally Researcher integrate this situation in inflation backlog rate to give the real touch of the model. Singh & Rana (2023) investigated the model which touch the impact of sustainable with the production of inventory items with encompassing the trade credit environment and partial backlogging. Mohd et al. (2017) used a very unique concept lingo based three-tier approach in supply chain, which integrate the dynamics of supply chain in real life situation to achieve the desired optimum result. Bhattacharjee N, Sen N (2022) A multi-item sustainable production inventory-constrained model to study and analyze the effective green investment and replenishment quantity. Kumar et al. (2022) projects an inventory model when demands is multivariate and partial backlogging. Singh & Goel (2024a) developed the model with the impact of inflation with balancing waste which comes from textile industry, by doing this researcher try to develop the concept in solving problem relate to reverse logistics model where waste of items can be make reusable under supply chain process of inventory items. Kumar et al. (2023) discuses an inventory model non-instantaneous deteriorating items with permissible delay in payments under inflation. Singh et al. (2024) highlighted the concept of Expiration Dates Under Advance-Cash-Credit Payment Policy

1.3. Carbon emissions and energy consumption

Anupama et al. (2024) proposed an ideal optimal ordering strategy for the model through the lens of Carbon Emissions and Energy Consumption in two warehouse model of green inventory. Cheng et al. (2022) explored the model where carbon emission impact in total expenditure of organization is investigated in supply chain of deteriorated inventory. The studies by Sebatjane (2024a) emphasize the importance of sustainable inventory also integrate the green technology, stock-dependent demand and carbon emissions tax regulation. On the other hand, the focused investigation by Sebatjane (2024b) highlighted the carbon emissions regulations while discussing the effect of carbon in environment. Carbon emissions policies also developed and highlighted by Sebatjane (2024c) by encompassing the advertisement, expiration date and price-dependent demand in developing the supply chain model to gain effective result which suits to daily life competitive market. Sebatjane (2024d) created a model for imperfect quality product where impact of carbon on environment. Also, negative impact of emissions highlighted during the development of Three-echelon circular economic production–inventory model. Singh & Chaudhary (2023) studied the impact of inflation which include the multivariate demand and explore the impact of partial backlogging balance with carbon tax policy. Vipin Kumar et al. (2024) integrate the Carbon Emission and Inflationary Environment combined with the Ramp-Type Demand on economic order quantity model. Uncertain demand condition integrates with all possible impact of carbon emission highlighted by Wang et al. (2022) and encourages to improve efforts to reducing it for applying in supply chain inventory model.

1.4. Preservation technology

The impact of time value of money in inventory supply chain is developed by Rizwanullah et al. (2021). Also, integration of exponent time dependent function with backlog situation make this model more effective. On the other hand, unavoidable factor like deterioration in dealing with inventory has been the discussed by Sindhuja & Arathi (2023). The aim of this research paper is making better acceptable decision which match up with competitive life style. Also, the integration of concept of dealing with payment in advance to the supplier with Hybrid price and stock dependent inventory has been part of this research which is investigate by Rahman et al. (2021). Additionally, investment in preservation combined with model where production quantity been in initial stage developed by Chang et al. (2022). Aim of this research article is to highlight the multistage supply chain which make the management to take right decision about how much production should can be instigate and when to stop. Biuki et al. (2020) proposed the model of supply chain network with balance with the concept of location routing and sustainable design of a perishable products in managing the inventory which is deteriorated in nature. Also, balancing market strategies with trade credit and preservation factor is applied in this research article developed by Goel & Singh A (2024). Additionally, Expiration Dates Under Advance-Cash-Credit Payment Policy investigated and highlighted by Singh D. (2024) in inventory model. Kumar (2024) integrate the Reworking, Waste, and Sustainable, Production and Inflation and developing the model of inventory items. This paper explores and offer finding to gain optimum result to make organisation profitable.

Author(S)	Model Type	Demand Dependency	Preservation On Investment	Partially Backlogged	Green Investment	Carbon Emission
Jauhari et al. (2023)	Multi-echelon	Stochastic	No	No	No	Yes
Singh and Rana (2023)	EPQ	Time	No	Yes	No	No
Padiyar et al. (2023)	Multi-echelon	Ramp type and selling price	No	Yes	No	No
Poswal et al. (2022a)	EOQ	Quadratic	No	Yes	No	No
Shen et al. (2019)	Multi-echelon	Constant	Yes	No	No	Yes
Rahman et al. (2021)	EOQ	Hybrid price and stock dependent	Yes	Yes	No	No
Singh and Goel (2024)	EOQ	Market potential, consumer desire for green products, price	Yes	Yes	Yes	Yes
This paper	EOQ	Price, frequency of Advertisement, consumer desire for green products	Yes	Yes	Yes	Yes

The Section 2. Include the 'Assumptions and Notations' whereas, Section 3. Explains Mathematical Model Formulation, Section 4 Include the 'Optimal Solution Procedure', Section 5. Explain the Numerical Examples, Sensitivity Analysis and Graphically Representation, on the other hand, Section 6. Observations, Result Analysis and managerial implication and in last Section 7 explain the paper's "Conclusion" with research limitation of the paper and offers suggestions for more research for future scope

2. Assumptions and notations

2.1. Assumptions

This model was developed using the following presumptions.

1) The demand rate $D(A, p, G)$ is dependent on selling price (p) of an item, the frequency of advertisement (A) and consumer desire for green products. i.e. $D(A. p, G) = A^{\gamma}(\alpha p^{-\beta} + aG)$, where $\gamma, \alpha, \beta, a \geq 0$ and A is an integer.
2) Backlog function. $B(T - t) = e^{\delta(T-t)}$, where, $\delta > 0$
3) The investment in preservation technologies affects the degradation of products over time. i.e., decrease in deterioration is given as $m(\xi) = \theta(1 - e^{c\xi})$
4) Only one item is ordered.
5) Partially backlogged shortages are acceptable.
6) The policy of the carbon price is also included in the model. The transportation method and distance travelled during inventory orders have a big influence on emissions. Carbon emissions are caused by inventory storage, although the main sources of emissions are refrigeration and warehouse energy usage.

2.2. Notations

The following notations are used in the development of this model.

Parameter	Discerption	Unit
A	the frequency of advertisement	
O	Ordering cost	Per order
O′	The cost of carbon emissions related to ordering inventories	
Pc	Purchasing cost per unit	
H	Holding cost	
h_1	Unit holding cost	
h_2	The cost of carbon emissions related holding inventory	Per unit
Sc	Shortage cost	
Ls	lost sale cost	
P	Selling price	
α		Constant
β	Demand related constant	Constant
γ		Constant
G	Green degree	Constant
a	Coefficient of the customer's sensitivity about the green degree	
θ	Deterioration Rate	
m(ξ)	Reduced deterioration rate	
ξ	Investment in preservation technology	
ξ′	Carbon emission cost associated with preservation	Constant
Q	Stock level at t=0	
B	Stock level at t=T	
Q	Order Quantity	
T	Replacement Cycle Length	Time unit
t_1	Stock in time	
Abbreviations		
OC	Ordering cost	
HC	Holding cost	
PC	Purchasing cost	
SC	Shortages cost	
LC	Lost sales cost	
PT	Preservation investment	
TAC	Total Average cost	

3. Mathematical model formation

In this model we have assumed Q inventory level at$t = 0$, which declined during $(0, t_1)$ due to combined effect of demand and deterioration. Shortage period start from$t = t_1$, which is partially backlog with backlog rate δ. Shortage period remain till$t = T$. During time interval(t_1, T). Inventory level due to demand only. Here B is negative inventory level at$t = T$,

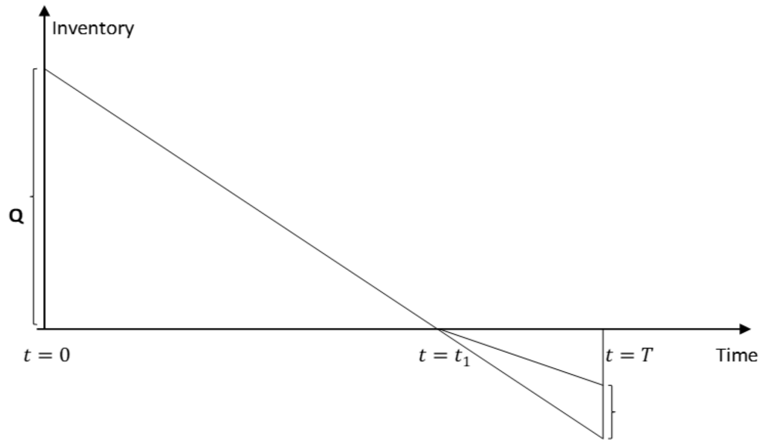


Fig. 1: Represent the Inventory Behaviour with Respect to Time.

$$\frac{dI_1(t)}{dt} + (\theta - m(\xi))I_1(t) = -A^{\gamma}(\alpha p^{-\beta} + aG),\ 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} = A^{\gamma}(\alpha p^{-\beta} + aG)e^{-\delta(T-t)}\ t_1 \le t \le T \tag{2}$$

With boundary conditions $I_1(0) = Q$

$$I_1(t) = \left[\frac{A^{\gamma}(aG-\alpha p^{-\beta})}{\theta(1-c\xi)} + Q\right] e^{-e^{-c\xi}t\theta} - \frac{A^{\gamma}(aG+\alpha p^{-\beta})}{\theta(1-c\xi)} \tag{3}$$

$I_1(t_1) = 0$

$$Q = \frac{A^{\gamma}(aG+\alpha p^{-\beta})}{\theta(1-c\xi)}\left(e^{(1-c\xi)t_1\theta} - 1\right) \tag{4}$$

The solution of the equation (2), is

$$I_2(t) = \frac{A^{\gamma}\left(e^{-(T-t_1)\delta} - e^{-(T-t)\delta}\right)(aG+\alpha p^{-\beta})}{\delta} \tag{5}$$

At $I_2(T) = -B$

$$B = \frac{A^{\gamma}(1-e^{-(T-t_1)\delta})(aG+\alpha p^{-\beta})}{\delta} \tag{6}$$

Ordering quantity

$$L = Q + B = A^{\gamma}\left[\frac{1}{\theta(1-c\xi)}\left(e^{(1-c\xi)t_1\theta} - 1\right) + \frac{1}{\delta}\left(1 - e^{-(T-t_1)\delta}\right)\right]\left(aG + \alpha p^{-\beta}\right) \tag{7}$$

3.1. Ordering cost

$$OC = (O + O') \tag{8}$$

3.2. Holding cost

$$HC = (h + h_1)\frac{\left((1-e^{-(1-c\xi)t_1\theta})Q\theta + A^{\gamma}(aG+p^{-\beta}\alpha)\left((1-e^{-(1-c\xi)t_1\theta})(1-c\xi) - t_1\theta\right)\right)}{\theta^2(1-c\xi)} \tag{9}$$

3.3. Purchasing cost

$$PC = P_cQ = P_cA^{\gamma}\left[\frac{1}{\theta(1-c\xi)}\left(e^{(1-c\xi)t_1\theta} - 1\right) + \frac{1}{\delta}\left(1 - e^{-(T-t_1)\delta}\right)\right]\left(aG + \alpha p^{-\beta}\right) \tag{10}$$

3.4. Shortages cost

$$SC = S_cA^{\gamma}\frac{(aG+\alpha p^{-\beta})(1-e^{-(T-t_1)\delta}(1+T\delta-t_1\delta))}{\delta^2} \tag{11}$$

3.5. Lost sales cost

$$LS = A^{\gamma}Ls(aG + p^{-\beta}\alpha)\left(T - t_1 - \frac{1-e^{-(T-t_1)\delta}}{\delta}\right) \tag{12}$$

3.6. Preservation investment

$$PT = (\xi + \xi') \tag{13}$$

Total Average cost

$$TAC = \frac{1}{T}[OC + HC + PC + SC + LSC + PA]$$

$$TAC = \frac{1}{T}\left[(O + O') + (h + h_1)\frac{\left((1-e^{-(1-c\xi)t_1\theta})Q\theta + A^{\gamma}(aG+p^{-\beta}\alpha)\left((1-e^{-(1-c\xi)t_1\theta})(1-c\xi) - t_1\theta\right)\right)}{\theta^2(1-c\xi)} + P_c A^{\gamma}\left[\frac{1}{\theta(1-c\xi)}\left(e^{(1-c\xi)t_1\theta} - 1\right) + \frac{1}{\delta}\left(1 - e^{-(T-t_1)\,\delta}\right)\right](aG + \alpha p^{-\beta}) + S_c A^{\gamma}\frac{(aG+\alpha p^{-\beta})(1-e^{-(T-t_1)\delta}(1+T\delta-t_1\delta))}{\delta^2} + A^{\gamma}Ls(aG + p^{-\beta}\alpha)\left(T - t_1 - \frac{1-e^{-(T-t_1)\delta}}{\delta}\right) + (\xi + \xi')\right] \tag{14}$$

4. Solution methodology

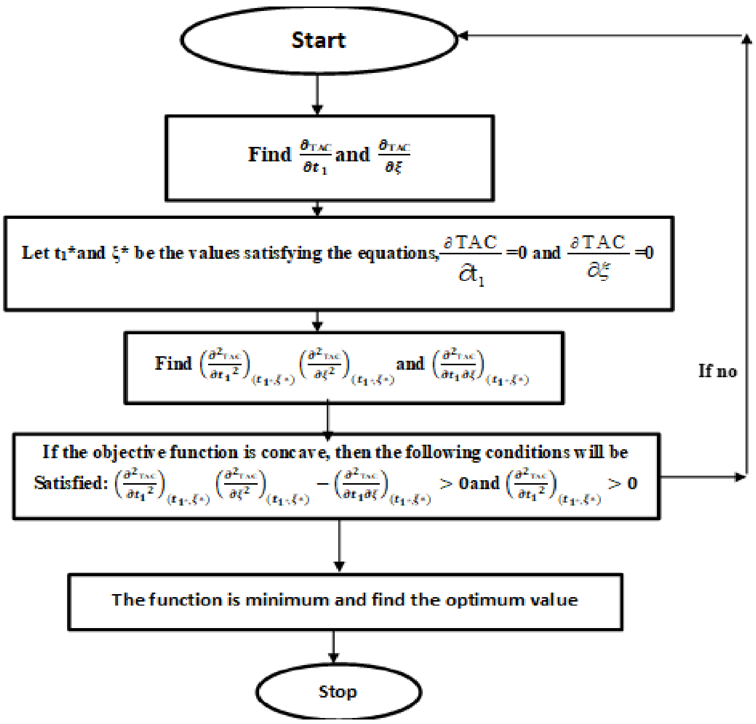
We will the following Mathematical Procedure to find optimal values:
STEP-1. Find $\frac{\partial TAC}{\partial t_1}and\frac{\partial TAC}{\partial \xi}$.

STEP -2. Let t1*and ξ* be the values satisfying the equations,$\frac{\partial TAC}{\partial t_1}$ =0 and $\frac{\partial TAC}{\partial \xi}$ =0

STEP-3. Find $\left(\frac{\partial^2 TAC}{\partial {t_1}^2}\right)_{(t_1*,\xi*)}\left(\frac{\partial^2 TAC}{\partial \xi^2}\right)_{(t_1*,\xi*)}$ and $\left(\frac{\partial^2 TAC}{\partial t_1\,\partial \xi}\right)_{(t_1*,\xi*)}$

STEP-4. If the objective function is concave, then the following conditions will be

Satisfied: $\left(\frac{\partial^2 TAC}{\partial {t_1}^2}\right)_{(t_1*,\xi*)}\left(\frac{\partial^2 TAC}{\partial \xi^2}\right)_{(t_1*,\xi*)} - \left(\frac{\partial^2 TAC}{\partial t_1\,\partial \xi}\right)_{(t_1*,\xi*)} > 0$and $\left(\frac{\partial^2 TAC}{\partial {t_1}^2}\right)_{(t_1*,\xi*)} > 0$



5. Numerical example, sensitivity analysis and graphically representation

5.1. Numerical example

We have use below numerical value for the parameter and coefficient used in this research
Paper to fund optimal value of Total cost function. We used Mathematica- 7 for finding
Optimum result.

$O = 400; O' = 0.5; h = 40; h1 = 0.3; \delta = 0.4; \alpha = 3; a = 0.04; T = 19; \theta = 0.5;$

$c = 0.09; Sc = 4; Ls = 3; \beta = 2; p = 2; \xi 1 = 0.9; Pc = 10; A = 5; \gamma = .4; G = .85;$

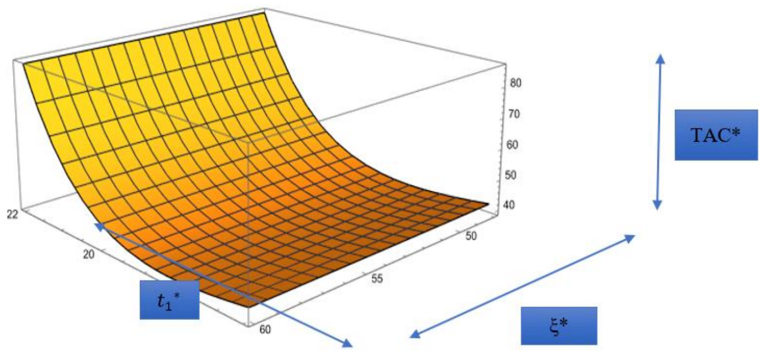


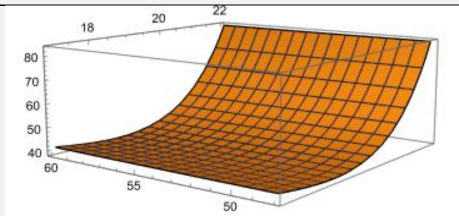
Fig. 2: Graphically Representation of TAC* with Respect to T_1^* & Ξ*.

Demand=$[A^{\gamma}(\alpha * p^{-\beta} + a * G)]$=1.492	Postiveinventory(Q1)=29.34
Optimal costs are,	Negativeinventory(B)=0.0305
OD*=21.3157,PC*=3.695,HC*=15.1398	Optimal order quantity., Q*=(Q1 + B)=29.376
SC*=0.251092,LSC*=0.0796951	

Some special cases

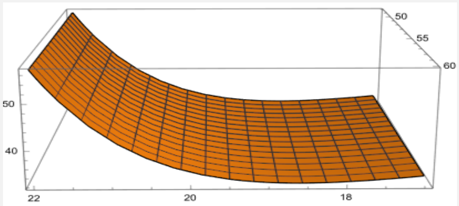
Special case-1

$G = 0$, i.e, No consumer desire for green inventory product involved then optimal decision variables are
$TAC * 38.705$, $t_1^* = 18.56$, $\xi *= 52.89$
Q*=., Demand =1.4277



Special case-2

$\gamma = 0$; i.e., No advertisement involved in demand
$TAC * 32.05$, $t_1^* = 18.55$, $\xi *= 46.66$, Q*=., Demand=0.784



5.2. Sensitivity analysis

Here, we have developed sensitivity analysis based on various parameter and coefficient for the total average cost. Each parameter and coefficient go through the change with the range (-20% to -20%).

Parameter & Coefficients	% change	Change In values	t_1^*	ξ*	TAC*
	-20%	320	18.5702	53.3651	37.3911
	-10%	360	18.5702	53.3651	38.3231
O	0	400	18.5702	53.3651	39.3601
	10%	440	18.5702	53.3651	41.2285
	20%	480	18.5702	53.3651	43.3383
	-20%	0.4	18.5702	53.3651	39.3618
	-10%	0.45	18.5702	53.3651	39.3606
O'	0	0.5	18.5702	53.3651	39.3601
	10%	0.55	18.5702	53.3651	39.3279
	20%	0.6	18.5702	53.3651	39.3255
	-20%	32	18.5705	53.3651	39.3681
	-10%	36	18.5704	53.3651	39.3624
h	0	40	18.5702	53.3651	39.3601
	10%	44	18.5701	53.3651	39.318
	20%	48	18.57	53.3651	39.3118
	-20%	0.24	18.5992	53.3651	39.4001
	-10%	0.27	18.5712	53.3651	39.3992
h'	0	0.3	18.5702	53.3651	39.3601
	10%	0.33	18.5601	53.3651	39.318
	20%	0.36	18.5599	53.3651	39.3118
	-20%	3.2	18.5601	71.09	39.3701
	-10%	3.6	18.5699	54.38	39.3699
δ	0	4	18.5702	53.3651	39.3601
	10%	4.4	18.5782	53.3743	39.1339
	20%	4.8	18.5858	53.3832	39.1223
	-20%	2.4	18.5702	53.3651	39.1011
α	-10%	2.7	18.5702	53.3651	39.1012
	0	3	18.5702	53.3651	39.3601

		10%	3.3	18.5702	53.3651	40.1118
		20%	3.6	18.5702	53.3651	40.212
		-20%	0.032	18.5701	53.2728	39.2283
		-10%	0.036	18.5702	53.319	39.2942
	a	0	0.04	18.5702	53.3651	39.3601
		10%	0.044	-	-	-
		20%	0.048	-	-	-
		-20%	17.2	-	-	-
		-10%	18.1	-	-	-
	T	0	19	18.5702	53.3651	39.3601
		10%	19.9	-	-	-
		20%	20.8	20.3719	55.3748	37.3431
		-20%	0.042	18.571	51.4728	39.2581
		-10%	0.045	18.5709	52.1994	39.2985
	θ	0	0.05	18.5702	53.3651	39.3601
		10%	0.055	18.57	54.4241	39.4158
		20%	0.06	18.5692	55.3909	39.4667
		-20%	3.2	18.6902	53.0299	74.238
		-10%	3.6	18.6721	53.3012	39.3985
	Sc	0	4	18.5702	53.3651	39.3601
		10%	4.4	18.4599	55.4029	39.37
		20%	4.8	18.4091	55.422	39.3698
		-20%	1.4	18.5608	54.212	39.2112
		-10%	1.6	18.5601	54.332	39.3512
	Ls	0	3	18.5702	53.3651	39.3601
		10%	2.2	18.5816	53.221	39.368
		20%	2.4	18.5893	53.22	39.379
		-20%	1.6	18.5798	53.988	103.47
		-10%	1.8	18.5791	53.871	41.515
	β	0	2	18.5702	53.3651	39.3601
		10%	2.2	18.5639	53.201	35.368
		20%	2.4	18.563	53.2	35.824
		-20%	1.6	18.576	57.96	39.4021
		-10%	1.8	18.573	55.51	39.402
p		0	2	18.5702	53.36	39.3601
		10%	2.2	18.566	51.45	38.001
		20%	2.4	18.566	51.01	37.001
		-20%	0.72	18.5702	53.365	39.3506
		-10%	0.81	18.5702	53.365	39.3553
	ξ1	0	0.9	18.5702	53.3651	39.3601
		10%	0.99	18.5702	53.3652	39.364
		20%	1.8	18.5702	53.3652	39.407
		-20%	0.072	18.3982	58.0589	39.6989
		-10%	0.081	18.3982	58.0574	39.6689
	c	0	0.09	18.5702	53.3651	39.3601
		10%	0.099	18.572	49.4334	39.1024
		20%	1.08	18.5877	46.70658	36.3823
		-20%	0.68	18.5701	53.2751	39.3301
		-10%	0.765	18.5701	53.3151	39.3501
	G	0	0.85	18.5702	53.3651	39.3601
		10%	0.935	18.5703	53.3751	39.3721
		20%	1.02	18.5712	53.3851	39.3782
		-20%	8	18.6582	51.121	36.365
		-10%	9	18.6142	52.3056	38.86919
	Pc	0	10	18.5702	53.3651	39.3601
		10%	11	18.57	54.322	39.9499
		20%	12	18.5257	55.1935	42.3066
		-20%	4	18.5685	52.4214	36.061
		-10%	4.5	18.5694	52.919	38.7328
	A	0	5	18.5702	53.3651	39.3601
		10%	5.5	18.5709	53.7692	39.9499
		20%	6	18.5716	54.1388	40.5076
		-20%	0.32	18.5676	52.0052	37.5207
		-10%	0.36	18.569	52.6846	38.4119
	γ	0	0.4	18.5702	53.3651	39.3601
		10%	0.44	18.5714	54.377	40.3688
		20%	0.45	18.5725	54.388	41.4423

5.3. Graphically representation

Here, bases on above sensitivity in section 5.2, graphically representation has been developed for each factor used in paper.

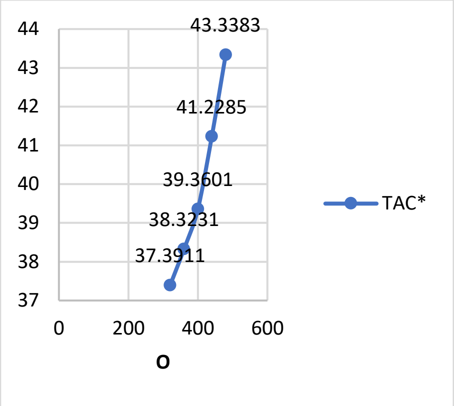


Fig. 2: Represent Variation In TAC* vs O.

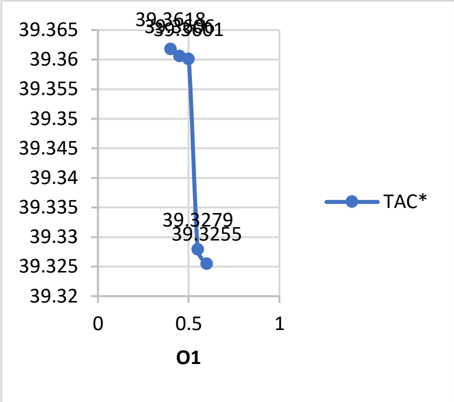


Fig. 3: Represent Variation In TAC* vs O′.

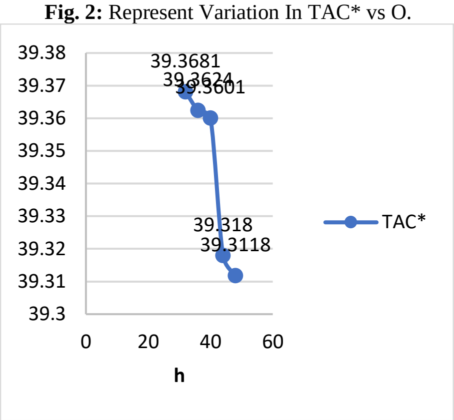


Fig. 3: Represent Variation in TAC* vs

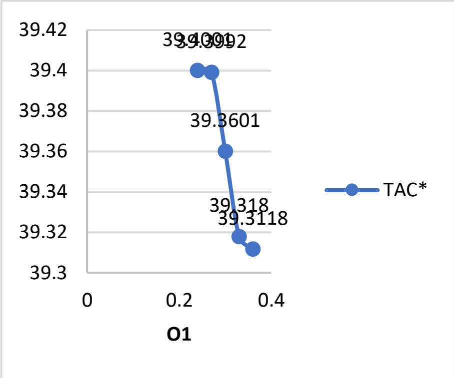


Fig. 3: Represent Variation in TAC* vs.

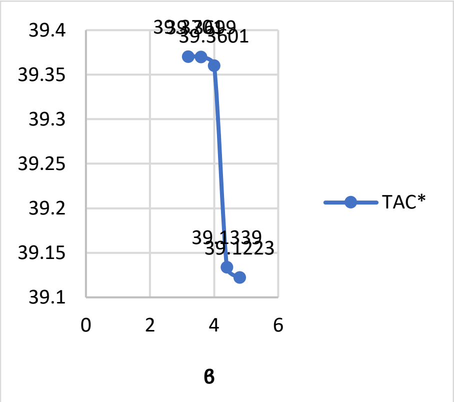


Fig. 3: Represent Variation in TAC* vs.

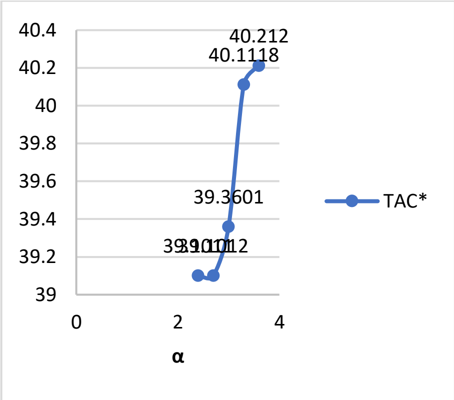


Fig. 3: Represent Variation in TAC* Vs.

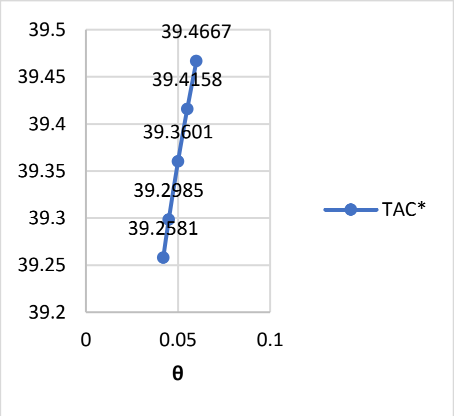


Fig. 3: Represent Variation in TAC* vs.

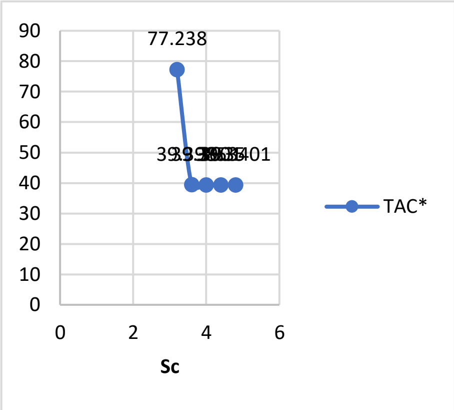


Fig. 3: Represent Variation in TAC* vs.

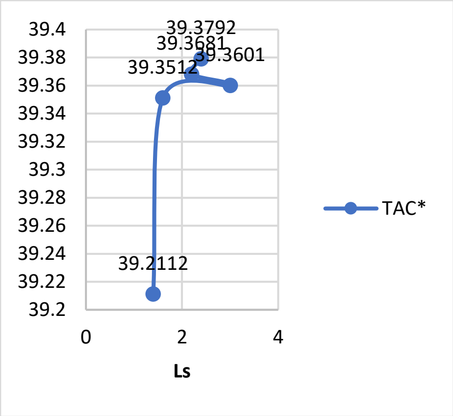


Fig. 3: Represent Variation in TAC* vs.

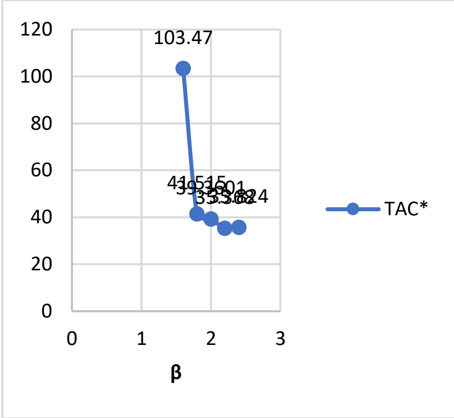


Fig. 3: Represent Variation in TAC* vs.

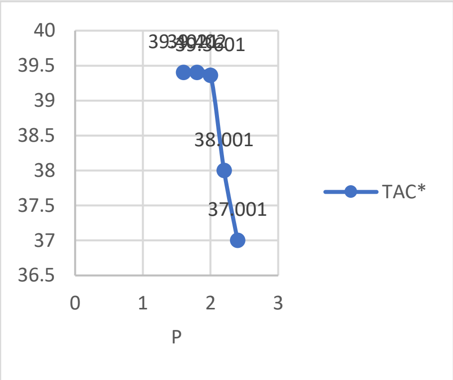


Fig. 3: Represent Variation in TAC* vs.

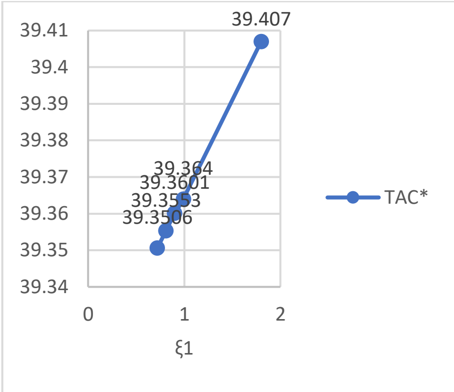


Fig. 3: Represent Variation in TAC* vs.

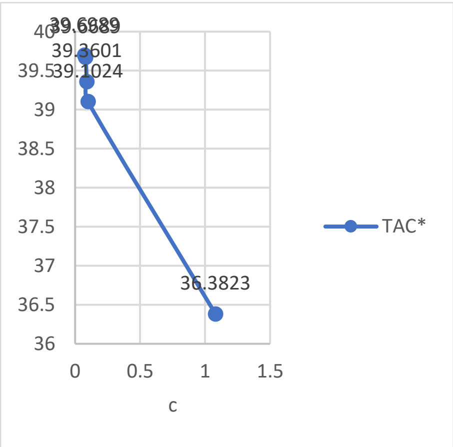


Fig. 3: Represent Variation in TAC* vs. C.

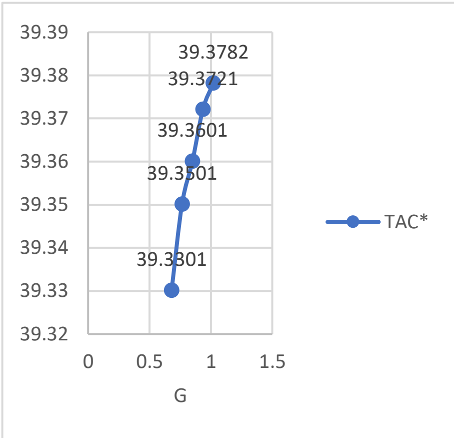


Fig. 3: Represent Variation in TAC* vs. G.

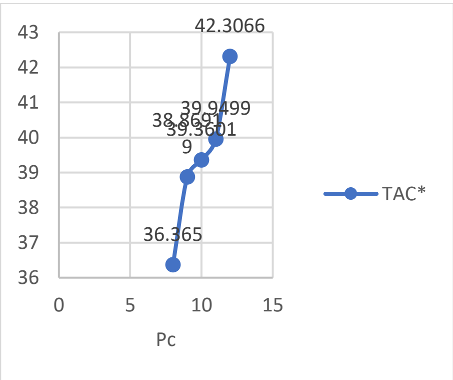


Fig. 3: Represent Variation in TAC* vs. Pc.

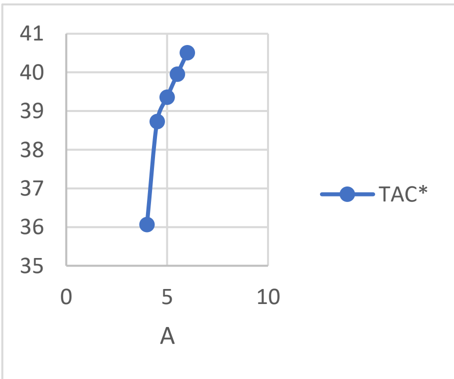


Fig. 3: Represent Variation in TAC* vs. A.

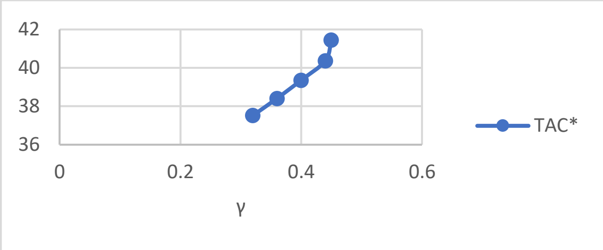


Fig. 3: Represent Variation in TAC* vs. Γ.

6. Result analysis and managerial implication

6.1. Result analysis

We observed from section 5 and notices some observation which have explained below.
Result-1. For incremental change in (O) & (O′), we noticed the NO change in stock in time (t_1^*) & (ξ*) but incremented change in Total Average cost (TAC*).
Result-2. For incremental change in (h) & (h'), we noticed the decremented change in stock in time (t_1^*) & NO change in (ξ*) but decremented change Total Average cost (TAC*).
Result-3. For incremental change in (δ), we noticed the incremental change in stock in time (t_1^*), but decremented change (ξ*) and Total Average cost (TAC*).
Result-4. For incremental change in (α), we noticed the No change in stock in time (t_1^*) & (ξ*) but incremented change in Total Average cost (TAC*).
Result-5. For incremental change in (θ), we noticed the decremented change in stock in time (t_1^*), & (ξ*) but incremented change in Total Average cost (TAC*).
Result-6. For Incremental change in shortage cost coefficient (Sc), we noticed the decremented change in stock in time (t_1^*) and Total Average cost (TAC*) but incremental change (ξ*)
Result-7. For Incremental change in shortage cost coefficient (Ls), we noticed the incremental change in stock in time (t_1^*) and Total Average cost (TAC*), *) but decremented change (ξ*)
Result-8. For incremental change in (β), we noticed the decremented change in stock in time (t_1^*) & (ξ*) but Incremental change in Total Average cost (TAC*).
Result-9. For incremental change in (p), we noticed the decremented change in stock in time (t_1^*), (ξ*) &Total Average cost (TAC*).
Result-10. For incremental change in (ξ1), we noticed then NO change in stock in time (t_1^*) But incremental change (ξ*) &Total Average cost (TAC*).
Result-11. For incremental change in (c), we noticed then incremented change in stock in time (t_1^*) But decremented change (ξ*) &Total Average cost (TAC*).
Result-12. For incremental change in (G), we noticed then incremented change in stock in time (t_1^*), (ξ*) &Total Average cost (TAC*).
Result-13. For incremental change in (Pc), we noticed then decremented change in stock in time (t_1^*) but incremented change in (ξ*) &Total Average cost (TAC*).
Result-14. For incremental change in (A), we noticed then incremented change in stock in time (t_1^*), (ξ*) &Total Average cost (TAC*).
Result-15. For incremental change in (γ), we noticed then incremented change in stock in time (t_1^*), (ξ*) &Total Average cost (TAC*).

6.2. Managerial implication

We observe that increase in some coefficients & parameters leads to the higher total expenditure cost, in this situation as a good manger need to identify the factor which can be the cause of this situation and try to opt best possible way to get solution so that overall expenditure can minimize.
Impact of demand parameter
A good manger must observe the reason of hike in total cost corresponding to increase in, hike in demand parameter (α). Cause of this situation may be investing more in production, labour, capital, or marketing, manger need to work on this and try to take specific actions and best way to deal it.
Impact of increasing in parameter p.
As we increase in price in hence the total cost in this case manager try to negotiate with supplier at low price, which help in reducing the total cost and expenditure.
Impact of increasing in parameter A.
Investing too much without analysis the market and target customer. Manger need to make sure and calculative about the amount of money spent on ads. Also make specific target customer, make budget according to them. Also, can use some easy way of advertisement like digital platform. By Appling these step managers and make reduce expenditure.
Impact of cost coefficient,
Impact of ordering cost,
If rise in cost of ordering leads to rise in total cost, in this situation investigation for the reason must be done and provide the solution to gain profit.as a good manger no of frequent orders must reduce, he must focus on large amount of inventory but less frequency.
Impact of purchasing cost,
If the model facing this scenario in this situation better negotiation with suppliers and make better professional relation help to risk of total waste on expenditure.
Impact of Shortage cost,
Better forecasting and having better planning of inventory can be best possible strategies which can be accept by the manger. Also keep sufficient amount of buffer stock might be another initiative that a manager can applied in this situation
Impact of Deterioration rate θ,

If deterioration rates are increasing, even with preservation technology in place, a good manager should take care about another factor which might impact total expenditure.in this situation. Efforts can be made to reducing holding inventory time by developing practice so that inventory goods move faster. Also improve better storage conditions is better way with the aim of reduction overall expenditure.
Impact of carbon emission rate $\xi 1$,
Integration of Adoption of cleaner technologies to reduce emissions e.g., renewable energy, energy-efficient equipment, improve process efficiency to reduce resource waste and emissions and redesigning of products to be eco-friendlier and energy-efficient can be fruitful and help in reducing overall expenditure.

7. Conclusion

This research paper tries to solve the problem related to used assumption in which incorporation of factor like carbon taxes policies with demand based on green product, sustainable environment of preservation technology with consideration of advertisement frequency has been included. Aim of this model to optimize the objective function which represent and explained by sensitivity and graphically representation. Also result analyses done and explained in last of the research appear in section ().
Limitations of the Paper:
Due to used assumption has some limitation, which leads the limitation of this research article also as the market is so much robust and and full of uncertain challenge.so this model may not cover all aspect of challenging factors from market.

Future scope

This research paper can extend and explore the many factors which play important role in future market trend. Development of this paper can involve payment system from supplier to retailor under the impact of with post payment and advance payment environment. Also focus can be divert to inflation, time value of money, non-instantaneous deteriorated items fluctuating demand for development of the paper in multi-product supply chains for real-world applicability. etc.

References

[1] Bhattacharjee, N., & Sen, N. (2022). A multi-item sustainable production inventory constrained model to study and analyse the effective green investment and replenishment quantity. International Journal of Mathematics in Operational Research, 22(4), 528-554. https://doi.org/10.1504/IJMOR.2022.126050.

[2] Biuki, M., Kazemi, A., & Alinezhad, A. (2020). An integrated location-routing-inventory model for sustainable design of a perishable products supply chain network. Journal of Cleaner Production, 260, 120842. https://doi.org/10.1016/j.jclepro.2020.120842.

[3] Chang, C. C., Lu, C. J., & Yang, C. T. (2022). Multistage supply chain production-inventory model with collaborative preservation technology investment. Scientia Iranica, 29(4), 2099-2114.

[4] Cheng, P., Zhang, G., & Sun, H. (2022). The Sustainable Supply Chain Network Competition Based on Non-Cooperative Equilibrium under Carbon Emission Permits. Mathematics, 10(9), 1364. https://doi.org/10.3390/math10091364.

[5] Goel, A., & Singh, A. (2024). Decaying inventory model with different rates and varying costs under preservation technology investment. International Journal of Mathematics in Operational Research, 28(2), 230-252. https://doi.org/10.1504/IJMOR.2024.138900

[6] Khan, M. A. A., Shaikh, A. A., Konstantaras, I., Bhunia, A. K., & Cárdenas-Barrón, L. E. (2020). Inventory models for perishable items with advanced payment, linearly time-dependent holding cost and demand dependent on advertisement and selling price. International Journal of Production Economics, 230, 107804. https://doi.org/10.1016/j.ijpe.2020.107804.

[7] Kumar, P., Gupta, M. M., Yadav, D., & Kumar, V. (2024, April). Benefits of preservation technology, reworking, and waste management in sustainable production-inventory system in inflationary environment. In AIP Conference Proceedings (Vol. 3081, No. 1). AIP Publishing. https://doi.org/10.1063/5.0196349.

[8] Kumar, V., Kumar, A., Shekhar, C., & Gupta, C. B. (2023). A fuzzy inventory model with permissible delay in payments for non-instantaneous deteriorating items under inflation. In Modeling and Applications in Operations Research (pp. 139-154). CRC Press. 139-154.

[9] Kumar, V., Rani, M., & Kumar, S. (2022, December). An inventory model for deteriorating items when demand depends on advertisement and stock with partial backlogging. In AIP Conference Proceedings (Vol. 2597, No. 1). AIP Publishing. 060002-1–10.

[10] Kumar, V., Sharma, A., & Gupta, C. B. (2024). A multi-item EOQ model for deterioration items with ramp-type demand and partial backlogging under carbon emission and inflationary environment. International Journal of Operational Research, 51(2), 260-296. https://doi.org/10.1504/IJOR.2024.142255

[11] Li, C., Wang, J., Zheng, J., & Gao, J. (2022). Effects of carbon policy on carbon emission reduction in supply chain under uncertain demand. Sustainability, 14(9), 5548. https://doi.org/10.3390/su14095548.

[12] Mohd, R., Kaanodiya, K. K., & Sachin, K. V. (2017). Modeling of supply chain dynamics: A lingo based three-tier distribution approach. International Journal of Education & Applied Sciences Research, 4(1), 24-32.

[13] Poswal, P., Chauhan, A., Rajoria, Y. K., Boadh, R., & Goel, A. (2022). Fuzzy optimization model of two parameter weibull deteriorating rate with quadratic demand and variable holding cost under allowable shortages. Yugoslav Journal of Operations Research, 32(4), 453-470. https://doi.org/10.2298/YJOR220115021P

[14] Rahman, M. S., Khan, M. A. A., Halim, M. A., Nofal, T. A., Shaikh, A. A., & Mahmoud, E. E. (2021). Hybrid price and stock dependent inventory model for perishable goods with advance payment related discount facilities under preservation technology. Alexandria Engineering Journal, 60(3), 3455-3465. https://doi.org/10.1016/j.aej.2021.01.045

[15] Rathore, H. (2019). An inventory model with advertisement dependent demand and reliability consideration. International Journal of Applied and computational mathematics, 5(2), 33. https://doi.org/10.1007/s40819-019-0618-y

[16] Rizwanullah, M., Verma, S. K., & Junnaidi, M. H. (2021, May). Supply-chain two-warehouse inventory model for deteriorating items on exponential time function with shortage and partial backlogging in inflationary environment. In Journal of Physics: Conference Series (Vol. 1913, No. 1, p. 012130). IOP Publishing. https://doi.org/10.1088/1742-6596/1913/1/012130.

[17] Saha, S., & Chakrabarti, T. (2018). Two-echelon supply chain model for deteriorating items in an imperfect production system with advertisement and stock dependent demand under trade credit. International Journal of Supply and Operations Management, 5(3), 207-217.

[18] San-José, L. A., Sicilia, J., & Abdul-Jalbar, B. (2021). Optimal policy for an inventory system with demand dependent on price, time and frequency of advertisement. Computers & Operations Research, 128, 105169. https://doi.org/10.1016/j.cor.2020.105169

[19] Sebatjane, M. (2024a). A sustainable inventory model for a two-echelon cold chain with green technology investments and stock-dependent demand under carbon emissions tax regulation. Cleaner Logistics and Supply Chain, 13, 100173. https://doi.org/10.1016/j.clscn.2024.100173.

[20] Sebatjane, M. (2024b). Sustainable inventory models under carbon emissions regulations: taxonomy and literature review. Computers & Operations Research, 106865. https://doi.org/10.1016/j.cor.2024.106865

[21] Sebatjane, M. (2024c). Sustainable inventory, pricing, and marketing strategies for imperfect quality perishable items with inspection errors and advertisement-, expiration date-and price-dependent demand under carbon emissions policies. Sustainable Operations and Computers, 5, 239-256. https://doi.org/10.1016/j.susoc.2024.09.004.

[22] Sebatjane, M. (2024d). Three-echelon circular economic production–inventory model for deteriorating items with imperfect quality and carbon emissions considerations under various emissions policies. Expert Systems with Applications, 252, 124162. https://doi.org/10.1016/j.eswa.2024.124162

[23] Shah, N. H., & Vaghela, C. R. (2017). Economic order quantity for deteriorating items under inflation with time and advertisement dependent demand. Opsearch, 54(1), 168-180. https://doi.org/10.1007/s12597-016-0274-5.

[24] Shaikh, A. A., & Cárdenas-Barrón, L. E. (2020). An EOQ inventory model for non-instantaneous deteriorating products with advertisement and price sensitive demand under order quantity dependent trade credit. Investigación Operacional, 41(2), 168-187.

[25] Shaikh, A. A., Cárdenas–Barrón, L. E., Bhunia, A. K., & Tiwari, S. (2019). An inventory model of a three parameter Weibull distributed deteriorating item with variable demand dependent on price and frequency of advertisement under trade credit. RAIRO-Operations Research, 53(3), 903-916. https://doi.org/10.1051/ro/2017052.

[26] Sharma, A., Kumar, V., Singh, S. R., & Gupta, C. B. (2024). Green inventory model with two-warehouse system considering variable holding cost, time dependent demand, carbon emissions and energy consumption. International Journal of Procurement Management, 19(2), 274-296. https://doi.org/10.1504/IJPM.2024.136054.

[27] Sindhuja, S., & Arathi, P. (2023). An inventory model for deteriorating products under preservation technology with time-dependent quality demand. Cogent Engineering, 10(1), 2176968. https://doi.org/10.1080/23311916.2023.2176968

[28] Singh, A., & Goel, A. (2024). Supply Chain Optimization Decisions for Environmentally Conscious Products with Preservation Investment and Carbon Tax Policy. Process Integration and Optimization for Sustainability, 1-9. https://doi.org/10.1007/s41660-024-00467-7.

[29] Singh, A., & Goel, A. (2024a). Design of the supply chain network for the management of textile waste using a reverse logistics model under inflation. Energy, 292, 130615. https://doi.org/10.1016/j.energy.2024.130615

[30] Singh, A., & Goel, A. (2024b). Warehouse Inventory Model for Perishable Items with Hybrid Demand and Trade Credit Policy. Process Integration and Optimization for Sustainability, 1-15. https://doi.org/10.1007/s41660-024-00398-3.

[31] Singh, D., Kumar, A., Kumar, S., & Kumar, V. (2024). EOQ Model for Deteriorating Item with Expiration Dates Under Advance-Cash-Credit Payment Policy. Aligarh Journal of Statistics, 44, 179-185.

[32] Singh, D., Kumar, A., Kumar, S., & Kumar, V. (2024). EOQ Model for Deteriorating Item with Expiration Dates Under Advance-Cash-Credit Payment Policy. Aligarh Journal of Statistics, 44. 195-412.

[33] Singh, S. R., & Rana, K. (2023). A sustainable production inventory model for growing items with trade credit policy under partial backlogging. International Journal of Advanced Operations Management, 15(1), 64-81. https://doi.org/10.1504/IJAOM.2023.129523

[34] Singh, S., & Chaudhary, R. (2023). Effect of inflation on EOQ model with multivariate demand and partial backlogging and carbon tax policy. Journal of Future Sustainability, 3(1), 35-58. https://doi.org/10.5267/j.jfs.2022.11.004

[35] Verma, S. K., Rizwanullah, M., & Singh, C. (2018). An EOQ model for deteriorating items with selling price dependent exponential demand for time varying holding and deterioration costs. International Journal of Engineering & Technology, 7(4.41), 36-41. https://doi.org/10.14419/ijet.v7i4.41.24294.

[36] Verma, S. K., Rizwanullah, M., & Singh, C. (2021). Optimal ordering policy for deteriorating items with stock dependent demand, partial backlogging and trade credit period. International Journal of Logistics Systems and Management, 40(1), 95-120. https://doi.org/10.1504/IJLSM.2021.117709.

[37] Verma, S. K., Rizwanullah, M., Singh, C., & Kumar, V. An Inventory Model for Variable Holding Cost with Partial Backlogging and Constant Deteriorating under Selling Price Demand Rate.

[38] Verma, S.K., Rizwanullah, Mohd. (2019). "An Optimal Policy for Deterministic Model for Time Proportional Deteriorated Inventory with Different Demand Rate Pattern". Jour of Adv Research in Dynamical & Control Systems, Vol. 11, 01-Special Issue.