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Fibonacci-Like Sequence

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Abstract

The Fibonacci and Lucas sequences are well-known examples of second order recurrence sequences, which belong to particular class of recursive sequences. In this article, Fibonacci-Like sequence is introduced and defined by $H_n = 2H_{n-1} + H_{n-2}$ for $n \geq 2$ with $H_0 = 2$, $H_1 = 1$. The Binet's formula and generating function of Fibonacci-Like sequence are presented with some identities and connection formulae.

Keywords: *Fibonacci sequence, Lucas sequence, Fibonacci-Like sequence.*

1 Introduction

The Fibonacci and Lucas sequences are well-known examples of second order recurrence sequences. The Fibonacci numbers are perhaps most famous for appearing in the rabbit breeding problem, introduced by Leonardo de Pisa in 1202 in his book called Liber Abaci. However, they also occur in Pascal's triangle, Pythagorean triples, computer algorithms, some areas of algebra, graph theory, quasicrystals and many other areas of mathematics. They occur in a variety of other fields such as finance, art, architecture, music, etc.

The Fibonacci sequence [7] is a sequence of numbers starting with integer 0 and 1, where each next term of the sequence calculated as the sum of the previous two.

i.e., $F_n = F_{n-1} + F_{n-2},\ n \geq 2$ with $F_0 = 0,\ F_1 = 1.$ (1.1)

The similar interpretation also exists for Lucas sequence. Lucas sequence [7] is defined by the recurrence relation,

$$L_n = L_{n-1} + L_{n-2},\ n \geq 2 \text{ with } L_0 = 2,\ L_1 = 1. \tag{1.2}$$

The Pell sequence [3] is defined by the recurrence relation,

$$P_n = 2P_{n-1} + P_{n-2},\ n \geq 2 \text{ with } P_0 = 0,\ P_1 = 1. \tag{1.3}$$

The Pell-Lucas sequence [3] is defined by the recurrence relation,

$$Q_n = 2Q_{n-1} + Q_{n-2},\ n \geq 2 \text{ with } Q_0 = 2,\ Q_1 = 2. \tag{1.4}$$

The modified Pell sequence [3] is defined by the recurrence relation,

$$q_n = 2q_{n-1} + q_{n-2},\ n \geq 2 \text{ with } q_0 = 1,\ q_1 = 1. \tag{1.5}$$

The Binet's formulae for Pell, Pell-Lucas and modified Pell sequence [3] are given by

$$P_n = \frac{\alpha^n - \beta^n}{\alpha - \beta},\ Q_n = \alpha^n + \beta^n,\ q_n = \frac{\alpha^n + \beta^n}{\alpha + \beta}. \tag{1.6}$$

In [4, 5] authors have generalized the Fibonacci sequence by preserving the recurrence relation and altering the first two terms of the sequence, while in [1, 2, 6, 8] authors have generalized the Fibonacci sequence by preserving the first two terms of the sequence but altering the recurrence relation slightly.

Many authors defined Fibonacci pattern based sequences which are known as Fibonacci-like sequence. The Fibonacci-Like sequence [10] is defined by recurrence relation,

$$S_n = S_{n-1} + S_{n-2},\ n \geq 2 \text{ with } S_0 = 2,\ S_1 = 2. \tag{1.7}$$

The associated initial conditions S_0 and S_1 are the sum of initial conditions of Fibonacci and Lucas sequence respectively.

i.e. $S_0 = F_0 + L_0$ and $S_1 = F_1 + L_1$.

In this paper, a new type of Fibonacci-Like sequence is introduced. The Binet's formula and generating function of Fibonacci-Like sequence are present and establish some identities and connection formulae. Also determinants identities are states and derive.

2 Fibonacci-like sequence

Fibonacci-Like sequence $\{H_n\}_{n=0}^{\infty}$ is defined by the recurrence relation,

$$H_n = 2H_{n-1} + H_{n-2} \text{ for } n \geq 2 \text{ with } H_0 = 2,\ H_1 = 1. \tag{2.1}$$

The first few terms of Fibonacci-Like sequence are 2, 1, 4, 9, 22, 53, . . . and so on.

The characteristic equation of recurrence relation (2.1) is $t^2 - 2t - 1 = 0.$ which has two real roots; $\alpha = 1+\sqrt{2}$ and $\beta = 1-\sqrt{2}.$

Also, $\alpha\beta = -1,\ \alpha+\beta = 2,\ \alpha-\beta = 2\sqrt{2},\ \alpha^2+\beta^2 = 6.$ (2.2)

Generating function of Fibonacci-Like sequence is defined by

$$\sum_{n=0}^{\infty} H_n t^n = H(t) = H_n(t) = \frac{2-t}{1-2t-t^2}. \tag{2.3}$$

Hypergeometric representation of generating function of Fibonacci-Like sequence is given by

$$\sum_{n=0}^{\infty} \frac{H_n}{n!} t^n = (2-t)e^{2t}\,{}_2F_1(n+1,\ 1;\ 1;\ t^2). \tag{2.4}$$

Binet's formula of Fibonacci-Like sequence is defined by

$$H_n = A\alpha^n + B\beta^n = A\left(1+\sqrt{2}\right)^n + \mathrm{B}\left(1-\sqrt{2}\right)^n. \tag{2.5}$$

Here, $A = \dfrac{1-2\beta}{\alpha-\beta}$ and $B = \dfrac{2\alpha-1}{\alpha-\beta}.$

Relation between Fibonacci-Like, Pell and Pell-Lucas sequences can be defined as

$$H_n = Q_n - P_n. \tag{2.6}$$

3 Some identities of Fibonacci-Like sequence

In this section, some identities of Fibonacci-Like sequence are presented which can be easily derived by Binet's formula and generating function.

Explicit Sum Formula:

Theorem (3.1): Let H_n be the n^{th} Fibonacci-Like number, then

$$H_n = a\sum_{k=0}^{\left[\frac{n}{2}\right]} \binom{n-k}{\mathrm{k}} 2^{n-2k+1} - 3\sum_{k=0}^{\left[\frac{n-1}{2}\right]} \binom{n-k-1}{\mathrm{k}} 2^{n-2k-1}. \tag{3.1}$$

Sum of First n terms:

Theorem (3.2): Let H_n be the n^{th} Fibonacci-Like number, then sum of first n terms of Fibonacci-Like numbers is

$$\sum_{k=0}^{n-1} H_k = \frac{1}{2}(H_n + H_{n-1} + 1). \tag{3.2}$$

Sum of First n terms with odd indices:

Theorem (3.3): Let H_n be the n^{th} Fibonacci-Like number, then sum of first n terms with odd indices of Fibonacci-Like numbers is

$$\sum_{k=0}^{n-1} H_{2k+1} = \frac{1}{2}(H_{2n} - 2). \tag{3.3}$$

Sum of First n terms with even indices:

Theorem (3.4): Let H_n be the n^{th} Fibonacci-Like number, then sum of first n terms with even indices of Fibonacci-Like numbers is

$$\sum_{k=0}^{n-1} H_{2k} = \frac{1}{2}(H_{2n-1} + 3). \tag{3.4}$$

Theorem (3.5): (Catalan's Identity) Let H_n be the n^{th} Fibonacci-Like number, then

$$H_n^2 - H_{n+r}H_{n-r} = \frac{(-1)^{n-r}}{-7}\left[H_r - 2H_{r+1}\right]^2, \; n > r \geq 1. \tag{3.5}$$

All above identities can be derived by rearranging the terms or Binet's formula.

Corollary (3.6): (Cassini's Identity) Let H_n be the n^{th} Fibonacci-Like number, then

$$H_n^2 - H_{n+1}H_{n-1} = 7(-1)^n, \; n \geq 1. \tag{3.6}$$

By taking $r = 1$ in the Catalan's identity, it can be derived easily.

Theorem (3.7): (d'Ocagne's Identity) Let H_n be the n^{th} Fibonacci-Like number, then

$$H_m H_{n+1} - H_{m+1}H_n = (-1)^n\left[H_{m-n} - 2H_{m-n+1}\right], \; m > n \geq 0. \tag{3.7}$$

Theorem (3.8): (Generalized Identity) Let H_n be the n^{th} Fibonacci-Like number, then

$$H_m H_n - H_{m-r}H_{n+r} = \frac{(-1)^{m-r}}{7}(H_r - 2H_{r+1})(H_{n-m+r} - 2H_{n-m+r+1}), \; n > m \geq r \geq 1. \tag{3.8}$$

It is generalization of Catalan's, Cassini's and d'Ocagne's identities. These can be determined by substituting $m = n, \; m = n, \; r = 1$ and $n = m, \; m = n+1, \; r = 1$ in identity (3.8).

4 Connection formulae

In this section, connection formulae of Fibonacci-Like sequence, Pell and Pell-Lucas sequences are presented.

Theorem (4.1): If H_n, P_n, Q_n are n^{th} Fibonacci-Like, Pell and Pell-Lucas numbers respectively. For all positive integers n,

$$H_n^2 = \frac{1}{8}\left[9Q_{2n} - 16P_{2n} + 14(-1)^n\right].$$

Proof: By (1.6) and (2.6), we have

$$\begin{aligned}
H_n^2 &= (Q_n - P_n)^2 \\
&= \left[(\alpha^n + \beta^n) - \frac{(\alpha^n - \beta^n)}{(\alpha - \beta)}\right]^2 \\
&= (\alpha^n + \beta^n)^2 + \frac{(\alpha^n - \beta^n)^2}{(\alpha - \beta)^2} - 2(\alpha^n + \beta^n)\frac{(\alpha^n - \beta^n)}{(\alpha - \beta)}
\end{aligned}$$

$$=(\alpha^{2n}+\beta^{2n})+2(\alpha\beta)^n+\frac{\alpha^{2n}+\beta^{2n}-2(\alpha\beta)^n}{8}-2\frac{(\alpha^{2n}-\beta^{2n})}{(\alpha-\beta)}$$

$$=Q_{2n}+2(-1)^n+\frac{Q_{2n}-2(-1)^n}{8}-2P_{2n}$$

Therefore, $H_n^2=\frac{1}{8}\left[9Q_{2n}-16P_{2n}+14(-1)^n\right]$.

Theorem (4.2): If H_n, P_n, Q_n are n^{th} Fibonacci-Like, Pell and Pell-Lucas numbers. For all positive integers n,

$H_{2n}^2=\frac{1}{8}\left[9Q_{4n}-16P_{4n}+14\right]$ and $H_{2n+1}^2=\frac{1}{8}\left[9Q_{4n+2}-16P_{4n+2}-14\right]$.

These can be obtained by taking $n=2n$ and $n=2n+1$ respectively in theorem (4.1).

Theorem (4.3): If H_n, P_n, Q_n and q_n are n^{th} Fibonacci-Like, Pell, Pell-Lucas and modified Pell numbers. For all positive integers n,

$H_nq_n=\frac{1}{2}\left[Q_{2n}-P_{2n}+2(-1)^n\right]=\frac{1}{2}\left[H_{2n}+2(-1)^n\right]$.

Proof: By (1.6) and (2.6), we have

$$H_nq_n=\left[(\alpha^n+\beta^n)-\frac{(\alpha^n-\beta^n)}{(\alpha-\beta)}\right]\left[\frac{(\alpha^n+\beta^n)}{(\alpha+\beta)}\right]$$

$$=\frac{(\alpha^n+\beta^n)^2}{(\alpha+\beta)}-\frac{(\alpha^{2n}-\beta^{2n})}{(\alpha+\beta)(\alpha-\beta)}$$

$$=\frac{1}{(\alpha+\beta)}\left[(\alpha^{2n}+\beta^{2n})+2(\alpha\beta)^n-\frac{(\alpha^{2n}-\beta^{2n})}{(\alpha-\beta)}\right]$$

$$=\frac{1}{2}\left[Q_{2n}+2(-1)^n-P_{2n}\right].$$

Therefore, $H_nq_n=\frac{1}{2}\left[Q_{2n}-P_{2n}+2(-1)^n\right]=\frac{1}{2}\left[H_{2n}+2(-1)^n\right]$.

Theorem (4.4): If H_n, P_n, Q_n and q_n are n^{th} Fibonacci-Like, Pell, Pell-Lucas and modified Pell numbers. For all positive integers n,

$H_{2n}q_{2n}=\frac{1}{2}\left[Q_{4n}-P_{4n}+2\right]=\frac{1}{2}H_{4n}+1$ and $H_{2n+1}q_{2n+1}=\frac{1}{2}\left[Q_{4n+1}-P_{4n+1}-2\right]=\frac{1}{2}H_{4n+2}-1$.

These can be obtained by taking $n=2n$ and $n=2n+1$ respectively in theorem (4.3).

Theorem (4.5): If H_n and q_n are n^{th} Fibonacci-Like and modified Pell numbers. For all positive integers n,

$H_nq_{n+2}=\frac{1}{2}\left[H_{2n+2}+8(-1)^n\right]=\frac{1}{2}H_{2n+2}+4(-1)^n$.

Proof: By Binet's formula of Fibonacci-Like and modified Pell numbers,

$$H_nq_{n+2}=\left[(\alpha^n+\beta^n)-\frac{(\alpha^n-\beta^n)}{(\alpha-\beta)}\right]\left[\frac{(\alpha^{n+2}+\beta^{n+2})}{(\alpha+\beta)}\right]$$

$$=\frac{(\alpha^n+\beta^n)^2}{(\alpha+\beta)}-\frac{(\alpha^{2n}-\beta^{2n})}{(\alpha+\beta)(\alpha-\beta)}$$

$$=\frac{1}{(\alpha+\beta)}\left[(\alpha^n+\beta^n)(\alpha^{n+2}+\beta^{n+2})-\frac{(\alpha^n-\beta^n)(\alpha^{n+2}-\beta^{n+2})}{(\alpha-\beta)}\right]$$

$$=\frac{1}{(\alpha+\beta)}\left[\alpha^{2n+2}+\beta^{2n+2}+(\alpha\beta)^n(\alpha^2+\beta^2)-\left\{\frac{(\alpha^{2n+2}-\beta^{2n+2})-(\alpha\beta)^n(\alpha^2-\beta^2)}{(\alpha-\beta)}\right\}\right]$$

$$=\frac{1}{\alpha+\beta}\left[Q_{2n+2}+(-1)^nQ_2-P_{2n+2}+(-1)^nP_2\right].$$
$$=\frac{1}{2}\left[H_{2n+2}+8(-1)^n\right].$$

Therefore, $H_nq_{n+2}=\frac{1}{2}\left[H_{2n+2}+8(-1)^n\right]=\frac{1}{2}H_{2n+2}+4(-1)^n$.

Theorem (4.6): If H_n and q_n are n^{th} Fibonacci-Like and modified Pell numbers. For all positive integers n,

$H_{2n}q_{2n+2}=\frac{1}{2}H_{4n+2}+4$ and $H_{2n+1}q_{2n+3}=\frac{1}{2}H_{4n+4}-4.$

Proof: Taking $n=2n$ and $n=2n+1$ in theorem (4.5) respectively, we obtain required result.

5 Some determinant identities

Determinants have played a significant part in various areas in mathematics. There are different perspectives on the study of determinants. Problems on determinants of Fibonacci sequence and Lucas sequence are appeared in various issues of Fibonacci Quarterly. Many determinant identities of generalized Fibonacci sequence are discussed in [9]. In this section some determinant identities of Fibonacci-Like sequence are derived. Entries of determinants are satisfying the recurrence relation of Fibonacci-Like sequence and other sequences.

Theorem(5.1): For any integer $n\geq 0$, prove that

$$\begin{vmatrix} H_{n+1} & H_{n+2} & H_{n+3} \\ H_{n+4} & H_{n+5} & H_{n+6} \\ H_{n+7} & H_{n+8} & H_{n+9} \end{vmatrix}=0.$$

Proof: Let $\Delta = \begin{vmatrix} H_{n+1} & H_{n+2} & H_{n+3} \\ H_{n+4} & H_{n+5} & H_{n+6} \\ H_{n+7} & H_{n+8} & H_{n+9} \end{vmatrix}.$

Applying $C_2\to C_2(2)$ and $C_1\to C_1+C_2$ respectively and then two columns are identical and obtained the required result.

Theorem (5.2): For any integer $n\geq 0$, prove that

$$\begin{vmatrix} H_n-H_{n+1} & H_{n+1}-H_{n+2} & H_{n+2}-H_n \\ H_{n+1}-H_{n+2} & H_{n+2}-H_n & H_n-H_{n+1} \\ H_{n+2}-H_n & H_n-H_{n+1} & H_{n+1}-H_{n+2} \end{vmatrix}=0.$$

Proof: Let $\Delta = \begin{vmatrix} H_n-H_{n+1} & H_{n+1}-H_{n+2} & H_{n+2}-H_n \\ H_{n+1}-H_{n+2} & H_{n+2}-H_n & H_n-H_{n+1} \\ H_{n+2}-H_n & H_n-H_{n+1} & H_{n+1}-H_{n+2} \end{vmatrix}.$

Applying $C_1\to C_1+C_2+C_3$ and expand along first row to get required result.

Theorem (5.3): For any integer $n\geq 0$, prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ H_{n+2} & 2H_{n+1} & H_n \\ H_n+2H_{n+1} & H_{n+1}+H_{n+2} & 2H_{n+1}+H_{n+2} \end{vmatrix}=0.$$

Proof: Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ H_{n+2} & 2H_{n+1} & H_n \\ H_n+2H_{n+1} & H_{n+1}+H_{n+2} & 2H_{n+1}+H_{n+2} \end{vmatrix}.$

Applying $R_3 \to R_3 + R_2$, we get

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ H_{n+2} & 2H_{n+1} & H_n \\ 2H_{n+2} & 2H_{n+2} & 2H_{n+2} \end{vmatrix}.$$

Common out H_{n+2} from third row then two rows are identical and obtained required result.

Theorem (5.4): For any integer $n \geq 0$, prove that

$$\begin{vmatrix} H_n & H_n + 2H_{n+1} & H_n + 2H_{n+1} + H_{n+2} \\ 2H_n & 3H_n + 4H_{n+1} & 2H_n + 6H_{n+1} + 2H_{n+2} \\ 3H_n & 6H_n + 6H_{n+1} & 6H_n + 12H_{n+1} + 3H_{n+2} \end{vmatrix} = 3H_n^3.$$

Proof: Let $\Delta = \begin{vmatrix} H_n & H_n + 2H_{n+1} & H_n + 2H_{n+1} + H_{n+2} \\ 2H_n & 3H_n + 4H_{n+1} & 2H_n + 6H_{n+1} + 2H_{n+2} \\ 3H_n & 6H_n + 6H_{n+1} & 6H_n + 12H_{n+1} + 3H_{n+2} \end{vmatrix}.$

Applying $R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 3R_1$, we get

$$\Delta = \begin{vmatrix} H_n & H_{n+2} & 2H_{n+2} \\ 0 & H_n & 2H_{n+1} \\ 0 & 3H_n & 3H_n + 6H_{n+1} \end{vmatrix}.$$

Applying $R_3 \to R_3 - 3R_2$ and expand along first row, we obtain

$$\Delta = \begin{vmatrix} H_n & H_{n+2} & 2H_{n+2} \\ 0 & H_n & 2H_{n+1} \\ 0 & 0 & 3H_n \end{vmatrix} = 3H_n^3.$$

Theorem (5.5): For any integer $n \geq 0$, prove that

$$\begin{vmatrix} 0 & H_n H_{n+1}^2 & H_n H_{n+2}^2 \\ H_n^2 H_{n+1} & 0 & H_{n+1} H_{n+2}^2 \\ H_n^2 H_{n+2} & H_{n+2} H_{n+1}^2 & 0 \end{vmatrix} = 2H_n^3 H_{n+1}^3 H_{n+2}^3.$$

Proof: Let $\Delta = \begin{vmatrix} 0 & H_n H_{n+1}^2 & H_n H_{n+2}^2 \\ H_n^2 H_{n+1} & 0 & H_{n+1} H_{n+2}^2 \\ H_n^2 H_{n+2} & H_{n+2} H_{n+1}^2 & 0 \end{vmatrix}.$

Taking common outs $H_n^2,\ H_{n+1}^2,\ H_{n+2}^2$ from first, second and third column respectively, we have

$$\Delta = H_n^2 H_{n+1}^2 H_{n+2}^2 \begin{vmatrix} 0 & H_n & H_n \\ H_{n+1} & 0 & H_{n+1} \\ H_{n+2} & H_{n+2} & 0 \end{vmatrix}.$$

Taking common outs $H_n,\ H_{n+1},\ H_{n+2}$ from first, second and third rows respectively and expand along first row, we obtain

$\Delta = 2H_n^3 H_{n+1}^3 H_{n+2}^3.$

Theorem(5.6): For any integer $n \geq 0$, prove that

$$\begin{vmatrix} H_n & P_n & 1 \\ H_{n+1} & P_{n+1} & 1 \\ H_{n+2} & P_{n+2} & 1 \end{vmatrix} = 2(P_n H_{n+1} - H_n P_{n+1}).$$

Proof: Let $$\Delta = \begin{vmatrix} H_n & P_n & 1 \\ H_{n+1} & P_{n+1} & 1 \\ H_{n+2} & P_{n+2} & 1 \end{vmatrix}.$$

Applying $R_3 \to R_3 - 2R_2$, we get

$$\Delta = \begin{vmatrix} H_n & P_n & 1 \\ H_{n+1} & P_{n+1} & 1 \\ H_n & P_n & -1 \end{vmatrix}.$$

Applying $R_1 \to R_1 - R_3$ and expand along first row, we obtain

$$\Delta = \begin{vmatrix} H_n & P_n & 1 \\ H_{n+1} & P_{n+1} & 1 \\ H_{n+2} & P_{n+2} & 1 \end{vmatrix} = 2(P_n H_{n+1} - H_n P_{n+1}).$$

Proofs of following can be given same as above.

Theorem (5.7): For any integer $n \geq 0$, prove that

$$\begin{vmatrix} H_n & Q_n & 1 \\ H_{n+1} & Q_{n+1} & 1 \\ H_{n+2} & Q_{n+2} & 1 \end{vmatrix} = 2(Q_n H_{n+1} - H_n Q_{n+1}).$$

Theorem (5.8): For any integer $n \geq 0$, prove that

$$\begin{vmatrix} H_n & q_n & 1 \\ H_{n+1} & q_{n+1} & 1 \\ H_{n+2} & q_{n+2} & 1 \end{vmatrix} = 2(q_n H_{n+1} - H_n q_{n+1}).$$

6 Conclusion

In this paper, Fibonacci pattern based sequence introduced which is known as Fibonacci-Like sequence. Further Binet's formula and generating function of Fibonacci-Like sequence are presented and derived some identities and connection formulae. Some determinant identities also obtained and derived by standard method.

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