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New application of sine-cosine method for the generalized (2+1)-dimensional nonlinear evolution equations

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Abstract

By means of the idea of sine-cosine method, some analytic solutions for the generalized (2+1)-dimensional nonlinear evolution equations are presented. (2+1)-dimensional breaking soliton equation and (2+1)-dimensional Calogero-Bogoyavlenskii-Schiff (CBS) equation and (2+1)-dimensional Bogoyavlenskii's Breaking soliton equation are chosen to illustrate the effectiveness of the method.

Keywords: *Sine-cosine method, (2+1)-dimensional CBS equation, (2+1)-dimensional Breaking soliton equation, (2+1)-dimensional Bogoyavlenskii's Breaking soliton equation.*

1 Introduction

In this work, we will study the generalized (2+1)-dimensional nonlinear evolution equations

$$u_{xt} + au_x u_{xy} + bu_{xx}u_y + u_{xxxy} = 0, \tag{1}$$

where a and b are parameters. For example, namely the (2+1)-dimensional Calogero-Bogoyavlenskii-Schiff (CBS) equation for which $a = 4$ and $b = 2$,

$$u_{xt} + 4u_x u_{xy} + 2u_{xx}u_y + u_{xxxy} = 0, \tag{2}$$

and the (2+1)-dimensional breaking soliton equation for which $a = -4$ and $b = -2$,

$$u_{xt} - 4u_x u_{xy} - 2u_{xx}u_y + u_{xxxy} = 0, \tag{3}$$

and the (2+1)-dimensional Bogoyavlenskii's Breaking Soliton equation for which $a = 4$ and $b = 4$,

$$u_{xt} + 4u_x u_{xy} + 4u_{xx}u_y + u_{xxxy} = 0, \tag{4}$$

In this paper we solve equation (1) by the sine-cosine method method and obtain some exact and new solutions for (2),(3) and (4).

2 The Sine-cosine method

1. We introduce the wave variable $\xi = x - ct$ into the PDE

$$P(u, u_t, u_x, u_{tt}, u_{xx}, u_{tx}, \ldots) = 0, \tag{5}$$

where $u(x, t)$ is traveling wave solution. This enables us to use the following changes:

$$\frac{\partial}{\partial t} = -c\frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial t^2} = c^2\frac{\partial^2}{\partial \xi^2}, \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial \xi^2}, \ldots. \tag{6}$$

One can immediately reduce the nonlinear PDE (5) into a nonlinear ODE

$$Q(u, u_\xi, u_{\xi\xi}, u_{\xi\xi\xi}, \ldots) = 0. \tag{7}$$

The ordinary differential equation (7) is then integrated as long as all terms contain derivatives, where we neglect integration constants.
2. The solutions of many nonlinear equations can be expressed in the form [1]

$$u(x,t) = \begin{cases} \lambda\, sin^{\beta}(\mu\,\xi), & |\xi| \le \dfrac{\pi}{\mu}, \\ 0 & \text{otherwise}, \end{cases} \tag{8}$$

or in the form

$$u(x,t) = \begin{cases} \lambda\, cos^{\beta}(\mu\,\xi), & |\xi| \le \dfrac{\pi}{2\mu}, \\ 0 & \text{otherwise}, \end{cases} \tag{9}$$

where λ, μ and$\beta \neq 0$ are parameters that will be determined, μ and c are the wave number and the wave speed respectively. We use

$$\begin{aligned} &u(\xi) = \lambda\, sin^{\beta}(\mu\,\xi), \\ &u^n(\xi) = \lambda^n\, sin^{n\,\beta}(\mu\,\xi), \\ &(u^n)_\xi = n\mu\beta\lambda^n \cos(\mu\xi)\sin^{n\beta-1}(\mu\xi), \\ &(u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \sin^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\sin^{n\beta-2}(\mu\xi), \end{aligned} \tag{10}$$

and the derivatives of 9 becoms

$$\begin{aligned} &u(\xi) = \lambda\, cos^{\beta}(\mu\,\xi), \\ &u^n(\xi) = \lambda^n\, cos^{n\,\beta}(\mu\,\xi), \\ &(u^n)_\xi = -n\mu\beta\lambda^n \sin(\mu\xi)\cos^{n\beta-1}(\mu\xi), \\ &(u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \cos^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\cos^{n\beta-2}(\mu\xi), \end{aligned} \tag{11}$$

and so on for other derivatives.
3. We substitute (10) or (11) into the reduced equation obtained above in (7), balance the terms of the cosine functions when (11) is used, or balance the terms of the sine functions when (10) is used, and solving the resulting system of algebraic equations by using the computerized symbolic calculations. We next collect all terms whit same power in $\cos^k(\mu\xi)$ or $\sin^k(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknowns μ, β and λ. We obtained all possible value of the parameters μ, β and λ [2].

3 New application Sine-cosine method

In this section we apply the sine-cosine method to the generalized (2+1)-dimensional nonlinear evolution equations

$$u_{xt} + a u_x u_{xy} + b u_{xx} u_y + u_{xxxy} = 0, \tag{12}$$

We use the wave transformation

$$u(\xi) = u(x,y,t)\,,\ \xi = x + y - c\,t, \tag{13}$$

where c is constant to be determined later. Substituting (13) into system (12), we obtain an ordinary differential equation:

$$-c\,u^{''} + a\,u^{'}\,u^{''} + b\,u^{'}\,u^{''} + u^{(4)} = 0, \tag{14}$$

or equivalently

$$-c\,u^{''} + (a+b)\,u^{'}\,u^{''} + u^{(4)} = 0, \tag{15}$$

where prime denotes the differential with respect to ξ. Integrating (15) with respect to ξ and taking the integration constant as zero yields

$$-c\,u^{'} + \frac{(a+b)}{2}\,(u^{'})^2 + u^{(3)} = 0, \tag{16}$$

setting $u^{'}(\xi) = v(\xi)$, Eq. (16) becomes

$$-c\,v + \frac{(a+b)}{2}\,v^2 + v^{''} = 0. \tag{17}$$

Substituting (8) into (17) gives

$$-c\,\lambda\,sin^{\beta}(\mu\,\xi) - \mu^2\beta^2\lambda\sin^{\beta}(\mu\xi) + \mu^2\lambda\beta(\beta-1)\sin^{\beta-2}(\mu\xi)) + \frac{a+b}{2}\,\lambda^2\,sin^{2\beta}(\mu\,\xi) = 0. \tag{18}$$

Equating the exponents and the coefficients of each pair of the sine functions we find the following system of algebraic equations:

$$\begin{aligned}
&(\beta-1) \neq 0,\\
&\beta - 2 = 2\,\beta,\\
&-c\lambda - \mu^2\beta^2\lambda = 0,\\
&\lambda\,\mu^2\beta\,(\beta-1) + \frac{a+b}{2}\,\lambda^2 = 0,
\end{aligned} \tag{19}$$

Solving the system (19) yields

$$\beta = -2 \quad , \quad \mu = \frac{1}{2}\,\sqrt{-c} \quad , \quad \lambda = \frac{3\,c}{a+b}, \tag{20}$$

where c is a free parameter. Hence, for $c < 0$, the following periodic solutions

$$v_1(\xi) = \frac{3\,c}{a+b}\,\csc^2\left[\frac{\sqrt{-c}}{2}\,\xi\right], \tag{21}$$

where $0 < \frac{1}{2}\,\sqrt{-c}\,\xi < \pi$, and

$$v_2(\xi) = \frac{3\,c}{a+b}\,\sec^2\left[\frac{\sqrt{-c}}{2}\,\xi\right], \tag{22}$$

where $|\frac{1}{2}\,\sqrt{-c}\,\xi| < \frac{\pi}{2}$. In view of these results, and recall that $u^{'}(\xi) = v(\xi)$, integrating (21) and (22) with respect to ξ and considering the zero constants for integration we obtain

$$\begin{aligned}
u_1(\xi) &= -\frac{6\,c}{\sqrt{-c}\,(a+b)}\,\cot\left[\frac{\sqrt{-c}}{2}\,\xi\right],\\
u_2(\xi) &= \frac{6\,c}{\sqrt{-c}\,(a+b)}\,\tan\left[\frac{\sqrt{-c}}{2}\,\xi\right],
\end{aligned} \tag{23}$$

using $u(x,y,t) = u(\xi)$ and $\xi = x + y - c\,t$ we get

$$\begin{aligned}
u_1(x,y,t) &= -\frac{6\,c}{\sqrt{-c}\,(a+b)}\,\cot\left[\frac{\sqrt{-c}}{2}\,(x+y-c\,t)\right],\\
u_2(x,y,t) &= \frac{6\,c}{\sqrt{-c}\,(a+b)}\,\tan\left[\frac{\sqrt{-c}}{2}\,(x+y-c\,t)\right].
\end{aligned} \tag{24}$$

4 Exact solution of (2+1)-dimensional CBS equation

In this section, we investigate explicit formula of solutions of the following (2+1)-dimensional Calogero-Bogoyavlenskii-Schiff (CBS) equation given in [3]

$$u_{xt} + 4u_x u_{xy} + 2u_{xx} u_y + u_{xxxy} = 0, \tag{25}$$

by using section (3), we have following exact solutions:

Exact solution I:

$$u_1(x, y, t) = -\frac{c}{\sqrt{-c}} \cot\left[\frac{\sqrt{-c}}{2}(x + y - c\,t)\right], \tag{26}$$

where $0 < \frac{1}{2}\sqrt{-c}(x + y - c\,t) < \pi$.

Exact solution II:

$$u_2(x, y, t) = \frac{c}{\sqrt{-c}} \tan\left[\frac{\sqrt{-c}}{2}(x + y - c\,t)\right], \tag{27}$$

where $|\frac{1}{2}\sqrt{-c}(x + y - c\,t)| < \frac{\pi}{2}$.

5 Exact solution of (2+1)-dimensional Breaking soliton equation

In this section, we investigate explicit formula of solutions of the following (2+1)-dimensional Breaking soliton equation given in [4]

$$u_{xt} - 4u_x u_{xy} - 2u_{xx} u_y + u_{xxxy} = 0, \tag{28}$$

by using section (3), we have following exact solutions:

Exact solution I:

$$u_1(x, y, t) = \frac{c}{\sqrt{-c}} \cot\left[\frac{\sqrt{-c}}{2}(x + y - c\,t)\right], \tag{29}$$

where $0 < \frac{1}{2}\sqrt{-c}(x + y - c\,t) < \pi$.

Exact solution II:

$$u_2(x, y, t) = -\frac{c}{\sqrt{-c}} \tan\left[\frac{\sqrt{-c}}{2}(x + y - c\,t)\right], \tag{30}$$

where $|\frac{1}{2}\sqrt{-c}(x + y - c\,t)| < \frac{\pi}{2}$.

6 Exact solution of (2+1)-dimensional Bogoyavlenskii's Breaking soliton equation

In this section, we investigate explicit formula of solutions of the following (2+1)-dimensional Bogoyavlenskii's Breaking soliton equation given in [5]

$$u_{xt} + 4u_x u_{xy} + 4u_{xx} u_y + u_{xxxy} = 0, \tag{31}$$

by using section (3), we have following exact solutions:

Exact solution I:

$$u_1(x,y,t) = \frac{3\,c}{4\sqrt{-c}}\,\cot\left[\frac{\sqrt{-c}}{2}\,(x+y-c\,t)\right], \tag{32}$$

where $0 < \frac{1}{2}\sqrt{-c}\,(x+y-c\,t) < \pi$.

Exact solution II:

$$u_2(x,y,t) = -\frac{3\,c}{4\sqrt{-c}}\,\tan\left[\frac{\sqrt{-c}}{2}\,(x+y-c\,t)\right], \tag{33}$$

where $|\frac{1}{2}\sqrt{-c}\,(x+y-c\,t)\,| < \frac{\pi}{2}$.

7 Conclusion

In this paper, by using the sine-cosine method, we obtained some explicit formulas of solutions for the generalized (2+1)-dimensional nonlinear evolution equations. Those solutions were similar to the solutions obtained in other paper. The study reveals the power of the method.

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