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Research paper

Fourier Coefficients of a Class of Eta Quotients of Weight 4

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Abstract

Recently, there have been several works on the coefficients of the Fourier series expansions of a class of eta quotients by Williams, Yao, Xia and Jin, Kendirli, and Alaca. Some important explicit formulas have been discovered. Williams expressed all coefficients of one hundred and twenty-six eta quotients in terms of $\sigma(n), \sigma(\frac{n}{2}), \sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$, and Yao, Xia and Jin, following the method of proof of Williams, expressed only even coefficients of one hundred and four eta quotients in terms of $\sigma_3(n), \sigma_3(\frac{n}{2}), \sigma_3(\frac{n}{3})$ and $\sigma_3(\frac{n}{6})$. The author has expressed the even and odd coefficients of the Fourier series expansions of a class of eta quotients in terms of $\sigma_{k-1}(n), \sigma_{k-1}(\frac{n}{2}), \sigma_{k-1}(\frac{n}{3})$ and $\sigma_{k-1}(\frac{n}{6})$ for $k = 6,8,10,12,14,16,18,20,22,24$. Meanwhile, Alaca has obtained the coefficients of the Fourier series expansions of a class of eta quotients in $M_2(\Gamma_0, \chi)$ in terms of $\sigma(n), \sigma(\frac{n}{2}), \sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$. Here, we will express the coefficients of the Fourier series expansions of a class of eta quotients in $M_4(\Gamma_0, \chi)$ in terms of $\sigma_3(n), \sigma_3(\frac{n}{2}), \sigma_3(\frac{n}{3}), \sigma_3(\frac{n}{6})$ and Fourier coefficients of the four eta quotients.

Keywords: *Dedekind eta function; Eisenstein series; Eta quotients; Fourier coefficients; Modular forms.*

1. Introduction

The divisor function $\sigma_i(n)$ is defined for a positive integer i by

$\sigma_i(n) := \sum_{d \text{ positive integer}, d|n} d^i$, if n is a positive integer, and

$\sigma_i(n) := 0$, if n is not a positive integer.

The Dedekind eta function is defined by

$\eta(z) := q^{1/24} \prod_{n=1}^{\infty}(1-q^n)$,

where

$q := e^{2\pi i z}, z \in H = \{x + iy : y > 0\}$

and, an eta quotient of level n is defined by

$f(z) := \prod_{m|n} \eta(mz)^{a_m}, n, m \in \mathbb{N}, a_m \in \mathbb{Z}$.

There have been considerable effort to determine explicit formulas for the Fourier coefficients of eta quotients since they are the building blocks of modular forms of level n and weight k. The book of Köhler [1] (Chapter 3, pg.39) describes such expansions by means of Hecke Theta series, and it develops algorithms for the determination of suitable eta quotients. One can find more information in [2], [3], [4], [5] and [6]. The author has also determined the Fourier coefficients of the theta series associated with some quadratic forms, see [8], [9], [10], [11], [12] and [13].
Recently, Williams [14] discovered explicit formulas for the coefficients of Fourier series expansions of a class of one hundred and twenty-six eta quotients in terms of $\sigma(n), \sigma(\frac{n}{2}), \sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$. One example is as follows:

$$\frac{\eta^2(2z)\eta^4(4z)\eta^6(6z)}{\eta^2(z)\eta^2(3z)\eta^4(12z)} = 1 + \sum_{n=1}^{\infty} c(n) q^n,$$

where

$$c(n) = 2\sigma(n) - 3\sigma(n/2) + 4\sigma(n/4) + 9\sigma(n/6) - 36\sigma(n/12).$$

Then Yao, Xia and Jin [15] expressed the even Fourier coefficients of one hundred and four eta quotients in terms of $\sigma_3(n), \sigma_3(\frac{n}{2}), \sigma_3(\frac{n}{3})$ and $\sigma_3(\frac{n}{6})$. One example is as follows:

$$\frac{\eta^{25}(2z)\eta^4(3z)}{\eta^{12}(z)\eta^5(4z)\eta^3(6z)\eta(12z)} = 1 + \sum_{n=1}^{\infty} c(n) q^n,$$

where

$$c(2n) = 65\sigma_3(n) - 68\sigma_3(n/2) - 81\sigma_3(n/3) + 324\sigma_3(n/6).$$

Motivated by these two results, we find that we can express the even and odd coefficients of the Fourier series expansions of a class of eta quotients in terms of $\sigma_{k-1}(n), \sigma_{k-1}(\frac{n}{2}), \sigma_{k-1}(\frac{n}{3})$ and $\sigma_{k-1}(\frac{n}{6})$ for k = 6[16],8[17],10[18],12[19],14[20],16[21], 18[22],20[23],22[24],24[25]. Meanwhile, Alaca [26] has obtained the coefficients of the Fourier series expansions of a class of eta quotients in $M_2(\Gamma_0, \chi)$ in terms of $\sigma(n), \sigma(\frac{n}{2}), \sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$. Here, we will express the coefficients of the Fourier series expansions of a class of one hundred and fifty-six eta quotients in $M_4(\Gamma_0, \chi)$ in terms of $\sigma_3(n), \sigma_3(\frac{n}{2}), \sigma_3(\frac{n}{3}), \sigma_3(\frac{n}{6})$ and the Fourier coefficients of the four eta quotients.
Here we give the following Lemma, see[[27] Theorem 1.64] about the modularity of an eta quotient.

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Lemma 1. An eta quotient of level N is a meromorphic modular form of weight $\frac{1}{2}\sum_{m|N} a_m$ on $\Gamma_0(N)$, with Dirichlet character χ, having rational coefficients with respect to q if

a) $\sum_{m|N} a_m$ is even,

b) $\sum_{m|N} m\, a_m \equiv \sum_{m|N} \frac{N}{m} a_m \equiv 0 \bmod 24$,

c) $\chi(m) = \left(\frac{(-1)^k \prod_{m|N} m^{a_m}}{m}\right)$.

In particular, since

$2^{a_2}3^{a_3}4^{a_4}6^{a_6}12^{a_{12}} = 2^{a_2+2a_4+a_6+2a_{12}}3^{a_3+a_6+a_{12}}$,

an eta quotient of level 12 is a meromorphic modular form of weight 4 if

$$a_1 + a_2 + a_3 + a_4 + a_6 + a_{12} = 8$$

and

$$a_1 + 2a_2 + 3a_3 + 4a_4 + 6a_6 + 12a_{12} \equiv 12a_1 + 6a_2 + 4a_3 + 3a_4 + 2a_6 + a_{12} \equiv 0 \bmod 24.$$

Since $a_2 + a_6$ is an even integer, we conclude that it is a meromorphic modular form iff $a_3 + a_6 + a_{12}$ is an even integer, and it is a meromorphic modular form with χ_3 iff $a_3 + a_6 + a_{12}$ is an odd integer, where χ_3 is the unique primitive Dirichlet character mod12. On the other hand, the modular forms are holomorphic iff its order at cusps, $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}$ and $\frac{1}{12}$ are nonnegative, see [1] i.e.,

$$\sum_{m|12} \frac{(\gcd(c,m))^2}{m} a_m \geq 0 \text{ for } c|12.$$

Theorem 1: *Let χ_0 be the trivial character mod 1, i.e., it sends n to 1, χ_1 be the primitive Dirichlet character mod 4, and χ_2 is the primitive Dirichlet character mod 3. Then the Eisenstein subspace of $M_4(\Gamma_0(12), \chi_3)$ is generated by*

$E_4^{\chi_1,\chi_2} = \sum_{n=1}^{\infty}\left(\sum_{0<d|n} \chi_1\left(\frac{n}{d}\right)\chi_2(d)d^3\right)q^n = q - q^2 + 41q^3 + 113q^4 - 124q^5 + 155q^6 + 874q^7 + O(q^8)$,

$E_4^{\chi_2,\chi_1} = \sum_{n=1}^{\infty}\left(\sum_{0<d|n} \chi_2\left(\frac{n}{d}\right)\chi_1(d)d^3\right)q^n = q - q^2 - 27q^3 + q^4 + 124q^5 + 27q^6 - 342q^7 + O(q^8)$,

$E_4^{\chi_0,\chi_3} = -\frac{B_{4,\chi_3}}{8} + \sum_{n=1}^{\infty}\left(\sum_{0<d|n} \chi_0\left(\frac{n}{d}\right)\chi_3(d)d^3\right)q^n = 23 + q + q^2 + q^3 + q^4 - 124q^5 + q^6 - 342q^7 + O(q^8)$,

$E_4^{\chi_3,\chi_0} = \sum_{n=1}^{\infty}\left(\sum_{0<d|n} \chi_3\left(\frac{n}{d}\right)\chi_0(d)d^3\right)q^n = q + 8q^2 + 27q^3 + 64q^4 + 124q^5 + 216q^6 + 342q^7 + O(q^8)$

and, the cuspidal subspace $S_4(\Gamma_0(12), \chi_3)$ is generated by

$$f_1 := \sum_{n=0}^{\infty} f_1(n)q^n = \frac{\eta^2(2z)\eta^{10}(3z)\eta(4z)\eta(12z)}{\eta^2(z)\eta^4(6z)},$$

$$f_2 := \sum_{n=0}^{\infty} f_2(n)q^n = \frac{\eta^2(2z)\eta^7(3z)\eta^4(12z)}{\eta(z)\eta^4(6z)},$$

$$f_3 := \sum_{n=0}^{\infty} f_3(n)q^n = \frac{\eta^4(z)\eta^7(4z)\eta^2(6z)}{\eta^4(2z)\eta(12z)},$$

$$f_4 := \sum_{n=0}^{\infty} f_4(n)q^n = \frac{\eta^4(z)\eta^2(4z)\eta^7(6z)}{\eta^3(2z)\eta^2(12z)}.$$

Proof:
Since

$\chi_3 = \chi_0 * \chi_3$, $\chi_3 = \chi_3 * \chi_0$, $\chi_3 = \chi_1 * \chi_2$, $\chi_3 = \chi_2 * \chi_1$,

the Eisenstein subspace of $M_4(\Gamma_0(12), \chi_3)$ is generated by $E_4^{\chi_1,\chi_2}, E_4^{\chi_2,\chi_1}, E_4^{\chi_0,\chi_3}$ and $E_4^{\chi_3,\chi_0}$.
Since f_1, f_2, f_3 and f_4 are in $S_4(\Gamma_0(12), \chi_3)$, linearly independent and $\dim(S_4(\Gamma_0(12), \chi_3)) = 4$, the result follows. Let

$$\delta(b_1) = \begin{cases} 0 \text{ if } b_1 \neq 0 \\ 1 \text{ if } b_1 = 0 \end{cases}.$$

Now we can state our main Conclusion:

Theorem 2: *The coefficients of the Fourier series of one hundred and fifty-six eta quotients*

$$\eta^{a_1}(z)\eta^{a_2}(2z)\eta^{a_3}(3z)\eta^{a_4}(4z)\eta^{a_6}(6z)\eta^{a_{12}}(12z) = \delta(b_1) + \sum_{n=1}^{\infty} c(n)q^n$$

in $M_4(\Gamma_0(12), \chi_3)$ are given in the form

$$c_1\sigma_3(n) + c_2\sigma_3\left(\tfrac{n}{2}\right) + c_3\sigma_3\left(\tfrac{n}{3}\right) + c_4\sigma_3\left(\tfrac{n}{4}\right) + c_6\sigma_3\left(\tfrac{n}{6}\right) + c_{12}\sigma_3\left(\tfrac{n}{12}\right)$$

$$+r_1f_1(n) + r_2f_2(n) + r_3f_3(n) + r_4f_4(n)$$

As in the Table in Appendix.

Remark 1: *The coefficients of the Fourier series of the 104 eta quotients in $M_4(\Gamma_0(12))$ are given in the form*

$$c_1\sigma_3(n) + c_2\sigma_3\left(\tfrac{n}{2}\right) + c_3\sigma_3\left(\tfrac{n}{3}\right) + c_4\sigma_3\left(\tfrac{n}{4}\right) + c_6\sigma_3\left(\tfrac{n}{6}\right) c_{12}\sigma_3\left(\tfrac{n}{12}\right)$$

$$+r_1g_1(n) + r_2g_2(n) + r_3h(n), g_1(2n) = g_2(2n) = h(2n-1) = 0,$$

exactly as in [15]. But our method is obviously much simpler than it since we didn't need to use (p,k) parametrizations.

Remark 2: *$S_4(\Gamma_0(12), \chi_3)$ is 4 dimensional, see [28] (Chapter 3, pg.87 and Chapter 5, pg.197), and it is generated by Δ_s, and its three conjugates, where $t = \sqrt{3}$, $s = \sqrt{-140 + 80\sqrt{3}}$ and*

$\Delta_s = q + \frac{1}{160}(-s^3 + 2s^2 - 300s + 280)q^2 + \frac{1}{80}(s^3 - 2s^2 + 260s - 280)q^3 + \frac{1}{40}(-s^3 - 260s - 80)q^4 + \frac{1}{40}(s^3 + 300s)q^5 + \frac{1}{160}(7s^3 + 10s^2 + 1940s + 440)q^6 + \frac{1}{40}(-s^3 - 260s)q^7 + \frac{1}{40}(-s^3 - 6s^2 - 300s - 840)q^8 + \frac{1}{40}(-3s^3 - 900s - 120)q^9 + \frac{1}{20}(s^3 + 260s + 400)q^{10} + \frac{1}{4}(s^2 + 140)q^{11} + O(q^{12})$.

Here, $s, -s, (4*t+7)*s$ and $-(4*t+7)*s$ are distinct four conjugate roots of the irreducible polynomial $x^4 + 280x^2 + 400$. By simple calculation, we see that

$f_1 = \left(\frac{1}{120}(7t+15)s + \frac{1}{12}(-t+3)\right)\Delta_s(z) + \left(\frac{1}{120}(-7t-15)s + \frac{1}{12}(-t+3)\right)\Delta_{-s}(z) + \left(\frac{1}{120}(11t+21)s + \frac{1}{12}(t+3)\right)\Delta_{(4*t+7)*s}(z) + \left(\frac{1}{120}(-11t-21)s + \frac{1}{12}(t+3)\right)\Delta_{-(4*t+7)*s}(z)$,

$f_2 = \left(\frac{1}{80}(3t+6)s - \frac{1}{24}t\right)\Delta_s(z) + \left(\frac{1}{80}(-3t-6)s - \frac{1}{24}t\right)\Delta_{-s}(z) + \left(\frac{1}{80}(3t+6)s + \frac{1}{24}t\right)\Delta_{(4*t+7)*s}(z) + \left(\frac{1}{80}(-3t-6)s + \frac{1}{24}t\right)\Delta_{-(4*t+7)*s}(z)$,

$f_3 = \left(\frac{1}{80}(-9t-16)s + \frac{1}{8}(-t+2)\right)\Delta_s(z) + \left(\frac{1}{80}(9t+16)s + \frac{1}{8}(-t+2)\right)\Delta_{-s}(z) + \left(\frac{1}{80}(-t-4)s + \frac{1}{8}(t+2)\right)\Delta_{(4*t+7)*s}(z) + \left(\frac{1}{80}(t+4)s + \frac{1}{8}(t+2)\right)\Delta_{-(4*t+7)*s}(z)$,

$f_4 = \left(\frac{1}{120}(-8t-15)s + \frac{1}{12}(-2t+3)\right)\Delta_s(z) + \left(\frac{1}{120}(8t+15)s + \frac{1}{12}(-2t+3)\right)\Delta_{-s}(z) + \left(\frac{1}{120}(-4t-9)s + \frac{1}{12}(2t+3)\right)\Delta_{(4*t+7)*s}(z) + \left(\frac{1}{120}(4t+9)s + \frac{1}{12}(2t+3)\right)\Delta_{-(4*t+7)*s}(z)$.

References

[1] G. Köhler, *Eta Products and Theta Series Identities*, Springer-Verlag, Berlin, 2011. http://dx.doi.org/10.1007/978-3-642-16152-0.

[2] B. Gordon, *Some identities in combinatorial analysis*, Quart. J. Math. Oxford Ser.12 (1961), 285-290.

[3] V. G. Kac, *Infinite-dimensional algebras, Dedekind's η-function, classical Möbius function and the very strange formula*, Adv. Math. 30 (1978), 85-136. http://dx.doi.org/10.1016/0001-8708(78)90033-6.

[4] I. G. Macdonald, *Affine root systems and Dedekind's η-function*, Invent. Math. 15 (1972), 91-143. http://dx.doi.org/10.1007/BF01418931.

[5] I. J. Zucker, *A systematic way of converting infinite series into infinite products*, J. Phys. A 20 (1987) L13-L17. http://dx.doi.org/10.1088/0305-4470/20/1/003.

[6] I. J. Zucker, *Further relations amongst infinite series and products:II. The evaluation of three-dimensional lattice sums*, J. Phys. A 23 (1990), 117-132. http://dx.doi.org/10.1088/0305-4470/23/2/009.

[7] B. Gordon and S. Robins, *Lacunarity of Dedekind η-products*, Glasgow Math. J. 37 (1995), 1-14. http://dx.doi.org/10.1017/S0017089500030329.

[8] B. Kendirli, *Evaluation of Some Convolution Sums by Quasimodular Forms*, European Journal of Pure and Applied Mathematics ISSN 13075543 Vol.8, No. 1 (2015), 81-110.

[9] B. Kendirli, *Evaluation of Some Convolution Sums and Representation Numbers of Quadratic Forms of Discriminant 135*, British Journal of Mathematics and Computer Science, Vol. 6 6 (2015), 494-531. http://dx.doi.org/10.9734/BJMCS/2015/13973.

[10] B. Kendirli, *Evaluation of Some Convolution Sums and the Representation numbers*, Ars Combinatoria Vol. CXVI (2014), 65-91.

[11] B. Kendirli, *Cusp Forms in $S_4(\Gamma_0(79))$ and the number of representations of positive integers by some direct sum of binary quadratic forms with discriminant -79*, Bulletin of the Korean Mathematical Society, Vol. 49 3 (2012), 529-572. http://dx.doi.org/10.4134/BKMS.2012.49.3.529.

[12] B. Kendirli, *Cusp Forms in $S_4(\Gamma_0(47))$ and the number of representations of positive integers by some direct sum of binary quadratic forms with discriminant -47*, International Journal of Mathematics and Mathematical Sciences Vol. (2012), Article ID 303492, 10 pages.

[13] B. Kendirli, *The Bases of $M_4(\Gamma_0(71)), M_6(\Gamma_0(71))$ and the Number of Representations of Integers*, Mathematical Problems in Engineering Vol. (2013), Article ID 695265, 34 pages.

[14] K. S. Williams, *Fourier series of a class of eta quotients*, Int. J. Number Theory 8 (2012), 993-1004. http://dx.doi.org/10.1142/S1793042112500595.

[15] Olivia X. M. Yao, Ernest X. W. Xia and J. Jin, Explicit Formulas for the Fourier coefficients of a class of eta quotients, International Journal of Number Theory Vol. 9 2 (2013), 487-503. http://dx.doi.org/10.1142/S179304211250145X.

[16] B. Kendirli, Fourier Coefficients of a Class of Eta Quotients, INTEGERS (to be published) (2016).

[17] B. Kendirli, Fourier Coefficients of Some Eta Quotients of Weight 8, European Journal of Pure and Applied Mathematics Vol. 8, No. 3, (2015), 395-416.

[18] B. Kendirli, Fourier Coefficients of a Class of Eta Quotients of Weight 10, J. Math. Comput. Sci. 5 (2015), No. 6, 780-810.

[19] B. Kendirli, Fourier Coefficients of a Class of Eta Quotients of Weight 12, British Journal of Mathematics and Computer Science, Vol. 10 2 (2015), 1-34. http://dx.doi.org/10.9734/BJMCS/2015/18442.

[20] B. Kendirli, Fourier Coefficients of a Class of Eta Quotients of Weight 14 with Level 12, Journal of Basic&Applied Sciences (2015), 11, 454-483. http://dx.doi.org/10.6000/1927-5129.2015.11.64.

[21] B. Kendirli, Fourier Coefficients of a Class of Eta Quotients of Weight 16 with Level 12 (2015), Applied Mathematics 6, quotients, International Journal of Number Theory Vol. 9 2 (2013), 487-503.

[22] B. Kendirli, Fourier Coefficients of a Class of Eta Quotients of Weight 18 with Level 12, Pure and Applied Mathematics Journal (2015), 4(4), 178-188. http://dx.doi.org/10.11648/j.pamj.20150404.17.

[23] B. Kendirli, Fourier Coefficients of a Class of Eta Quotients of Weight 20 with Level 12, International Journal of Advanced Mathematical Sciences, 3 (2) (2015), 121-146. http://dx.doi.org/10.14419/ijams.v3i2.5247.

[24] B. Kendirli, Fourier Coefficients of a Class of Eta Quotients of Weight 22 with Level 12, European Journal of Mathematics and Computer Science, Vol.2, No.1, (2015), 1-47. http://dx.doi.org/10.9734/BJMCS/2015/18442.

[25] B. Kendirli, Fourier Coefficients of a Class of Eta Quotients of Weight 24 with Level 12, International Journal of Electronics, Mechanical and Mechatronics Engineering (to be published) (2016).

[26] A. Alaca, S. Alaca and Z. S. Aygin, *Fourier Coefficients of a class of eta quotients of weight 2* Int. J. Number Theory 8 (2015).

[27] K. Ono, The Web of Modularity: Arithmetic of the Coefficients of Modular Forms and q-series, CBMS Regional Conf. Series in Math. 102, Amer. Math. Soc., Providence, R. I.,2004.

[28] F. Diamond, J. Shurman, *A First Course in Modular Forms*, Springer Graduate Texts in Mathematics 228, 2005.

Appendix

Table

No	a_1	a_2	a_3	a_4	a_6	a_{12}	c_1	c_2	c_3	c_4	r_1	r_2	r_3	r_4
1	-10	20	10	-9	-10	7	$\frac{16}{23}$	0	0	$-\frac{1}{23}$	0	$-\frac{166}{23}$	$\frac{12}{23}$	$-\frac{27}{23}$
2	-9	16	7	-2	-6	2	$\frac{36}{23}$	0	0	0	$-\frac{109}{184}$	$-\frac{205}{92}$	$-\frac{9}{23}$	$\frac{77}{184}$
3	-9	20	-1	-4	2	0	$\frac{27}{23}$	0	$\frac{1}{23}$	0	$-\frac{21}{46}$	$\frac{42}{23}$	$\frac{16}{23}$	$-\frac{21}{46}$
4	-8	14	4	1	-4	1	$\frac{27}{23}$	0	0	0	$-\frac{15}{92}$	$-\frac{51}{46}$	$-\frac{1}{23}$	$\frac{3}{92}$
5	-8	9	12	2	-5	-2	$\frac{96}{23}$	$\frac{1}{23}$	0	0	$\frac{111}{23}$	$\frac{282}{23}$	$-\frac{40}{23}$	$\frac{16}{23}$
6	-7	10	1	8	0	-4	$\frac{725}{276}$	0	$\frac{9}{92}$	$\frac{7}{69}$	$\frac{3769}{736}$	$-\frac{3371}{368}$	$-\frac{51}{92}$	$-\frac{289}{736}$
7	-7	13	9	-7	-9	9	$\frac{8}{69}$	0	0	$-\frac{1}{69}$	$\frac{5}{46}$	-2	$\frac{14}{69}$	$-\frac{19}{46}$
8	-7	17	1	-9	-1	7	$\frac{2}{23}$	0	0	$\frac{1}{23}$	$-\frac{43}{184}$	$\frac{21}{92}$	$-\frac{5}{23}$	$\frac{59}{184}$
9	-7	18	9	-8	-8	4	0	0	0	$\frac{3}{23}$	$-\frac{22}{23}$	$\frac{376}{23}$	$-\frac{48}{23}$	$-\frac{90}{23}$
10	-7	7	9	5	-3	-3	$\frac{72}{23}$	$\frac{1}{23}$	0	0	$\frac{221}{46}$	$-\frac{244}{23}$	$-\frac{34}{23}$	$\frac{1}{2}$
11	-7	8	9	0	2	-4	$\frac{64}{23}$	$\frac{1}{23}$	0	0	$\frac{118}{23}$	$-\frac{216}{23}$	$-\frac{32}{23}$	$\frac{10}{23}$
12	-6	9	6	0	-5	4	$\frac{6}{23}$	0	0	0	$-\frac{113}{368}$	$-\frac{53}{184}$	$-\frac{3}{46}$	$\frac{41}{368}$
13	-6	13	-2	-2	3	2	$\frac{9}{46}$	0	$\frac{1}{46}$	0	$-\frac{133}{368}$	$\frac{167}{184}$	$\frac{9}{46}$	$-\frac{19}{368}$
14	-6	16	6	-5	-6	3	$\frac{390}{29}$	0	$\frac{7}{87}$	$\frac{66}{29}$	$-\frac{1288}{87}$	$-\frac{13\,869}{29}$	$-\frac{1035}{29}$	$-\frac{109}{29}$
15	-6	20	-2	-7	2	1	$-\frac{806}{29}$	0	$\frac{1}{87}$	$-\frac{32}{29}$	$\frac{2600}{87}$	$\frac{29\,289}{29}$	$\frac{3009}{29}$	$\frac{117}{29}$
16	-6	6	6	3	4	-5	$\frac{48}{23}$	$\frac{1}{23}$	0	0	$\frac{110}{23}$	$-\frac{206}{23}$	$-\frac{28}{23}$	$\frac{7}{23}$
17	-6	6	14	-3	-8	5	$\frac{64}{207}$	0	0	$-\frac{1}{207}$	$-\frac{5}{23}$	$-\frac{32}{23}$	0	$-\frac{2}{23}$

18	−6	7	6	−2	9	−6	$\frac{128}{69}$	$\frac{1}{23}$	0	$-\frac{1}{69}$	$\frac{120}{23}$	$-\frac{224}{23}$	$-\frac{64}{69}$	$-\frac{4}{23}$
19	−6	7	6	4	−3	0	$\frac{18}{23}$	0	0	0	$\frac{121}{368}$	$-\frac{251}{184}$	$-\frac{9}{46}$	$\frac{31}{368}$
20	−5	7	3	3	−3	3	0	0	0	0	$\frac{1}{2}$	7	0	$\frac{1}{2}$
21	−5	12	3	2	−2	−2	$-\frac{1}{69}$	0	$\frac{2}{23}$	$\frac{7}{69}$	$\frac{1075}{184}$	$-\frac{629}{92}$	$-\frac{9}{23}$	$-\frac{5}{8}$
22	−5	5	3	1	11	−7	$\frac{32}{23}$	$\frac{1}{23}$	0	0	$\frac{102}{23}$	$-\frac{196}{23}$	$-\frac{24}{23}$	$\frac{4}{23}$
23	−5	5	3	7	−1	−1	$\frac{27}{46}$	0	0	0	$\frac{423}{736}$	$-\frac{549}{368}$	$-\frac{25}{92}$	$\frac{81}{736}$
24	−5	6	3	2	4	−2	$\frac{12}{23}$	0	0	0	$\frac{117}{184}$	$-\frac{99}{92}$	$-\frac{3}{23}$	$-\frac{5}{184}$
25	−5	6	11	−4	−8	8	$\frac{16}{207}$	0	0	$-\frac{1}{207}$	0	$-\frac{21}{23}$	$\frac{4}{69}$	$-\frac{3}{23}$
26	−4	3	0	10	1	−2	$\frac{81}{184}$	0	$\frac{1}{184}$	0	$\frac{549}{736}$	$-\frac{639}{368}$	$-\frac{39}{92}$	$\frac{171}{736}$
27	−4	4	0	−1	18	−9	$\frac{64}{69}$	$\frac{1}{23}$	0	$\frac{1}{69}$	$\frac{84}{23}$	$-\frac{168}{23}$	$-\frac{80}{69}$	$\frac{12}{23}$
28	−4	4	0	5	6	−3	$\frac{9}{23}$	0	0	0	$\frac{151}{184}$	$-\frac{149}{92}$	$-\frac{8}{23}$	$\frac{25}{184}$
29	−4	4	8	−1	−6	7	$\frac{4}{69}$	0	0	0	$-\frac{5}{92}$	$-\frac{17}{46}$	$-\frac{1}{69}$	$\frac{1}{92}$
30	−4	4	16	−1	−6	−1	0	$\frac{1}{23}$	0	$\frac{1}{69}$	$\frac{104}{23}$	$-\frac{32}{23}$	$-\frac{64}{69}$	$\frac{8}{23}$
31	−4	5	0	0	11	−4	$\frac{8}{23}$	0	0	0	$\frac{85}{92}$	$-\frac{79}{46}$	$-\frac{2}{23}$	$-\frac{17}{92}$
32	−4	5	0	6	−1	2	$\frac{27}{184}$	0	$-\frac{1}{184}$	0	$-\frac{63}{368}$	$\frac{45}{184}$	$\frac{7}{46}$	$-\frac{45}{368}$
33	−4	6	8	−5	−8	11	$\frac{4}{207}$	0	0	$-\frac{1}{207}$	$\frac{5}{92}$	$-\frac{25}{46}$	$\frac{5}{69}$	$-\frac{13}{92}$
34	−4	10	0	−7	0	9	$\frac{1}{69}$	0	0	$\frac{1}{69}$	$-\frac{1}{23}$	$\frac{1}{23}$	$\frac{1}{69}$	0
35	−4	10	0	5	0	−3	$-\frac{1}{69}$	0	$\frac{3}{23}$	$\frac{7}{69}$	$\frac{407}{92}$	$-\frac{283}{46}$	$-\frac{15}{23}$	$\frac{1}{92}$
36	−4	11	8	−6	−7	6	0	0	0	$\frac{1}{23}$	$-\frac{15}{23}$	$\frac{110}{23}$	$-\frac{16}{23}$	$\frac{30}{23}$
37	−4	15	0	−8	1	4	0	0	0	$-\frac{3}{23}$	$\frac{19}{92}$	$\frac{7}{46}$	$\frac{2}{23}$	$-\frac{15}{92}$
38	−3	2	5	2	−4	6	$\frac{1}{23}$	0	0	0	$-\frac{53}{736}$	$-\frac{33}{368}$	$-\frac{1}{92}$	$\frac{29}{736}$
39	−3	3	−3	3	13	−5	$\frac{6}{23}$	0	0	0	$\frac{185}{184}$	$-\frac{199}{92}$	$-\frac{13}{23}$	$\frac{55}{184}$
40	−3	3	5	−3	1	5	$\frac{8}{207}$	0	0	$\frac{1}{207}$	$-\frac{5}{46}$	$\frac{4}{23}$	$-\frac{2}{23}$	$\frac{7}{46}$
41	−3	4	−3	−2	18	−6	$-\frac{16}{69}$	0	0	$\frac{1}{69}$	$\frac{49}{46}$	$-\frac{51}{23}$	$-\frac{20}{69}$	$-\frac{1}{46}$
42	−3	4	−3	4	6	0	$\frac{9}{92}$	0	$-\frac{1}{92}$	0	$-\frac{71}{736}$	$\frac{133}{368}$	$\frac{21}{92}$	$-\frac{7}{32}$
43	−3	4	5	−2	−6	10	$\frac{1}{69}$	0	0	0	$\frac{13}{736}$	$-\frac{103}{368}$	$-\frac{1}{276}$	$-\frac{21}{736}$
44	−3	4	13	−2	−6	2	0	0	0	$\frac{1}{69}$	$\frac{18}{23}$	$\frac{52}{23}$	$-\frac{16}{69}$	$\frac{10}{23}$
45	−3	6	−3	0	4	4	$\frac{3}{92}$	0	$\frac{1}{92}$	0	$-\frac{65}{736}$	$\frac{67}{368}$	$-\frac{1}{92}$	$\frac{41}{736}$
46	−3	9	5	−3	−5	5	0	0	0	0	$-\frac{1}{4}$	$\frac{5}{2}$	0	$\frac{1}{4}$
47	−3	13	−3	−5	3	3	0	0	$\frac{2}{23}$	0	$\frac{7}{184}$	$\frac{103}{92}$	$\frac{57}{23}$	$-\frac{479}{184}$
48	−3	14	5	−4	−4	0	$-\frac{1}{69}$	0	$\frac{2}{23}$	$\frac{7}{69}$	$\frac{503}{92}$	$-\frac{1177}{46}$	$\frac{14}{23}$	$-\frac{13}{4}$
49	−2	0	2	5	−2	5	$\frac{3}{92}$	0	0	0	$-\frac{57}{736}$	$\frac{27}{368}$	$\frac{5}{92}$	$-\frac{7}{736}$
50	−2	2	−6	1	20	−7	$\frac{4}{23}$	0	0	0	$\frac{25}{23}$	$-\frac{60}{23}$	$-\frac{24}{23}$	$\frac{18}{23}$
51	−2	2	2	1	−4	9	$\frac{1}{92}$	0	0	0	$\frac{1}{184}$	$-\frac{15}{92}$	$-\frac{3}{46}$	$\frac{9}{184}$
52	−2	2	10	1	−4	1	0	0	0	0	1	0	0	0
53	−2	3	−6	2	13	−2	$\frac{3}{46}$	0	$-\frac{1}{46}$	0	$-\frac{3}{368}$	$\frac{33}{184}$	$\frac{11}{46}$	$-\frac{101}{368}$
54	−2	3	2	−4	1	8	$\frac{2}{207}$	0	0	$\frac{1}{207}$	$-\frac{7}{368}$	$-\frac{3}{184}$	$-\frac{11}{138}$	$\frac{31}{368}$
55	−2	3	2	8	1	−4	0	$\frac{1}{23}$	0	0	$\frac{801}{368}$	$-\frac{603}{184}$	$-\frac{9}{46}$	$-\frac{9}{368}$
56	−3	18	−3	−6	4	−2	$-\frac{1}{69}$	0	$-\frac{14}{23}$	$\frac{7}{69}$	$\frac{245}{92}$	$-\frac{715}{46}$	$-\frac{534}{23}$	$\frac{2215}{92}$
57	−2	5	2	4	−1	0	0	0	0	0	$\frac{21}{16}$	$-\frac{15}{8}$	$-\frac{1}{2}$	$\frac{3}{16}$
58	−2	12	2	−1	−2	−1	$-\frac{1}{69}$	0	$\frac{2}{23}$	$\frac{7}{69}$	$\frac{227}{92}$	$-\frac{625}{46}$	$-\frac{78}{23}$	$\frac{11}{4}$
59	−1	−4	−1	12	2	0	$\frac{27}{368}$	0	$\frac{1}{368}$	0	$\frac{3}{46}$	$-\frac{6}{23}$	$-\frac{19}{92}$	$\frac{3}{46}$
60	−1	1	−9	−1	27	−9	$\frac{8}{69}$	0	0	$-\frac{1}{69}$	$\frac{51}{46}$	−3	$-\frac{124}{69}$	$\frac{73}{46}$
61	−1	1	−1	−1	3	7	$\frac{1}{138}$	0	0	0	$-\frac{5}{736}$	$-\frac{17}{368}$	$-\frac{35}{276}$	$\frac{93}{736}$
62	−1	1	−1	11	3	−5	$-\frac{11853}{5336}$	$\frac{1}{23}$	$-\frac{209}{5336}$	$\frac{27}{2668}$	$\frac{93}{29}$	$\frac{1620}{29}$	$\frac{729}{116}$	$-\frac{459}{667}$
63	−1	1	7	−1	3	−1	0	0	0	$-\frac{1}{69}$	$\frac{28}{23}$	$-\frac{52}{23}$	$\frac{16}{69}$	$-\frac{10}{23}$
64	−1	2	−9	0	20	−4	$\frac{1}{23}$	0	$-\frac{1}{23}$	0	$\frac{3}{46}$	$-\frac{2}{23}$	0	$-\frac{3}{46}$
65	−1	2	−1	0	−4	12	$\frac{1}{368}$	0	$-\frac{1}{368}$	0	$\frac{3}{736}$	$-\frac{25}{368}$	0	$-\frac{3}{736}$
66	−1	2	−1	6	8	−6	0	$\frac{1}{23}$	0	0	$\frac{159}{184}$	$-\frac{129}{92}$	$\frac{7}{23}$	$-\frac{39}{184}$
67	−1	2	7	0	−4	4	0	0	0	0	0	1	0	0
68	−1	3	−1	−5	1	11	$\frac{1}{414}$	0	0	$\frac{1}{207}$	$-\frac{9}{736}$	$\frac{11}{368}$	$\frac{13}{276}$	$-\frac{31}{736}$
69	−1	3	−1	7	1	−1	0	0	$\frac{1}{46}$	0	$\frac{927}{736}$	$-\frac{909}{368}$	$-\frac{81}{92}$	$\frac{441}{736}$
70	−1	4	7	−4	−6	8	0	0	0	$\frac{1}{69}$	$-\frac{5}{23}$	$\frac{29}{23}$	$-\frac{16}{69}$	$\frac{10}{23}$
71	−1	8	−1	−6	2	6	0	0	0	$-\frac{1}{23}$	$\frac{5}{184}$	$-\frac{3}{92}$	$-\frac{7}{23}$	$\frac{59}{184}$
72	0	−1	−4	2	5	6	$\frac{1}{184}$	0	$\frac{1}{184}$	0	$-\frac{1}{368}$	$-\frac{5}{184}$	$-\frac{3}{46}$	$\frac{21}{368}$

73	0	0	−4	−3	10	5	$\frac{1}{207}$	0	0	$-\frac{1}{207}$	$\frac{1}{184}$	$-\frac{7}{92}$	$-\frac{4}{23}$	$\frac{31}{184}$
74	0	0	−4	9	10	−7	0	$\frac{1}{23}$	$-\frac{1}{23}$	0	$-\frac{63}{184}$	$\frac{153}{92}$	$\frac{36}{23}$	$-\frac{225}{184}$
75	0	0	4	3	−2	3	0	0	0	0	$\frac{1}{8}$	$\frac{1}{4}$	0	$-\frac{1}{8}$
76	0	1	−12	−2	27	−6	$\frac{2}{69}$	0	$-\frac{2}{23}$	$-\frac{1}{69}$	$\frac{7}{46}$	$-\frac{13}{23}$	$-\frac{52}{69}$	$\frac{31}{46}$
77	0	1	−4	−2	3	10	$\frac{1}{552}$	0	$-\frac{1}{184}$	0	$-\frac{3}{736}$	$-\frac{7}{368}$	$-\frac{17}{276}$	$\frac{51}{736}$
78	0	1	−4	4	15	−8	0	$\frac{1}{23}$	0	0	$-\frac{47}{92}$	$\frac{85}{46}$	$\frac{30}{23}$	$-\frac{77}{92}$
79	0	1	4	−2	3	2	0	0	0	$-\frac{1}{69}$	$\frac{5}{23}$	$-\frac{6}{23}$	$\frac{16}{69}$	$-\frac{10}{23}$
80	0	2	4	−1	−4	7	0	0	0	0	$-\frac{1}{8}$	$\frac{3}{4}$	0	$\frac{1}{8}$
81	0	6	−4	−3	4	5	0	0	$\frac{1}{23}$	0	$-\frac{1}{23}$	$\frac{10}{23}$	$\frac{17}{23}$	$-\frac{17}{23}$
82	0	7	4	−2	−3	2	0	0	0	0	2	−8	0	1
83	0	11	−4	−4	5	0	0	0	$-\frac{8}{23}$	0	$\frac{147}{92}$	$-\frac{321}{46}$	$-\frac{182}{23}$	$\frac{705}{92}$
84	0	14	4	−7	−4	1	$-\frac{1}{69}$	0	$\frac{2}{23}$	$\frac{88}{69}$	$-\frac{841}{92}$	$\frac{1727}{46}$	$-\frac{418}{23}$	$\frac{2389}{92}$
85	1	−2	1	6	0	2	0	0	0	0	$\frac{7}{32}$	$-\frac{5}{16}$	$-\frac{1}{4}$	$\frac{1}{32}$
86	1	−1	−7	7	17	−9	0	$\frac{1}{23}$	$-\frac{2}{23}$	0	$-\frac{285}{184}$	$\frac{435}{92}$	$\frac{65}{23}$	$-\frac{411}{184}$
87	1	−1	1	1	5	1	0	0	0	0	$\frac{1}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{4}$
88	1	0	−7	−4	10	8	$\frac{1}{828}$	0	$-\frac{1}{92}$	$-\frac{1}{207}$	$\frac{3}{736}$	$-\frac{25}{368}$	$-\frac{47}{276}$	$\frac{133}{736}$
89	1	0	−7	2	22	−10	0	$\frac{1}{23}$	0	0	$-\frac{81}{46}$	$\frac{123}{23}$	$\frac{76}{23}$	$-\frac{119}{46}$
90	1	0	1	2	−2	6	0	0	0	0	$-\frac{3}{32}$	$\frac{9}{16}$	$\frac{1}{4}$	$-\frac{5}{32}$
91	1	0	9	2	−2	−2	0	$\frac{1}{23}$	0	0	$-\frac{6}{23}$	$-\frac{84}{23}$	$-\frac{16}{23}$	$-\frac{2}{23}$
92	1	5	1	1	−1	1	0	0	0	0	$\frac{3}{2}$	−6	−2	$\frac{3}{2}$
93	2	−4	−2	9	2	1	0	0	$\frac{1}{92}$	0	$\frac{111}{368}$	$-\frac{141}{184}$	$-\frac{13}{23}$	$\frac{93}{368}$
94	2	−2	−10	5	24	−11	0	$\frac{1}{23}$	$-\frac{4}{23}$	0	$-\frac{119}{46}$	$\frac{175}{23}$	$\frac{100}{23}$	$-\frac{167}{46}$
95	2	−2	−2	−1	12	−1	0	0	0	$\frac{1}{69}$	$\frac{13}{46}$	$-\frac{17}{23}$	$-\frac{16}{69}$	$-\frac{3}{46}$
96	2	−2	−2	5	0	5	0	0	$-\frac{1}{92}$	0	$-\frac{61}{736}$	$\frac{167}{368}$	$\frac{29}{92}$	$-\frac{163}{736}$
97	2	−2	6	5	0	−3	0	$\frac{1}{23}$	0	0	$-\frac{29}{23}$	$\frac{8}{23}$	$-\frac{16}{23}$	$-\frac{2}{23}$
98	2	−1	−10	0	29	−12	0	$\frac{1}{23}$	0	$\frac{1}{23}$	$-\frac{67}{23}$	$\frac{210}{23}$	$\frac{152}{23}$	$-\frac{133}{23}$
99	2	−1	−2	0	5	4	0	0	0	0	$-\frac{1}{16}$	$\frac{3}{8}$	$\frac{1}{2}$	$-\frac{7}{16}$
100	2	−1	6	0	5	−4	0	$\frac{1}{23}$	0	$\frac{1}{23}$	$-\frac{44}{23}$	$\frac{72}{23}$	$-\frac{32}{23}$	$\frac{28}{23}$
101	2	1	−2	−4	3	8	$\frac{97}{116}$	0	$\frac{5}{1044}$	$\frac{7}{174}$	$-\frac{230}{261}$	$-\frac{897}{29}$	$-\frac{175}{58}$	$-\frac{2}{29}$
102	2	1	−2	8	3	−4	$-\frac{12\,033}{2668}$	$\frac{1}{23}$	$-\frac{277}{2668}$	$-\frac{711}{1334}$	$\frac{90}{29}$	$\frac{6291}{29}$	$\frac{1161}{58}$	$\frac{162}{667}$
103	3	−3	−13	3	31	−13	0	$\frac{1}{23}$	$-\frac{8}{23}$	0	$-\frac{157}{46}$	$\frac{227}{23}$	$\frac{124}{23}$	$-\frac{215}{46}$
104	3	−3	−5	3	7	3	0	0	$-\frac{1}{46}$	0	$-\frac{53}{736}$	$\frac{127}{368}$	$\frac{35}{92}$	$-\frac{211}{736}$
105	3	−3	3	3	7	−5	0	$\frac{1}{23}$	0	0	$-\frac{52}{23}$	$\frac{100}{23}$	$-\frac{16}{23}$	$-\frac{2}{23}$
106	3	−2	−5	−2	12	2	0	0	0	$\frac{1}{69}$	$-\frac{17}{184}$	$\frac{47}{92}$	$\frac{53}{69}$	$-\frac{127}{184}$
107	3	−2	3	4	0	0	0	0	0	0	$\frac{1}{2}$	−2	0	$\frac{1}{2}$
108	3	−1	−5	−1	5	7	0	0	$\frac{1}{46}$	0	$\frac{7}{736}$	$\frac{11}{368}$	$\frac{11}{92}$	$-\frac{111}{736}$
109	3	0	3	0	−2	4	0	0	0	0	$\frac{1}{2}$	−2	0	$-\frac{1}{2}$
110	3	4	−5	−2	6	2	0	0	$-\frac{4}{23}$	0	$\frac{55}{184}$	$-\frac{137}{92}$	$-\frac{45}{23}$	$\frac{337}{184}$
111	3	7	3	−5	−3	3	0	0	0	$\frac{9}{23}$	$-\frac{20}{23}$	$\frac{116}{23}$	$-\frac{144}{23}$	$\frac{178}{23}$
112	3	11	−5	−7	5	1	0	0	$-\frac{32}{23}$	$-\frac{27}{23}$	3	$-\frac{438}{23}$	$-\frac{848}{23}$	$\frac{861}{23}$
113	4	−4	−16	1	38	−15	0	$\frac{1}{23}$	$-\frac{16}{23}$	$-\frac{1}{23}$	$-\frac{90}{23}$	$\frac{244}{23}$	$\frac{96}{23}$	$-\frac{82}{23}$
114	4	−4	−8	1	14	1	0	0	$-\frac{1}{23}$	0	$-\frac{15}{184}$	$\frac{29}{92}$	$\frac{6}{23}$	$-\frac{25}{184}$
115	4	−4	0	1	14	−7	0	$\frac{1}{23}$	0	$-\frac{1}{23}$	$-\frac{60}{23}$	$\frac{128}{23}$	0	$-\frac{32}{23}$
116	4	−4	0	7	2	−1	0	0	0	0	0	0	1	0
117	4	−3	0	2	7	−2	0	0	0	0	0	0	0	1
118	4	−2	0	3	0	3	0	0	0	0	$\frac{1}{2}$	−2	−1	$\frac{1}{2}$
119	4	3	0	2	1	−2	$-\frac{1}{69}$	0	$\frac{2}{23}$	$\frac{7}{69}$	$-\frac{601}{92}$	$\frac{1031}{46}$	$\frac{198}{23}$	$-\frac{25}{4}$
120	5	−6	−3	10	4	−2	0	0	$-\frac{1}{23}$	0	$-\frac{405}{736}$	$\frac{783}{368}$	$\frac{231}{92}$	$-\frac{675}{736}$
121	5	−5	−11	−1	21	−1	0	0	$-\frac{2}{23}$	$-\frac{1}{69}$	$-\frac{13}{184}$	$\frac{11}{92}$	$-\frac{17}{69}$	$\frac{77}{184}$
122	5	−5	−3	5	9	−3	0	0	0	0	$-\frac{1}{2}$	2	2	$-\frac{1}{2}$
123	5	−4	−3	0	14	−4	0	0	0	$-\frac{1}{23}$	$-\frac{8}{23}$	$\frac{28}{23}$	$\frac{16}{23}$	$\frac{16}{23}$
124	5	−4	−3	6	2	2	0	0	$\frac{1}{23}$	0	$\frac{405}{736}$	$-\frac{783}{368}$	$-\frac{139}{92}$	$\frac{675}{736}$
125	5	1	−3	5	3	−3	$-\frac{1}{69}$	0	$-\frac{6}{23}$	$\frac{7}{69}$	$-\frac{148}{23}$	$\frac{493}{23}$	$\frac{246}{23}$	$-\frac{209}{23}$
126	6	−6	−6	3	16	−5	0	0	0	0	−1	4	4	−2
127	6	−5	−6	4	9	0	0	0	$\frac{2}{23}$	0	$\frac{221}{368}$	$-\frac{415}{184}$	$-\frac{93}{46}$	$\frac{491}{368}$
128	6	−3	−6	0	7	4	0	0	$-\frac{2}{23}$	0	$-\frac{37}{368}$	$\frac{47}{184}$	$\frac{1}{46}$	$\frac{61}{368}$

129	6	0	2	−3	−2	5	0	0	0	$\frac{3}{23}$	$\frac{1}{23}$	$\frac{8}{23}$	$-\frac{48}{23}$	$\frac{44}{23}$
130	6	4	−6	−5	6	3	0	0	$-\frac{16}{23}$	$-\frac{9}{23}$	$\frac{49}{46}$	−7	$-\frac{312}{23}$	$\frac{625}{46}$
131	6	5	2	−4	−1	0	$-\frac{1}{69}$	0	$\frac{2}{23}$	$-\frac{74}{69}$	$-\frac{361}{92}$	$\frac{335}{46}$	$\frac{446}{23}$	$-\frac{1883}{92}$
132	6	9	−6	−6	7	−2	$-\frac{1}{69}$	0	$\frac{130}{23}$	$\frac{250}{69}$	$-\frac{1465}{92}$	$\frac{3839}{46}$	$\frac{3102}{23}$	$-\frac{12347}{92}$
133	7	−7	−9	1	23	−7	0	0	0	$\frac{1}{23}$	$-\frac{38}{23}$	$\frac{156}{23}$	$\frac{168}{23}$	$-\frac{108}{23}$
134	7	−6	−9	2	16	−2	0	0	$\frac{4}{23}$	0	$\frac{129}{184}$	$-\frac{231}{92}$	$-\frac{47}{23}$	$\frac{215}{184}$
135	7	−6	−1	8	4	−4	0	$\frac{1}{23}$	0	0	$-\frac{81}{46}$	$\frac{54}{23}$	$-\frac{108}{23}$	$-\frac{27}{46}$
136	7	−4	−1	4	2	0	0	0	0	0	$-\frac{3}{2}$	6	4	$-\frac{3}{2}$
137	7	3	−1	−1	1	−1	$-\frac{1}{69}$	0	$\frac{2}{23}$	$\frac{7}{69}$	$-\frac{601}{92}$	$\frac{1031}{46}$	$\frac{750}{23}$	$-\frac{133}{4}$
138	8	−8	−4	11	6	−5	0	$\frac{1}{23}$	$\frac{4}{23}$	0	$\frac{81}{184}$	$-\frac{567}{92}$	$-\frac{270}{23}$	$\frac{567}{184}$
139	8	−7	−12	0	23	−4	0	0	$\frac{8}{23}$	$\frac{1}{23}$	$\frac{3}{4}$	$-\frac{103}{46}$	$-\frac{18}{23}$	$-\frac{33}{92}$
140	8	−7	−4	6	11	−6	0	$\frac{1}{23}$	0	0	$-\frac{6}{23}$	$-\frac{84}{23}$	$-\frac{200}{23}$	$\frac{21}{23}$
141	8	−6	−4	7	4	−1	0	0	$-\frac{4}{23}$	0	$-\frac{405}{184}$	$\frac{783}{92}$	$\frac{162}{23}$	$-\frac{675}{184}$
142	9	−9	−7	9	13	−7	0	$\frac{1}{23}$	$\frac{8}{23}$	0	$\frac{243}{92}$	$-\frac{675}{46}$	$-\frac{432}{23}$	$\frac{27}{4}$
143	9	−8	−7	4	18	−8	0	$\frac{1}{23}$	0	0	$\frac{40}{23}$	$-\frac{268}{23}$	$-\frac{384}{23}$	$\frac{136}{23}$
144	9	−3	−7	−3	7	5	0	0	$-\frac{8}{23}$	$-\frac{3}{23}$	$\frac{5}{92}$	$-\frac{61}{46}$	$-\frac{88}{23}$	$\frac{17}{4}$
145	9	−2	1	−2	0	2	0	0	0	$-\frac{9}{23}$	$-\frac{26}{23}$	$\frac{68}{23}$	$\frac{144}{23}$	$-\frac{86}{23}$
146	9	2	−7	−4	8	0	0	0	$\frac{64}{23}$	$\frac{27}{23}$	$-\frac{78}{23}$	$\frac{528}{23}$	$\frac{1024}{23}$	$-\frac{1014}{23}$
147	10	−4	−2	1	2	1	0	0	0	0	−3	12	16	−12
148	10	−10	−10	7	20	−9	0	$\frac{1}{23}$	$\frac{16}{23}$	0	$\frac{255}{46}$	$-\frac{591}{23}$	$-\frac{664}{23}$	$\frac{579}{46}$
149	10	−9	−10	2	25	−10	0	$\frac{1}{23}$	0	$-\frac{3}{23}$	$\frac{108}{23}$	−24	$-\frac{704}{23}$	16
150	11	−11	−13	5	27	−11	0	$\frac{1}{23}$	$\frac{32}{23}$	0	$\frac{215}{23}$	$-\frac{914}{23}$	$-\frac{944}{23}$	$\frac{443}{23}$
151	11	−8	−5	8	6	−4	$-\frac{1}{69}$	0	$\frac{18}{23}$	$\frac{7}{69}$	$\frac{623}{92}$	$-\frac{1363}{46}$	$-\frac{726}{23}$	$\frac{1189}{92}$
152	12	−12	−16	3	34	−13	0	$\frac{1}{23}$	$\frac{64}{23}$	$\frac{3}{23}$	14	$-\frac{1276}{23}$	$-\frac{1184}{23}$	$\frac{518}{23}$
153	12	−5	−8	−2	9	2	0	0	$\frac{33}{23}$	$\frac{9}{23}$	$\frac{17}{23}$	$\frac{22}{23}$	$\frac{216}{23}$	$-\frac{274}{23}$
154	13	−6	−3	2	4	−2	$-\frac{1}{69}$	0	$\frac{2}{23}$	$\frac{7}{69}$	$\frac{1055}{92}$	$-\frac{2281}{46}$	$-\frac{1458}{23}$	$\frac{155}{4}$
155	14	−8	−6	5	6	−3	$-\frac{1}{69}$	0	$\frac{66}{23}$	$\frac{7}{69}$	$\frac{1811}{92}$	$-\frac{3577}{46}$	$-\frac{1842}{23}$	$\frac{3997}{92}$
156	16	−6	−4	−1	4	−1	$-\frac{1}{69}$	0	$\frac{2}{23}$	$-\frac{236}{69}$	$\frac{3431}{92}$	$-\frac{7681}{46}$	$-\frac{4578}{23}$	$\frac{13717}{92}$