International Journal of Advanced Mathematical Sciences, 3 (2) (2015) 103-107
www.sciencepubco.com/index.php/IJAMS
©Science Publishing Corporation
doi: 10.14419/ijams.v3i2.5019
Research Paper

Some types of fuzzy open sets in fuzzy topological groups

Mohamad Thigeel Hmod*, Munir A. Al-khafaji, Taghreed Hur Majeed

Department of Mathematics , College of Education, AL-Mustinsiryah University , Baghdad, Iraq
**Corresponding author E-mail: mohamad.thijeel@yahoo.com*

Abstract

The aim of this work is to introduce the definitions and study the concepts of fuzzy open (resp, fuzzy α- open, fuzzy semi- open, fuzzy pre- open, fuzzy regular- open, fuzzy b- open, fuzzy β- open) sets in fuzzy topological groups, and devote to study and discuss some of the basic concepts of some types of fuzzy continuous, fuzzy connected and fuzzy compact spaces in fuzzy topological groups with some theorems and Proposition are proved.

Keywords: *Fuzzy continuous; Fuzzy Compact; Fuzzy Connected in Fuzzy Topological Groups.*

1. Introduction

The concept of fuzzy sets was introduced by zadeh [1]. Chang [2] introduced the definition of fuzzy topological spaces and extended in a straight-forward manner some concepts of crisp topological spaces to fuzzy topological spaces. Rosenfeld [3] formulated the elements of a theory of fuzzy groups. A notion of a fuzzy topological group was proposed by foster [4].in In this paper, we introduce some types of fuzzy open sets in fuzzy topological groups, and study some relations between some types of fuzzy continuous, fuzzy connected and fuzzy compact spaces.

2. On fuzzy topological groups

Definition 2.1: [1] [7] [8]Let X be a non-empty set, a fuzzy set $\tilde{A}$ in X is characterized by a function $M_{\tilde{A}}: X \to I$, where $I = [0,1]$ which is written as $\tilde{A} = \{(x , M_{\tilde{A}}(x)) : x \in X , 0 \leq M_{\tilde{A}}(x) \leq 1\}$, The collection of all fuzzy sets in X will be denoted by I^X,that is
$I^X = \{\tilde{A}: \tilde{A}$ is a fuzzy sets in X$\}$ where $M_{\tilde{A}}$ is called the membership function.
Definition 2.2: [3] [6] Let X is a group and let $\tilde{G}$be fuzzy set of X. A fuzzy set $\tilde{G}$is called a fuzzy group of X if

1) $M_{\tilde{G}}(xy) \geq \min\{M_{\tilde{G}}(x), M_{\tilde{G}}(y)\}$ for all $x, y \in X$.

2) $M_{\tilde{G}}(x^{-1}) \geq M_{\tilde{G}}(x)$For all $x \in X$.

Definition 2.3: [2] [4] [5] A collection $\tilde{T}$ of a fuzzy subsets of $\tilde{A}$, such that $\tilde{T} \subseteq P(\tilde{A})$ is said to be fuzzy topology on $\tilde{A}$ if it satisfied the following conditions
1) $\tilde{A} , \tilde{\emptyset} \in T$

2) If $\tilde{B} , \tilde{C} \in T$ then $\tilde{B} \cap \tilde{C} \in T$

3) If $\tilde{B}_\alpha \in T$ then $\bigcup_\alpha \tilde{B}_\alpha \in T$, $\alpha \in \Lambda$

$(\tilde{A}, T)$ is said to be Fuzzy topological space and every member of T is called fuzzy open set in $\tilde{A}$ and its complement is a fuzzy closed set.

Definition 2.4: [6] Let G be a fuzzy group and $(G, \widetilde{T})$ be a fuzzy topological space. $(G, \widetilde{T})$ is called a fuzzy topological group if the maps

g: $(G, \widetilde{T}) \times (G, \widetilde{T}) \rightarrow (G, \widetilde{T})$, defined by $g(x, y) = xy$ and

h: $(G, \widetilde{T}) \rightarrow (G, \widetilde{T})$, defined by $h(x) = x^{-1}$ are fuzzy continuous.

3. Some types of fuzzy open sets

Definition 3.1: A fuzzy set $\tilde{A}$ in fuzzy topological group $(G, \widetilde{T})$ is called

1) Fuzzy α −open set if $\tilde{A} \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(\tilde{A})))$.

2) Fuzzy semi −open set if $\tilde{A} \subseteq \mathrm{cl}(\mathrm{int}(\tilde{A}))$.

3) Fuzzy pre −open set if $\tilde{A} \subseteq \mathrm{int}(\mathrm{cl}(\tilde{A}))$.

4) Fuzzy regular −open set if $\tilde{A} = \mathrm{int}(\mathrm{cl}(\tilde{A}))$.

5) Fuzzy b −open set if $\tilde{A} \subseteq (\mathrm{int\,cl}(\tilde{A}) \cup \mathrm{cl\,int}(\tilde{A}))$.

6) Fuzzy β − open set if $\tilde{A} \subseteq (\mathrm{cl}(\mathrm{int}(\mathrm{cl}(\tilde{A})))$.

Proposition 3.2:

1) Every fuzzy open (resp, fuzzy closed) set is fuzzy b − open (resp, fuzzy b − closed) set.
2) Every fuzzy α − open (resp, fuzzy α − closed) set is fuzzy b − open (resp, fuzzy b − closed) set.
3) Every fuzzy semi − open (resp, fuzzy semi − closed) set is fuzzy b − open (resp, fuzzy b − closed) set.
4) Every fuzzy pre − open (resp, fuzzy pre − closed) set is fuzzy b − open (resp, fuzzy b − regular closed) set.
5) Every fuzzy regular − open (resp, fuzzy regular − closed) set is fuzzyb − open (resp, fuzzy b − closed) set.
6) Every fuzzy b − open (resp, fuzzy b − closed) set is fuzzy β −open (resp, fuzzy β −closed) set.

Proof:

1) $M_{\tilde{A}}(x) \leq M_{\mathrm{cl}\,\tilde{A}}(x)$

$M_{\mathrm{int}\,\tilde{A}}(x) \leq M_{\mathrm{int\,cl}\,\tilde{A}}(x)$.

$$M_{\tilde{A}}(x) \leq M_{\mathrm{int\,cl}\,\tilde{A}}(x) \quad (1)$$

$M_{\mathrm{int}\,\tilde{A}}(x) \leq M_{\mathrm{cl\,int}\,\tilde{A}}(x)$.

$$M_{\tilde{A}}(x) \leq M_{\mathrm{cl\,int}\,\tilde{A}}(x) \quad (2)$$

From (1) and (2) we gait.

$M_{\tilde{A}}(x) \leq \max\{M_{\mathrm{int\,cl}\,\tilde{A}}(x), M_{\mathrm{cl\,int}\,\tilde{A}}(x)\}$.

2) $M_{\tilde{A}}(x) \leq M_{\mathrm{int\,cl\,int}\,\tilde{A}}(x)$.

$M_{\tilde{A}}(x) \leq M_{\mathrm{cl\,int}\,\tilde{A}}(x)$.

$M_{\mathrm{cl}\,\tilde{A}}(x) \leq M_{\mathrm{cl\,int}\,\tilde{A}}(x)$.

$M_{\mathrm{int\,cl}\,\tilde{A}}(x) \leq M_{\mathrm{int\,cl\,int}\,\tilde{A}}(x)$.

$M_{int\,cl\,\tilde{A}}(x) \leq M_{cl\,int\,\tilde{A}}(x)$.

$max\{M_{int\,cl\,\tilde{A}}(x), M_{cl\,int\,\tilde{A}}(x)\} = M_{cl\,int\,\tilde{A}}(x)$.

$max\{M_{int\ cl\ \tilde{A}}(x), M_{cl\ int\ \tilde{A}}(x)\} = M_{int\ cl\ int\ \tilde{A}}(x)$.
$M_{\tilde{A}}(x) \leq max\{M_{int\ cl\ \tilde{A}}(x), M_{cl\ int\ \tilde{A}}(x)\}$.

Similarly we prove 2, 3, 4, 5 and 6 ■

Definition 3.3: Let $(G, \tilde{T})$ be a fuzzy topological group and let $\tilde{W} = \{\tilde{C}_\alpha, \alpha \in \mu\}$ be a collection of fuzzy open(resp, fuzzy $\alpha -$ open , fuzzy semi $-$ open , fuzzy pre $-$ open , fuzzy regular $-$ open , fuzzy b $-$ open, fuzzy $\beta -$open) sets in G is said to be fuzzy open (resp, fuzzy $\alpha -$ open , fuzzy semi $-$ open , fuzzy pre $-$ open , fuzzy regular $-$ open , fuzzy b $-$ open , fuzzy $\beta -$open) cover of fuzzy set $\tilde{B}$ of G if and only if $M_G(x) = sup\{M_{\tilde{C}_\alpha}(x): \alpha \in \mu\}\ \forall x \in S(\tilde{B})$. and fuzzy open (resp, fuzzy $\alpha -$ open , fuzzy semi $-$ open , fuzzy pre $-$ open , fuzzy regular $-$ open , fuzzy b $-$ open , fuzzy $\beta -$open) cover of fuzzy set $\tilde{B}$ of G is said to have a finite sub cover if and only if finite cub collection $\tilde{C} = \{\tilde{C}_1, \dots\dots\dots, \tilde{C}_n\}$ of $\tilde{W}$ such that $M_{\tilde{B}}(x) \leq max\{M_{\tilde{C}_1}(x), \dots\dots\dots, M_{\tilde{C}_n}(x)\}\ \forall x \in S(\tilde{B})$.

Theorem 3.4: *Let $(G, \tilde{T})$ be a fuzzy topological group*

1) Every fuzzy open cover is fuzzy $b -$ opencover.
2) Every fuzzy $\alpha -$open cover is fuzzy $b -$ opencover.
3) Every fuzzy $semi -$open cover is fuzzy $b -$ opencover.
4) Every fuzzy $pre -$open cover is fuzzy $b -$ opencover.
5) Every fuzzy $regular -$open cover is fuzzy $b -$ opencover.
6) Every $b -$fuzzy open cover is fuzzy $\beta -$ open cover.

Proof:

3) Let $\tilde{W} = \{\tilde{C}_\alpha, \alpha \in \mu\}$ be a collection of fuzzy semi $-$ open sets of G.

And $\tilde{W}$ is fuzzy $semi -$ open cover of fuzzy set $\tilde{B}$ in G.

$\because\ \tilde{W} = \{\tilde{C}_\alpha, \alpha \in \mu\}$ Be a collection of fuzzy open $semi -$ sets in G.

And every fuzzy $semi -$ open set is Fuzzy $b -$ open set.

$\therefore \tilde{W}$Be a collection of fuzzy $b -$ open sets of G.

$\because \tilde{W}$Isfuzzy $semi -$ open cover of fuzzy set $\tilde{B}$ in G.

$\therefore M_G(x) = sup\{M_{\tilde{C}\alpha}(x): \alpha \in \mu\}\ \forall x \in S(\tilde{B})$.

Then$\tilde{W}$ is fuzzy $b -$ open cover of fuzzy set $\tilde{B}$ in G.

Similarly we prove 1,2,4,5 and 6 ■

Definition 3.5: Let $(G, \tilde{T})$ be a fuzzy topological group is said to be fuzzy (resp, fuzzy $\alpha -$, fuzzy semi $-$, fuzzy pre $-$, fuzzy regular $-$, fuzzy b $-$, fuzzy $\beta -$) compact if each fuzzy open (resp, fuzzy $\alpha -$ open , fuzzy semi $-$ open , fuzzy pre $-$ open , fuzzy regular $-$ open , fuzzy b $-$ open , fuzzy $\beta -$open) cover has a finite sub cover .

Theorem 3.6: *Let $(G, \tilde{T})$ be a fuzzy topological group*

1) Every fuzzy $b -\ compact$ is fuzzy open$compact$.
2) Every fuzzy $b -\ compact$ is fuzzy $\alpha - compact$.
3) Every fuzzy $b -\ compact$ is fuzzy $semi - compact$.
4) Every fuzzy $b -\ compact$ is fuzzy $pre - compact$.
5) Every fuzzy $b -\ compact$ is fuzzy $regular -\ compact$.
6) Every fuzzy $\beta -\ compact$ is fuzzy $b - compact$.

Proof:

4) Let $(G, \tilde{T})$ be a fuzzy topological group and is said to be fuzzy $-$compact .

Let $\tilde{C}$ be a fuzzy $pre -$open cover of G.

$\therefore \tilde{C}$ be a fuzzy $b -$open cover of G .

$\because (G, \tilde{T})$ Be a fuzzy $b - compact$.

$\therefore \tilde{C}$ has a finite sub cover of G.

$\therefore\ \forall \tilde{C}$is fuzzy $pre -$open cover of G has a finite sub cover of G.

Then $(G, \tilde{T})$ is fuzzy $pre - compact$.

Similarly we prove1, 2, 3, 5 and 6 ■

Definition 3.7: A mapping $f \in (x, y)$ where $(x, y) \in G \times K$ from fuzzy topological group $(G, \tilde{T})$ to fuzzy topological group $(K, \tilde{\delta})$ is said to be fuzzy (resp, fuzzy $\alpha -$, fuzzy semi $-$, fuzzy pre $-$, fuzzy regular $-$, fuzzy b $-$, fuzzy $\beta -$) continuous if $f^{-1}(\tilde{C})$ is fuzzy open (resp, fuzzy $\alpha -$ open , fuzzy semi $-$ open , fuzzy pre $-$ open , fuzzy regular $-$ open , fuzzy b $-$ open , fuzzy $\beta -$open) set in G , for each fuzzy open set $\tilde{C}$ in K.

Theorem 3.8:

1) Every fuzzy continuous is fuzzy $b -$ continuous.
2) Every fuzzy $\alpha -$continuous is fuzzy $b -$ continuous.
3) Every fuzzy $semi -$continuous is fuzzy $b -$ continuous.
4) Every fuzzy $pre -$continuous is fuzzy $b -$ continuous.
5) Every fuzzy $regular -$continuous is fuzzy $b -$ continuous.
6) Every fuzzy $b -$continuous is fuzzy $\beta -$ continuous.

Proof:
5) Let $f: (G, \tilde{T}) \to (K, \tilde{\delta})$ is fuzzy regular $-$continuous.
Let $\tilde{C}$ de a fuzzy open set in K.
Then $f^{-1}(\tilde{C})$ is fuzzy $regular -$ open set in G.
$\because$ Every fuzzy $regular -$ open set is Fuzzy $b - regular$ open set.
$\therefore f^{-1}(\tilde{C})$ isfuzzy $b -$ open set in G.
$\therefore \forall\ \tilde{C}$ is fuzzy open in K, $f^{-1}(\tilde{C})$ is fuzzy $b -$ open set in G.
Then $f: (G, \tilde{T}) \to (K, \tilde{\delta})$ is fuzzy $b -$continuous.
Similarly we prove 1,2,3,4 and 6 ■

Definition 3.9: Let $(G, \tilde{T})$ be a fuzzy topological group and $\tilde{A}$ is a fuzzy set in G then
1) $cl(\tilde{A}) = \cap \{\tilde{F} : \tilde{F} is\ closed\ fuzzy\ set\ , \tilde{A} \subseteq \tilde{F}\}$.
2) $\alpha cl(\tilde{A}) = \cap \{\tilde{F} : \tilde{F} is\ \alpha - closed\ fuzzy\ set\ , \tilde{A} \subseteq \tilde{F}\}$.
3) $semi\ cl(\tilde{A}) = \cap \{\tilde{F} : \tilde{F} is\ semi - closed\ fuzzy\ set\ , \tilde{A} \subseteq \tilde{F}\}$.
4) $pre\ cl(\tilde{A}) = \cap \{\tilde{F} : \tilde{F} is\ pre - closed\ fuzzy\ set\ , \tilde{A} \subseteq \tilde{F}\}$.
5) $bcl(\tilde{A}) = \cap \{\tilde{F} : \tilde{F} is\ b - closed\ fuzzy\ set\ , \tilde{A} \subseteq \tilde{F}\}$.
6) $\beta cl(\tilde{A}) = \cap \{\tilde{F} : \tilde{F} is\ \beta - closed\ fuzzy\ set\ , \tilde{A} \subseteq \tilde{F}\}$.

Definition 3.10: Let $(G, \tilde{T})$ be a fuzzy topological group and $\tilde{B}, \tilde{C}$ are fuzzy set in G then$\tilde{B}$ and $\tilde{C}$ are said to be
1) Fuzzy $separated$ iff $\{cl(\tilde{B}) \cap \tilde{C}\} = \emptyset$ and$\{cl(\tilde{C}) \cap \tilde{B}\} = \emptyset\ \forall x \in G$.
2) Fuzzy $\alpha -$separated iff $\{\alpha cl(\tilde{B}) \cap \tilde{C}\} = \emptyset$ and $\{\alpha cl(\tilde{C}) \cap \tilde{B}\} = \emptyset\ \forall x \in G$.
3) Fuzzy $semi -\ separated$ iff $\{semi\ cl(\tilde{B}) \cap \tilde{C}\} = \emptyset$ and $\{semi\ cl(\tilde{C}) \cap \tilde{B}\} = \emptyset\ \forall x \in G$.
4) Fuzzy $pre -\ separated$ iff $\{pre\ cl(\tilde{B}) \cap \tilde{C}\} = \emptyset$ and$\{pre\ cl(\tilde{C}) \cap \tilde{B}\} = \emptyset\ \forall x \in G$.
5) Fuzzy $regular -\ separated$ iff $\{regular\ cl(\tilde{B}) \cap \tilde{C}\} = \emptyset$ and $\{regular\ cl(\tilde{C}) \cap \tilde{B}\} = \emptyset\ \forall x \in G$.
6) Fuzzy $b -\ separated$ iff $\{bcl(\tilde{B}) \cap \tilde{C}\} = \emptyset$ and$\{bcl(\tilde{C}) \cap \tilde{B}\} = \emptyset\ \forall x \in G$.
7) Fuzzy $\beta -\ separated$ iff $\{\beta cl(\tilde{B}) \cap \tilde{C}\} = \emptyset$ and $\{\beta cl(\tilde{C}) \cap \tilde{B}\} = \emptyset\ \forall x \in G$.

Theorem 3.11: *Let $(G, \tilde{T})$ be a fuzzy topological group*

1) Every fuzzy $separated$is fuzzy $b -\ separated$.
2) Every fuzzy $\alpha -\ separated$ is fuzzy $b -\ separated$.
3) Every fuzzy $semi -\ separated$ is fuzzy $b -\ separated$.
4) Every fuzzy $pre -\ separated$ is fuzzy $b - separated$.
5) Every fuzzy $regular - separated$is fuzzy $b -\ separated$.
6) Every fuzzy $b - separated$ is fuzzy $\beta -\ separated$.

Proof:
Obvious ■

Definition 3.12: Let $(G, \tilde{T})$ be a fuzzy topological group is said to be fuzzy (resp, fuzzy $\alpha -$, fuzzy semi $-$, fuzzy pre $-$, fuzzy regular $-$, fuzzy b$-$, fuzzy $\beta -$)connected , if $\tilde{A}$ cannot be expressed as the union of two maximal fuzzy (resp, fuzzy $\alpha -$, fuzzy semi $-$, fuzzy pre $-$, fuzzy regular $-$, fuzzy b $-$, fuzzy $\beta -$)separated sets . Other wise $(G, \tilde{T})$ is fuzzy (resp, fuzzy $\alpha -$, fuzzy emi $-$, fuzzy pre $-$, fuzzy regular $-$, fuzzy b $-$, fuzzy $\beta -$)disconnected.

Theorem 3.13: *Let* $(G, \tilde{T})$ *be a fuzzy topological group*

1) Every fuzzy $b - connected$ is fuzzy open $connected$.
2) Every fuzzy $b -\ connected$ is fuzzy $\alpha - connected$.
3) Every fuzzy $b -\ connected$ is fuzzy $semi - connected$.
4) Every fuzzy $b - connected$ is fuzzy $pre - connected$.
5) Every fuzzy $b -\ connected$ is fuzzy $regular -\ connected$.
6) Every fuzzy $\beta -\ connected$ is fuzzy $b -\ connected$.

Proof:

6) Let $(\mathrm{G}, \tilde{\mathrm{T}})$ be a fuzzy topological group and $\tilde{\mathrm{A}}$ is fuzzy $\beta -$ connected.

Let $\tilde{A}$ is fuzzy $b -\ disconnected$ space.

Then there exists non-empty maximal fuzzy $b - separated\tilde{B}$ and $\tilde{C}$ in G such that $M_G(x) = max\{M_{\tilde{B}}(x), M_{\tilde{C}}(x)\}$.

By theorem (3.11) there exists non-empty maximal fuzzy $\beta - \mathrm{separated}\tilde{\mathrm{B}}$ and $\tilde{\mathrm{C}}$ in G such that $\mathrm{M_G(x)} = \max\{\mathrm{M}_{\tilde{\mathrm{B}}}(\mathrm{x}), \mathrm{M}_{\tilde{\mathrm{C}}}(\mathrm{x})\}$.

Then $\tilde{\mathrm{A}}$ is fuzzy $\beta -$ disconnected space, contradiction

Hence $\tilde{\mathrm{A}}$ is fuzzy $\mathrm{b} -$ connected space.

Similarly we prove1, 2, 3, 4 and 5 ■

References

[1] L. A. ZADEH, Fuzzy sets, Inform and Control, 8, 338-353 (1965).
[2] C.L.CHANG, Fuzzy topological spaces, 45, 182-190 (1968).
[3] A. ROSENFELD, Fuzzy groups, Journal of Mathematical Analyses and Applications. 35, 512-517 (1971).
[4] B. HUTTON, Normality in Fuzzy Topological Spaces, Journal of Mathematical Analyses and Applications. 50, 74-79 (1975).
[5] R. LOWEN, Fuzzy Topological Spaces and Fuzzy Compactness, Journal of Mathematical Analyses and Applications. 6, 621-633 (1976).
[6] D.H.FOSTER, Fuzzy topological groups, Journal of Mathematical Analyses and Applications. 67, 549-564 (1979).
[7] P. E. Klodenk, Fuzzy Dynamical Systems, Fuzzy Sets and Systems Vol.7, 275-296 (1982).
[8] G. J. KLIR. And Yuan, B., Fuzzy Set Theory: Foundations and Applications, Prentice Hall PTR, (1997).