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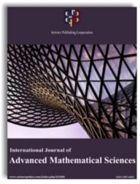

Theoretical analysis of the quartic autocatalytic reaction of a thermally radiative ternary hybrid nanofluid in a stratified porous medium

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Abstract

Quartic autocatalytic form of chemical reaction has valuable interference in catalysis, manufacturing of ceramics, and production of polymers. Motivated by this, the present work examined the quartic autocalytic chemical reaction of an hybrid nanofluid in the presence of thermal stratification, radiation, porosity. Similarity transformation method was employed to convert the governing equations into ordinary differential equations. The existence and uniqueness of a solution was examined, and numerical solution thereafter obtained. Results obtained were shown in figures.

Keywords: *Autocatalytic Chemical Reaction; Hybrid Nanofluid; Existence and Uniqueness; Thermal Stratification; Variable Thermal Conductivity; Thermal Radiation.*

1. Introduction

The superior performance and better thermophysical properties demonstrated by nanofluids have make them the preferred choice of developers, researchers and manufacturers in cooling system, solar reactor, air conditioning, freezing system, solar reactor and so on. By nanofluid, this denotes a conventional fluid infused with a nanosized particle. The first popular work on nanofluid can be attributed to Choi [1] when he examined enhancing thermal conductivity with nanoparticles. Later, using the Buongiorno's model, Xu et al. [2] stretched the work on nanofluid further by considering the homogeneous – heterogeneous reactions of a nanofluid flow within a region of stagnation point. Several other works on nanofluid in diverse geometry and forms have been conducted but they cannot all be mentioned. However, a few of such are listed here [2 – 7]. The nanofluids family recently witnessed the arrival of a new variant known as hybrid nanofluid. A notable subclass of this variant is the ternary hybrid nanofluid which has three distinct nanoparticles injected in a base fluid. The ternary hybrid nanofluid under the influence of chemical reaction and Arrhenius energy over a wedge was deliberated on by Sajid et al. [8]. The authors obtained a numerical solution by employing the Lobatto IIIA scheme. Algehyne et al [9], the authors conducted a numerical simulation on ternary the ternary hybrid nanofluid using variable diffusion and non-Fourier concept. Guedri et al [10] investigated a radiative ternary hybrid nanofluid on a nonlinear sheet subject to Darchy-Forchheimer phenomenon. The impact of maragoni convection and radiation on the flow of ternary nanofluid in a porous medium in the presence of mass transpiration was discussed by Maranna et al [11]. They used silver, SWCNT and graphene nanoparticles and thereafter obtained an analytical solution based on Laplace transform.

Many systems rely on chemical reactions (both homogeneous and heterogeneous) for their operations. Some of such systems include cooling towers, biological systems, catalysis, fog dispersion, manufacturing of ceramics, production of polymers and hydrometallurgical. By homogeneous reaction, this refers to a form of chemical reaction in which all constituents are in same state while the heterogeneous on the other involves substances of different state. Example of such is a reaction between a gas and a liquid. In order to successfully design systems that rely on this form of chemical reaction for their operation, it is necessary to have a good knowledge of how this chemical reaction works and this knowledge can only be obtained by experiment or theoretical simulation. One of the earliest notable work in this direction got to limelight in 1995, when Chaudhary and Merkin investigated the homogeneous heterogeneous reaction in boundary layer flow [12]. In [13], the investigation on homogeneous – heterogeneous reactions was extended to a nanofluid flowing on a porous sheet. The numerical solution to the problem was obtained and an analytical solution was also gotten for the momentum equation. In 2017, the effects of nonlinear thermal radiation and quartic autocatalytic chemical reaction on the flow of a three dimensional Eyring-Powell alumina water nanofluid was studied [14]. The stagnation flow of a SWCNT nanofluid towards a plane surface with heterogeneous-homogeneous reactions was examined by Sohail Ahmed [15]. Recently, the impact of homogeneous and heterogeneous reactions on the flow of hybrid nanofluid was examined on three different surfaces (cone, plate and wedge) by Haq et al [16].

Thermal radiation is a very important process which is applicable in nuclear reactor, cooling systems, gas turbines, missiles, satellites, space vehicles, food processing and preservation, medical treatment of diseases and so on. These and many other applications caught the attention of researchers and developers which motivated the research on radiation. Cess [17] discussed the effects of radiation on the boundary layer flow of an absorbing gas. Radiation in a reacting boundary layer was studied by Goldmann and Heyt [18]. Smith et al. [19]

using numerical approach analyzed the evolution of boundary layer during a radiation fog event. Azeem Shehzard et al. [20] discussed the effect of radiation on the boundary layer flow of absorbing gas. The thermal radiation with viscous dissipation for a Williamson fluid flow due to a nonlinearly stretching sheet was analyzed by Megahed [21]. Dogonchi et al [22] examined the effects of thermal radiation in company of homogeneous heterogeneous reactions on an MHD Cu – water nanofluid over an expanding flat sheet.
Motivated by the applications of this form of chemical reaction and hybrid nanofluids together with the fact that based on available literature no one have fully considered the quartic autocatalytic reaction of a thermally radiative ternary hybrid nanofluid in a stratified porous medium. Hence, the need to undertake the study.

2. Mathematical formulation

The present work assumed the steady, laminar flow of a ternary hybrid nanofluid with water base fluid through a stretching sheet. The ternary hybrid nanofluid is made up of Ag, Al_2O_3, SiO_2 nanopaticles. It is further presumed that the base fluid together with the nanoparticles are thermally balanced and the flow is irrotational and inviscid. Taking solace in the homogeneous-heterogeneous reaction model recommended in [12], [23], [24], the isothermal quartic autocatalytic reaction when chemical reactant B is of high concentration at the surface is given as

$$\mathrm{A} + 3\mathrm{B} \rightarrow 4\mathrm{B}, \text{rate of chemical reaction} = \mathrm{k}_1 \mathrm{al}^3 \quad (1)$$

and on porous surface, in the presence of catalyst, it is assumed that there exist a single isothermal reaction of first order in the form

$$\mathrm{A} \rightarrow \mathrm{B}, \text{rate of chemical reaction} = \mathrm{k}_s \mathrm{a}, \quad (2)$$

where $'\mathrm{a}'$ and $'\mathrm{b}'$ are the concentrations of chemical reactants A and B. The symbols k_1 and k_s stand for the reaction rate coefficients.

Table 1: Thermophysical Properties of Some Nanofluids [10], [11], [25], [27]

Material	Density $(\mathrm{kg/m^3})$	Specific Heat Capacity $\mathrm{C_p}(\mathrm{J/KgK})$	Electrical Conductivity $\sigma \times 10^{-5}(\mathrm{S/m})$	Thermal Conductivity $\mathrm{K(W/mk)}$
Aluminium Oxide (Al_2O_3)	3970	765	0.85	40
Blood	1050	3617	0.18	0.52
Copper (Cu)	8933	385	1.67	401
Gold (Au)	19300	129	4.1	318
Silver (Ag)	10500	235	18.9	429
Water(H_2O)	997.1	4179	0.05	0.613

The flow is assumed to take place in the presence of radiation, hence, the radiative heat flux q_r is incorporated into the energy equation. In the light of the above assumptions, together with the description in [8], [10], [24], [25], [27], the governing equations takes the form:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_{thnf}}\frac{\partial}{\partial y}\left(\mu_{thnf}(T)\frac{\partial u}{\partial y}\right) - \frac{\mu_{thnf}u}{\rho_{thnf}K_p} \quad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{(\rho C_p)_{hnf}}\frac{\partial}{\partial y}\left[k_{thnf}(T)\frac{\partial T}{\partial y}\right] + \tau\left[D_A\frac{\partial a}{\partial y}\frac{\partial T}{\partial y} + \left(\frac{\partial T}{\partial y}\right)^2\left(\frac{D_T}{T_\infty}\right)\right] - \frac{1}{(\rho C_p)_{thnf}}\frac{\partial q_r}{\partial y} \quad (4)$$

$$u\frac{\partial a}{\partial x} + v\frac{\partial a}{\partial y} = D_A\frac{\partial^2 a}{\partial y^2} - \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} - k_1 ab^3 \quad (5)$$

$$u\frac{\partial b}{\partial x} + v\frac{\partial b}{\partial y} = D_B\frac{\partial^2 b}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} + k_1 ab^3 \quad (6)$$

Subject to the following boundary conditions

$$u = U_0 x, v = 0, T = T_w = T_0 + d_1 x, D_A\frac{\partial a}{\partial y} = k_s a, D_B\frac{\partial b}{\partial y} = -k_s a \; at \; y = 0$$

$$u \rightarrow 0, T \rightarrow T_\infty = T_0 + d_2 x, a \rightarrow a_0, b \rightarrow 0 \; as \; y \rightarrow \infty \quad (7)$$

where u, v are velocity components in x and y directions, $K_p$is permeability of porous plate, k_{thnf} ternary hybrid nanofluid thermal conductivity, T is the fluid temperature,T_w represents the surface temperature, T_∞ represents the ambient temperature, specific capacity at constant pressure, ρ fluid density, D_T stands for thermophoretic diffusion coefficient, D_B stands for Brownian diffusion coefficient, $\tau = \frac{(\rho C_p)_{thnf}}{(\rho C_p)_f}$ represents the ratio of heat capacity of ternary nanofluid to heat capacity of base fluid.
The present study will invoke the variable viscosity and thermal conductivity models of the form specified [23], [26] below

$$\mu(T) = \mu[a_1 + b(T_w - T)], k(T) = K[b_1 + \gamma(T - T_\infty)] \quad (8)$$

where a_1, b_1 and γ are constant.

Table 2: Model for Ternary Hybrid Nanofluid [9], [10], [27]

μ_{thnf}	$\frac{\mu_f}{(1-\omega_1)^{2.5}(1-\omega_2)^{2.5}(1-\omega_3)^{2.5}}$
$(\rho C_p)_{thnf}$	$(1-\omega_1)\times\left\{(1-\omega_2)\begin{bmatrix}(1-\omega_3)(\rho C_p)_f \\ +\omega_3(\rho C_p)_{s_3}\end{bmatrix}+\omega_2(\rho C_p)_{s_2}\right\}+\omega_1(\rho C_p)_{s_1}$
$\frac{k_{thnf}}{k_{hnf}}$	$\frac{k_1+2k_{nf}-2\omega_1(k_{nf}-k_1)}{k_1+2k_{nf}+2\omega_1(k_{nf}-k_1)}$
$\frac{k_{hnf}}{k_{nf}}$	$\frac{k_2+2k_{nf}-2\omega_2(k_{nf}-k_2)}{k_2+2k_{nf}+2\omega_2(k_{nf}-k_2)}$
$\frac{k_{nf}}{k_f}$	$\frac{k_3+2k_{nf}-2\omega_3(k_{nf}-k_3)}{k_3+2k_{nf}+2\omega_3(k_{nf}-k_3)}$

3. Method of solution

The special form of similarity variable η, stream function ψ and variables (θ, a, b, u, v) represented as [24], [26]:

$$\eta = y\sqrt{\frac{U_0}{\vartheta}}, \psi = \sqrt{\vartheta U_0}\, x f(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w-T_0}, a = a_0 g(\eta), b = a_0 h(\eta), \tag{9}$$

Are considered to obtain the similarity solutions to the problem at hand. Based on the terms in equation (9) and table 2, the continuity equation is satisfied. The remaining equations from the governing equations are reduced to the following nonlinear ordinary differential equations:

$$\frac{A_1[1+\xi[1-s_t-\theta]]}{A_2}\frac{d^3f}{d\eta^3} = \frac{df}{d\eta}\frac{df}{d\eta} - f(\eta)\frac{d^2f}{d\eta^2} + \frac{\xi A_1}{A_2}\frac{d^2f}{d\eta^2}\frac{d\theta}{d\eta} + \frac{A_1 P_{or}}{A_2}\frac{df}{d\eta}, \tag{10}$$

$$\frac{\{3A_4[1+\epsilon x_4]+4R_a\}}{3A_3P_r}\frac{d^2\theta}{d\eta^2} = -f(\eta)\frac{d\theta}{d\eta} - A_3\left(N_b\frac{d\theta}{d\eta}\frac{dg}{d\eta} + N_t\frac{d\theta}{d\eta}\frac{d\theta}{d\eta}\right) + [S_t+\theta]\frac{df}{d\eta} - \frac{A_4}{A_3}\frac{\epsilon}{P_r}\frac{d\theta}{d\eta}\frac{d\theta}{d\eta}, \tag{11}$$

$$\frac{d^2g}{d\eta^2} = \frac{N_t}{N_b}\frac{d^2\theta}{\partial\eta^2} + Sc_A K_r g(\eta)h^3(\eta) - Sc_A f(\eta)\frac{dg}{d\eta}$$

$$\frac{d^2h}{d\eta^2} = -Sc_B f(\eta)\frac{dh}{d\eta} - \frac{N_t Sc_B}{PN_b SC_A}\frac{d^2\theta}{d\eta^2} - \frac{Sc_B K_r g(\eta)h^3(\eta)}{P} \tag{12}$$

Subject to

$$\frac{df(0)}{d\eta} = 1, f(0) = 0, \theta(0) = 1 - S_t, \frac{dg(0)}{d\eta} = \aleph g(0), \frac{dh(0)}{d\eta} = -\frac{\aleph g(0)}{P},$$

$$\frac{df(\infty)}{d\eta} \to 0, \theta(\infty) \to 0, g(\infty) \to 1, h(\infty) \to 0 \tag{13}$$

where thermal viscosity parameter $\xi = bd_1x$, thermal stratification parameter $s_t = \frac{d_2}{d_1}$, porous medium parameter $P_{or} = \frac{\vartheta}{K_1U_0}$, thermal conductivity parameter $\epsilon = b_2(T_w - T_\infty)$, Radiation parameter $R_a = \frac{4\sigma^* T_\infty^3}{k_f k_1}$, Prandtl number $P_r = \frac{(\rho C_p)_f \vartheta}{k_f}$, Brownian motion parameter $N_b = \frac{D_A a_0}{\vartheta}$, Thermophoretic parameter $N_t = \frac{D_T}{\vartheta T_\infty}d_1x$, Schmidt number for reactant A, $Sc_A = \frac{\vartheta}{D_A}$, Schmidt number for reactant B, $Sc_B = \frac{\vartheta}{D_B}$, homogeneous reaction strength $K_r = \frac{k_1 b_0^3}{U_0}$, $P = \frac{b_0}{a_0}$, $a = 1$, $\aleph = \frac{k_s}{D_A}\sqrt{\frac{\vartheta}{U_0}}$ is the heterogeneous reaction strength,

$$A_1 = \frac{1}{(1-\omega_1)^{2.5}(1-\omega_2)^{2.5}(1-\omega_3)^{2.5}}, A_2 = \begin{bmatrix}(1-\omega_1)\left\{(1-\omega_2)\left[(1-\omega_3)+\frac{\rho_3\omega_3}{\rho_f}\right]+\frac{\rho_2\omega_2}{\rho_f}\right\} \\ +\frac{\rho_1\omega_1}{\rho_f}\end{bmatrix},$$

$$A_3 = (1-\omega_1)\begin{Bmatrix}(1-\omega_2)\left[(1-\omega_3)+\frac{\omega_3(\rho C_p)_{s_3}}{(\rho C_p)_f}\right] \\ +\frac{\omega_2(\rho C_p)_{s_2}}{(\rho C_p)_f}\end{Bmatrix} + \frac{\omega_1(\rho C_p)_{s_1}}{(\rho C_p)_f}, A_4 = \frac{k_{thnf}}{k_f} = \frac{k_1+2k_{nf}-2\omega_1(k_{nf}-k_1)}{k_1+2k_{nf}+\omega_1(k_{nf}-k_1)} \times \frac{k_2+2k_{nf}-2\omega_2(k_{nf}-k_2)}{k_2+2k_{nf}+\omega_2(k_{nf}-k_2)}$$

$$\times \frac{k_3+2k_f-2\omega_3(k_{nf}-k_3)}{k_3+2k_f+\omega_3(k_{nf}-k_3)}.$$

3.1. Existence and uniqueness of solution

Here, the coupled boundary value problem (10) – (12) subject to the boundary conditions (13) is to be examined for whether it has a solution and if it has, is the solution unique or not?

The Existence and Uniqueness Theorem: Let f, θ and ϕ be continuous functions of η at all points in some neighbourhood, and $\xi > 0$, $s_t > 0$, $P_{or} > 0$, $\epsilon > 0$, $R_a > 0$, $P_r > 0$, $N_b > 0$, $N_t > 0$, $Sc_A > 0$, $Sc_B > 0$, $K_r > 0$, $P > 0$, $a = 1$, $\aleph > 0$, $\omega_2 > 0$, $\omega_1 > 0$, $\omega_3 > 0$, $\rho_1 >$

$0, \rho_2 > 0, \rho_3 > 0, (\rho C_p)_{s_2} > 0, (\rho C_p)_{s_1} > 0, (\rho C_p)_{s_3} > 0$,then there exists a unique solution for the coupled nonlinear boundary value problem (10) – (13) on some interval $\|\eta - \eta_0\| \leq a, \|\eta_0 - \eta\| \leq b$ provided there exist k such that $k = max(0,1,P_1,P_2,\dots P_{12})$ and $0 < k < \infty$

Proof

Imposing the identities similar to those of superimposition (shown in equation 24) on equations (10) – (12) and boundary conditions (13), then in compact form we have

$$\begin{pmatrix} \frac{dx_1}{d\eta} \\ \frac{dx_2}{d\eta} \\ \frac{dx_3}{d\eta} \\ \frac{dx_4}{d\eta} \\ \frac{dx_5}{d\eta} \\ \frac{dx_6}{d\eta} \\ \frac{dx_7}{d\eta} \\ \frac{dx_8}{d\eta} \\ \frac{dx_9}{d\eta} \end{pmatrix} = \begin{pmatrix} x_2 \\ x_3 \\ \frac{A_2\left\{x_2x_2 - x_1x_3 + \frac{A_1\xi x_3x_5}{A_2} + \frac{A_1P_{or}x_2}{A_2}\right\}}{A_1[1+\xi[1-s_t-x_4]]} \\ x_5 \\ \frac{3A_3P_r\left\{-x_1x_5 - A_3(N_bx_5x_7 + N_tx_5x_5) + [S_t + x_4]x_2 - \frac{A_4\frac{\epsilon}{P_r}x_5x_5}{A_3}\right\}}{3A_4[1+\epsilon x_4]+4R_a} \\ x_7 \\ \frac{N_t}{N_b}\frac{dx_5}{d\eta} + Sc_AK_1x_6x_8^3 - Sc_Ax_1x_7 \\ x_9 \\ -Sc_Bx_1x_9 - \frac{N_tSc_B}{PN_bSc_A}\frac{dx_5}{d\eta} - \frac{Sc_BK_rx_6x_8^3}{P} \end{pmatrix} \tag{14}$$

Satisfying the boundary condition

$$\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \\ x_5(0) \\ x_7(0) \\ x_8(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ \alpha \\ 1 - s_t \\ \beta \\ \aleph x_6 \\ -\frac{\aleph x_6}{P} \end{pmatrix} \tag{15}$$

We shall consider $\frac{\partial f_1}{\partial x_j}$ (such that $i, j = 1,2, \dots 7$) to represent the nonlinear functions on the right hand side of equation (14). When $i = 1$ and $j =$counts, we have

$f_1 = x_2$ then

$$\left|\frac{df_1}{dx_1}\right| = \left|\frac{df_1}{dx_3}\right| = \left|\frac{df_1}{dx_4}\right| = \left|\frac{df_1}{dx_5}\right| = \left|\frac{df_1}{dx_6}\right| = \left|\frac{df_1}{dx_7}\right| = 0 < \infty, \left|\frac{df_1}{dx_2}\right| = 1 < \infty \tag{16}$$

When $i = 2$ and $j =$counts, we have

$f_2 = x_3$

$$\left|\frac{df_2}{dx_1}\right| = \left|\frac{df_2}{dx_2}\right| = \left|\frac{df_2}{dx_4}\right| = \left|\frac{df_2}{dx_5}\right| = \left|\frac{df_2}{dx_6}\right| = \left|\frac{df_2}{dx_7}\right| = 0 < \infty, \left|\frac{df_2}{dx_3}\right| = 1 < \infty. \tag{17}$$

When $i = 3$ and $j =$ counts, we have

$f_3 = \frac{A_2\left\{x_2x_2 - x_1x_3 + \frac{A_1\xi x_3x_5}{A_2} + \frac{A_1P_{or}x_2}{A_2}\right\}}{A_1[1+\xi[1-s_t-x_4]]}$ then

Following the properties of absolute values of real numbers according to Wrede and Spiegel [29] which states that $|a + b| \leq |a| + |b|$, thus

$$\left|\frac{df_3}{dx_1}\right| = \left|\frac{|-A_2|\{x_3\}}{A_1[1+\xi[1-s_t-x_4]]}\right| \leq \frac{|-A_2||x_3|}{|A_1|[1+\xi|1-s_t-x_4|]} = P_1 < \infty,$$

$$\left|\frac{df_3}{dx_2}\right| \leq \left|\frac{A_2\left\{2x_2 + \frac{A_1P_{or}}{A_2}\right\}}{A_1[1+\xi[1-s_t-x_4]]}\right| \leq \frac{A_2|2A_2||x_2| + |A_1P_{or}|}{|A_1|[1+\xi|1-s_t-x_4|]} = P_2 < \infty,$$

$$\left|\frac{df_3}{dx_3}\right| \leq \left|\frac{\left\{-A_2x_1 + \frac{A_1A_2\xi x_5}{A_2}\right\}}{A_1[1+\xi[1-s_t-x_4]]}\right| \leq \frac{|-A_2||x_1| + A_1|\xi x_5|}{|A_1|[1+\xi|1-s_t-x_4|]} = P_3 < \infty,$$

$$\left|\frac{df_3}{dx_4}\right| = \left|\frac{A_2\left\{x_2x_2 - x_1x_3 + \frac{A_1\xi x_3x_5}{A_2} + \frac{A_1P_{or}x_2}{A_2}\right\}}{\{A_1[1+\xi[1-s_t-x_4]]\}^2}\right|,$$

$$\left|\frac{df_3}{dx_4}\right| \le \frac{|A_2||x_2||x_2|}{\left|\{A_1[1+\xi[1-S_t-x_4]]\}^2\right|} + \frac{|-A_2||x_1||x_3|}{\left|\{A_1[1+\xi[1-S_t-x_4]]\}^2\right|} + \frac{A_1A_2\xi|x_3||x_5|}{\left|\{A_1[1+\xi[1-S_t-x_4]]\}^2\right|} + \frac{|A_1P_{or}||x_2|}{\left|\{A_1[1+\xi[1-S_t-x_4]]\}^2\right|} = P_4 < \infty,$$

$$\left|\frac{df_3}{dx_5}\right| = \left|\frac{A_1\xi x_3}{A_1[1+\xi[1-S_t-x_4]]}\right| \le \frac{|A_1\xi||x_3|}{|A_1|[1+\xi|1-S_t-x_4|]} = P_5 < \infty,$$

$$\left|\frac{df_3}{dx_6}\right| = \left|\frac{df_3}{dx_7}\right| = \left|\frac{df_3}{dx_8}\right| = \left|\frac{df_3}{dx_9}\right| = 0 < \infty \tag{18}$$

When $i = 4$ and $j =$ counts, we have

$$f_4 = x_5$$

$$\left|\frac{df_4}{dx_1}\right| = \left|\frac{df_4}{dx_2}\right| = \left|\frac{df_4}{dx_3}\right| = \left|\frac{df_4}{dx_4}\right| = \left|\frac{df_4}{dx_6}\right| = \left|\frac{df_4}{dx_7}\right| = \left|\frac{df_4}{dx_8}\right| = \left|\frac{df_4}{dx_9}\right| = 0 < \infty, \left|\frac{df_4}{dx_5}\right| = 1 < \infty \tag{19}$$

When $i = 5$ and $j =$ counts, we have

$$f_5 = \left\{-\frac{3A_3P_rx_1x_5}{\{3A_4[1+\epsilon x_4]+4R_a\}} - \frac{3A_3P_rA_3(N_bx_5x_7+N_tx_5x_5)}{\{3A_4[1+\epsilon x_4]+4R_a\}} + \frac{3A_3P_r[S_t+x_4]x_2}{\{3A_4[1+\epsilon x_4]+4R_a\}} - \frac{3A_4\epsilon x_5x_5}{\{3A_4[1+\epsilon x_4]+4R_a\}}\right\}, \text{then}$$

$$\left|\frac{df_5}{dx_1}\right| = \left|-\frac{3A_3P_rx_5}{\{3A_4[1+\epsilon x_4]+4R_a\}}\right| \le \frac{|-3|A_3P_r|x_5|}{3R_a(1+\epsilon|x_4|)+4R_a} = P_6 < \infty,$$

$$\left|\frac{df_5}{dx_2}\right| = \left|\frac{3A_3P_r[S_t+x_4]}{\{3A_4[1+\epsilon x_4]+4R_a\}}\right| \le \frac{3A_3P_r[S_t+|x_4|]}{\{3A_4[1+\epsilon|x_4|]+4R_a\}} = P_7 < \infty,$$

$$\left|\frac{df_5}{dx_3}\right| = \left|\frac{df_5}{dx_6}\right| = \left|\frac{df_5}{dx_8}\right| = \left|\frac{df_5}{dx_9}\right| = 0,$$

$$\left|\frac{df_5}{dx_4}\right| = \left|\begin{matrix}\frac{9A_3A_4\epsilon P_rx_1x_5}{\{3A_4[1+\epsilon x_4]+4R_a\}^2} + \frac{9A_3P_rA_3A_4\epsilon(N_bx_5x_7+N_tx_5x_5)}{\{3A_4[1+\epsilon x_4]+4R_a\}^2}\\ +\frac{\{9A_4A_3P_rx_2(1-\epsilon S_t)+4R_ax_2\}}{\{3A_4[1+\epsilon x_4]+4R_a\}^2} + \frac{9A_4A_4\epsilon^2x_5x_5}{\{3A_4[1+\epsilon x_4]+4R_a\}^2}\end{matrix}\right|$$

$$\left|\frac{df_5}{dx_4}\right| \le \frac{9A_3A_4P_r\epsilon|x_1||x_5|}{(3A_4(1+|x_4|\epsilon)+4R_a)^2} + \frac{|9|A_3P_rA_3A_4\epsilon N_b|x_5||x_7|+9A_3P_rA_3A_4\epsilon N_t|x_5||x_5|}{(3A_4(1+|x_4|\epsilon)+4R_a)^2} + \frac{9A_4A_3P_r|x_2|(1-\epsilon S_t)+4R_a|x_2|}{(3A_4(1+|x_4|\epsilon)+4R_a)^2} + \frac{9A_4A_4\epsilon^2|x_5||x_5|}{(3A_4(1+|x_4|\epsilon)+4R_a)^2} = P_8 < \infty,$$

$$\left|\frac{df_5}{dx_5}\right| = \left|-\frac{3A_3P_rx_1}{\{3A_4[1+\epsilon x_4]+4R_a\}} - \frac{3A_3P_rA_3(N_bx_7+2N_tx_5)}{\{3A_4[1+\epsilon x_4]+4R_a\}} - \frac{6A_4\epsilon x_5}{\{3A_4[1+\epsilon x_4]+4R_a\}}\right|,$$

$$\left|\frac{df_5}{dx_5}\right| \le \frac{|-3|A_3P_r|x_1|}{(3A_4(1+|x_4|\epsilon)+4R_a)} + \frac{|-3|A_3A_3P_r(N_b|x_7|+2N_t|x_5|)}{(3A_4(1+|x_4|\epsilon)+4R_a)} + \frac{|-6|A_4\epsilon|x_5|}{(3A_4(1+|x_4|\epsilon)+4R_a)}$$

$$= P_9 < \infty, \left|\frac{df_5}{dx_7}\right| = \left|-\frac{3A_3P_rA_3N_bx_5}{\{3A_4[1+\epsilon x_4]+4R_a\}}\right| \le \frac{|-3|A_3A_3P_rN_b|x_5|}{(3A_4(1+|x_4|\epsilon)+4R_a)} = P_{10} < \infty \tag{20}$$

When $i = 6$ and $j =$ counts, we have

$$f_6 = x_7,$$

$$\left|\frac{df_6}{dx_1}\right| = \left|\frac{df_6}{dx_2}\right| = \left|\frac{df_6}{dx_3}\right| = \left|\frac{df_6}{dx_4}\right| = \left|\frac{df_6}{dx_5}\right| = \left|\frac{df_6}{dx_6}\right| = \left|\frac{df_6}{dx_8}\right| = \left|\frac{df_6}{dx_9}\right| = 0, \left|\frac{df_6}{dx_7}\right| = 1 < \infty \tag{21}$$

When $i = 7$ and $j =$ counts, we have

$$f_7 = -\frac{N_t}{N_b}\frac{3A_3P_rx_1x_5}{\{3A_4[1+\epsilon x_4]+4R_a\}} - \frac{N_t}{N_b}\frac{3A_3P_rA_3(N_bx_5x_7+N_tx_5x_5)}{\{3A_4[1+\epsilon x_4]+4R_a\}} + \frac{N_t}{N_b}\frac{3A_3P_r[S_t+x_4]x_2}{\{3A_4[1+\epsilon x_4]+4R_a\}} - \frac{N_t}{N_b}\frac{3A_4\epsilon x_5x_5}{\{3A_4[1+\epsilon x_4]+4R_a\}} + Sc_AK_1x_6x_8^3 - Sc_Ax_1x_7, \text{ then}$$

$$\left|\frac{df_7}{dx_1}\right| = \left|-\frac{N_t}{N_b}\frac{3A_3P_rx_5}{\{3A_4[1+\epsilon x_4]+4R_a\}} - Sc_Ax_7\right| \le \left|-\frac{N_t}{N_b}\right|3A_3P_r|x_5| + |-Sc_A||x_7| = P_{11} < \infty,$$

$$\left|\frac{df_7}{dx_2}\right| = \left|\frac{N_t}{N_b}\frac{3A_3P_r[S_t+x_4]}{\{3A_4[1+\epsilon x_4]+4R_a\}}\right| \le \frac{N_t}{N_b}\frac{3A_3P_r[S_t+|x_4|]}{\{3A_4[1+\epsilon|x_4|]+4R_a\}} = P_{12} < \infty, \left|\frac{df_7}{dx_3}\right| = \left|\frac{df_7}{dx_9}\right| = 0,$$

$$\left|\frac{df_7}{dx_4}\right| = \left|\begin{matrix}\frac{9A_3A_4\epsilon N_tP_rx_1x_5}{N_b\{3A_4[1+\epsilon x_4]+4R_a\}^2} + \frac{9A_3P_rA_3A_4\epsilon N_t(N_bx_5x_7+N_tx_5x_5)}{N_b\{3A_4[1+\epsilon x_4]+4R_a\}^2} + \frac{N_t\{9A_4A_3P_rx_2(1-\epsilon S_t)+4R_ax_2\}}{N_b\{3A_4[1+\epsilon x_4]+4R_a\}^2}\\ +\frac{9A_4A_4\epsilon^2N_tx_5x_5}{N_b\{3A_4[1+\epsilon x_4]+4R_a\}^2}\end{matrix}\right|$$

$$\left|\frac{df_7}{dx_4}\right| \le \frac{9A_3A_4N_tP_r\epsilon|x_1||x_5|}{N_b(3A_4(1+|x_4|\epsilon)+4R_a)^2} + \frac{|9|A_3P_rA_3A_4\epsilon N_bN_t|x_5||x_7| + 9A_3P_rA_3A_4\epsilon N_tN_t|x_5||x_5|}{N_b(3A_4(1+|x_4|\epsilon)+4R_a)^2}$$

$$+\frac{9A_4A_3N_tP_r|x_2|(1-\epsilon S_t)+4N_tR_a|x_2|}{N_b(3A_4(1+|x_4|\epsilon)+4R_a)^2} + \frac{9A_4A_4\epsilon^2N_t|x_5||x_5|}{N_b(3A_4(1+|x_4|\epsilon)+4R_a)^2} = P_{13} < \infty,$$

$\left|\frac{df_7}{dx_5}\right| = \left|-\frac{N_t}{N_b}\frac{3A_3P_rx_1}{\{3A_4[1+\epsilon x_4]+4R_a\}} - \frac{N_t}{N_b}\frac{3A_3P_rA_3(N_bx_7+2N_tx_5)}{\{3A_4[1+\epsilon x_4]+4R_a\}} - \frac{6A_4\epsilon x_5N_t}{N_b\{3A_4[1+\epsilon x_4]+4R_a\}}\right|,$

$\left|\frac{df_7}{dx_5}\right| \leq \frac{A_3N_tP_r|-3||x_1|}{N_b(3A_4(1+|x_4|\epsilon)+4R_a)} + \frac{|-3|A_3A_3N_tP_r(N_b|x_7|+2N_t|x_5|)}{N_b(3A_4(1+|x_4|\epsilon)+4R_a)} + \frac{6A_4N_t\epsilon|x_5|}{N_b(3A_4(1+|x_4|\epsilon)+4R_a)} = P_{14} < \infty,$

$\left|\frac{df_7}{dx_6}\right| = |Sc_AK_1x_8^3| \leq Sc_AK_1x_8^2|x_8| = P_{15} < \infty, \left|\frac{df_7}{dx_7}\right| = \left|-\frac{3A_3P_rA_3N_bN_tx_5}{N_b\{3A_4[1+\epsilon x_4]+4R_a\}} - Sc_Ax_1\right| \leq |-Sc_A||x_1| + \frac{|-3|A_3A_3P_rN_bN_t|x_5|}{N_b(3A_4(1+|x_4|\epsilon)+4R_a)} = P_{16} < \infty,$

$\left|\frac{df_7}{dx_8}\right| = |3Sc_AK_1x_6x_8^2| \leq 3Sc_AK_1|x_6|x_8^2 = P_{17} < \infty,$

When $i = 8$ and $j =$ counts, we have

$f_8 = x_9$

$$\left|\frac{df_8}{dx_1}\right| = \left|\frac{df_8}{dx_2}\right| = \left|\frac{df_8}{dx_3}\right| = \left|\frac{df_8}{dx_4}\right| = \left|\frac{df_8}{dx_5}\right| = \left|\frac{df_8}{dx_6}\right| = \left|\frac{df_8}{dx_7}\right| = \left|\frac{df_8}{dx_8}\right| = 0, \left|\frac{df_8}{dx_9}\right| = 1 < \infty. \quad (22)$$

When $i = 9$ and $j =$ counts, we have

$$f_9 = -Sc_Bx_1x_9 + \frac{N_tSc_B}{PN_bSc_A}\frac{3A_3P_rx_1x_5}{\{3A_4[1+\epsilon x_4]+4R_a\}} + \frac{N_tSc_B}{PN_bSc_A}\frac{3A_3P_rA_3(N_bx_5x_7+N_tx_5x_5)}{\{3A_4[1+\epsilon x_4]+4R_a\}}$$

$-\frac{N_tSc_B}{PN_bSc_A}\frac{3A_3P_r[S_t+x_4]x_2}{\{3A_4[1+\epsilon x_4]+4R_a\}} + \frac{N_tSc_B}{PN_bSc_A}\frac{3A_4\epsilon x_5x_5}{\{3A_4[1+\epsilon x_4]+4R_a\}} - \frac{Sc_BK_1x_6x_8^3}{P}$, then

$\left|\frac{df_9}{dx_1}\right| = \left|-Sc_Bx_9 + \frac{3A_3N_tP_rSc_Bx_5}{PN_bSc_A\{3A_4[1+\epsilon x_4]+4R_a\}}\right| \leq |-Sc_B||x_9| + \frac{3A_3N_tP_rSc_B|x_5|}{PN_bSc_A\{3A_4[1+\epsilon|x_5|]+4R_a\}} = P_{18} < \infty,$

$\left|\frac{df_9}{dx_2}\right| = \left|-\frac{N_tSc_B}{PN_bSc_A}\frac{3A_3P_r[S_t+x_4]}{\{3A_4[1+\epsilon x_4]+4R_a\}}\right| \leq \frac{|-3|A_3N_tP_rSc_B[S_t+|x_4|]}{PN_bSc_A\{3A_4[1+\epsilon|x_4|]+4R_a\}} = P_{19} < \infty, \left|\frac{df_9}{dx_3}\right| = 0,$

$$\left|\frac{df_9}{dx_4}\right| = \left|\frac{9A_3A_4\epsilon N_tP_rSc_Bx_1x_5}{N_bPSc_A\{3A_4[1+\epsilon x_4]+4R_a\}^2} + \frac{9A_3P_rA_3A_4\epsilon N_tSc_B(N_bx_5x_7+N_tx_5x_5)}{N_bPSc_A\{3A_4[1+\epsilon x_4]+4R_a\}^2} + \frac{N_t\{9A_4A_3P_rSc_Bx_2(1-\epsilon S_t)+4R_ax_2\}}{N_bPSc_A\{3A_4[1+\epsilon x_4]+4R_a\}^2} + \frac{9A_4A_4\epsilon^2N_tSc_Bx_5x_5}{N_bPSc_A\{3A_4[1+\epsilon x_4]+4R_a\}^2}\right|$$

$\left|\frac{df_9}{dx_4}\right| \leq \frac{9A_3A_4\epsilon N_tP_rSc_B|x_1||x_5|}{N_bPSc_A(3A_4(1+|x_4|\epsilon)+4R_a)^2} + \frac{9A_3A_3A_4\epsilon N_bN_tP_rSc_B|x_5||x_7|+9A_3A_3A_4\epsilon N_tN_tP_rSc_B|x_5||x_5|}{N_bPSc_A(3A_4(1+|x_4|\epsilon)+4R_a)^2}$

$+\frac{9A_4A_3N_tP_rSc_B|x_2|(1-\epsilon S_t)+4N_tSc_BR_a|x_2|}{N_bPSc_A(3A_4(1+|x_4|\epsilon)+4R_a)^2} + \frac{9A_4A_4\epsilon^2N_tSc_B|x_5||x_5|}{N_bPSc_A(3A_4(1+|x_4|\epsilon)+4R_a)^2} = P_{20} < \infty,$

$$\left|\frac{df_9}{dx_5}\right| = \left|\frac{3A_3P_rN_tSc_Bx_1}{N_bPSc_A\{3A_4[1+\epsilon x_4]+4R_a\}} + \frac{3A_3P_rA_3N_tSc_B(N_bx_7+2N_tx_5)}{N_bPSc_A\{3A_4[1+\epsilon x_4]+4R_a\}} + \frac{6A_4\epsilon x_5N_tSc_B}{N_bPSc_A\{3A_4[1+\epsilon x_4]+4R_a\}}\right|$$

$\left|\frac{df_9}{dx_5}\right| \leq \frac{3A_3N_tP_rN_tSc_B|x_1|}{N_bPSc_A(3A_4(1+|x_4|\epsilon)+4R_a)} + \frac{3A_3A_3N_tP_rN_tSc_B(N_b|x_7|+2N_t|x_5|)}{N_bPSc_A(3A_4(1+|x_4|\epsilon)+4R_a)} + \frac{6A_4N_t\epsilon N_tSc_B|x_5|}{N_bPSc_A(3A_4(1+|x_4|\epsilon)+4R_a)} = P_{21} < \infty,$

$\left|\frac{df_9}{dx_6}\right| = \left|-\frac{Sc_BK_1x_6x_8^3}{P}\right| \leq \left|-\frac{Sc_A}{P}\right|K_1x_8^2|x_8| = P_{22} < \infty,$

$\left|\frac{df_9}{dx_7}\right| = \left|\frac{3A_3P_rA_3N_bN_tSc_Bx_5}{N_bPSc_A\{3A_4[1+\epsilon x_4]+4R_a\}}\right| \leq \frac{3A_3P_rA_3N_bN_tSc_B|x_5|}{N_bPSc_A(3A_4(1+|x_4|\epsilon)+4R_a)} = P_{23} < \infty,$

$\left|\frac{df_9}{dx_8}\right| = \left|-\frac{3Sc_BK_1x_6x_8^2}{P}\right| \leq \frac{|-3|Sc_AK_1|x_6|x_8^2}{P} = P_{24} < \infty,$

$$\left|\frac{df_9}{dx_9}\right| = |-Sc_Bx_1| \leq |-Sc_B||x_1| = P_{25} < \infty \quad (23)$$

Therefore, we have shown that $\frac{\partial f_i}{\partial x_j} \leq k$ such that $i,j = 1(1)7$. Clearly, $\left|\frac{\partial f_i}{\partial x_j}\right|_{1(1)7}$ is bounded and there exists k such that $k = max(0,1,P_1,P_2,P_3,P_4,P_5,P_6,P_7,P_8,P_9,\dots,P_{25})$ where $0 < k < \infty$. Hence, $f_i(x_1,x_2,x_3,x_4,x_5,x_6,x_7)$ are Lipschitz continuous and therefore the system of coupled differential equation considered has a unique solution.

3.2. Numerical solution

The set of equations (10) – (12) with the boundary conditions (13) are first transformed into a set of first order ordinary differential equations using the idea of superposition introduced by Na [28]. The following identities are essential for the method of superimposition

$$f = f_1, f' = f_2, f'' = f_3, f''' = f_3', \theta = f_4, \theta' = f_5, \theta'' = f_5', g = f_6, g' = f_7, g'' = f_7', h = f_8, h' = f_9, h'' = f_9' \quad (24)$$

Substituting (14) into equations (10) – (13) and simplifying yields:

$$f_3' = \frac{A_2x_2x_2 - A_2x_1x_3 + A_1\xi x_3x_5 + A_1P_{or}x_2}{A_1[1+\xi[1-s_t-x_4]]} \quad (25)$$

$$f_5' = \frac{3A_3P_r\left\{-x_1x_5 - A_3(N_bx_5x_7 + N_tx_5x_5) + [S_t+x_4]x_2 - \frac{A_4\frac{\epsilon}{P_r}x_5x_5}{A_3}\right\}}{3A_4[1+\epsilon x_4]+4R_a} \quad (26)$$

$$f_7' = \frac{N_t}{N_b}\frac{dx_5}{d\eta} + Sc_AK_1x_6x_8^3 - Sc_Ax_1x_7, \quad (27)$$

$$f_9' = -Sc_Bx_1x_9 - \frac{N_tSc_B}{PN_bSc_A}\frac{dx_5}{d\eta} - \frac{Sc_BK_rx_6x_8^3}{P} \quad (28)$$

Subject to

$$f_2(0) = 1, f_1(0) = 0, f_4(0) = 1 - s_t, f_7(0) = \aleph f_6(0), f_9(0) = -\frac{\aleph f_6(0)}{P},$$

$$f_2(\infty) \to 0, f_4(\infty) \to 0, f_6(\infty) \to 0, f_8(\infty) \to 0 \quad (29)$$

The coupled differential equations (25) – (29) are then solved numerically using the Shooting method embedded in o.d.e. solver matlab bvp4c. The values used for the thermophysical properties of nanoparticles are the ones shown on table 1. Except otherwise stated, default values of parameters are $\xi = 0.7, \epsilon = 0.7, P_r = 0.7, M = 1, R_a = 1, N_b = 1, N_t = 1, S_{c_B} = 0.2, S_{c_B} = 0.2,$
$k_r = 0.2, P_{or} = 0.5, P = 0.2, \aleph = 0.2, s_t = 0.2.$

4. Results

In order to analyze our results, numerical computation has been carried out for various values of Brownian motion parameter (N_b), homogenous fluid parameter (k_r), porosity parameter (P_{or}), Schmidt number (S_c),radiation parameter ,Prandtl number (P_r), thermal conductivity parameter (ϵ) and thermophoretic parameter (N_t), using the shooting approach discussed in the previous section.
The numerical values are plotted in Figs. 1– 8. The effect of porosity on fluid temperature is graphically represented in Fig.1. The figure portrayed that porosity triggers a rise in the ternary nanofluid temperature. A rise in both conventional and ternary nanofluid temperature is observed with the ternary nanofluid rising more than the conventional nanofluid. Deviation in radiation with fluid temperature is described in Fig.2. The figure showed that the fluid temperature increases with radiation. Increasing radiation parameter implies that more heat energy is injected into the system and additional heat results in a rise in the fluid temperature. Fig.3 is a graphical demonstration of the impact of radiation on the heterogeneous bulk concentration. The figure revealed that heterogeneous bulk fluid concentration reduces with increasing value of radiation. The influence of stratification on velocity is illustrated in Fig.4. The figure demonstrated the tendency of stratification to increase fluid flow. Variation in homogeneous fluid parameter (k_r) with homogeneous bulk fluid concentration is graphically represented in Fig.5. The figure exhibited that the homogeneous fluid parameter deflates the homogeneous fluid concentration. This observation is in good agreement with Fig.10 in [30] and Fig.7 of [22]. Fig.6 elucidated the effect of homogeneous parameter on the heterogeneous bulk fluid concentration. The figure established that the homogeneous parameter causes the heterogeneous bulk fluid concentration to reduce.
Variation in thermophoretic parameter with homogeneous bulk fluid concentration is elucidated in Figure 7. The figure showed that thermophoretic parameter reduces the concentration of the reactant fluid. Similar effect is observed in Fig.8 where the Brownian motion parameter deflates the homogeneous fluid concentration.

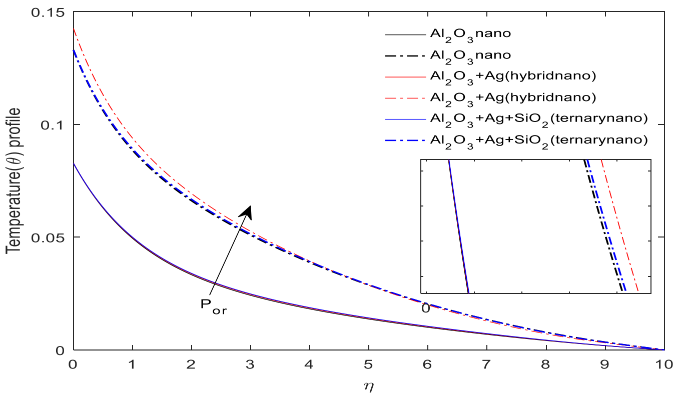

Fig. 1: Variation in Porous Parameter (P_{or}) with Temperature.

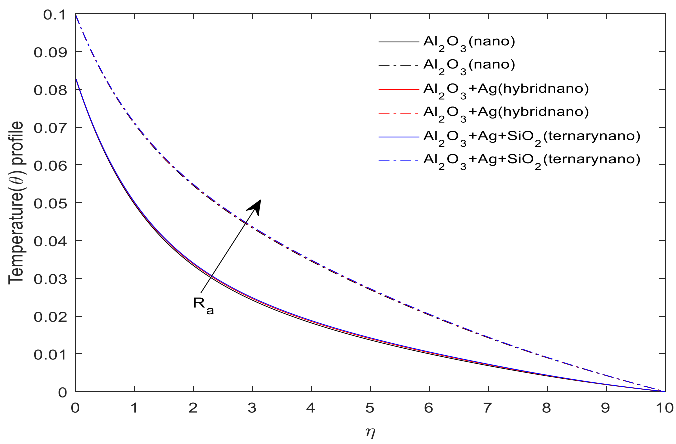

Fig. 2: Variation in Radiation Parameter (R_A) with Temperature.

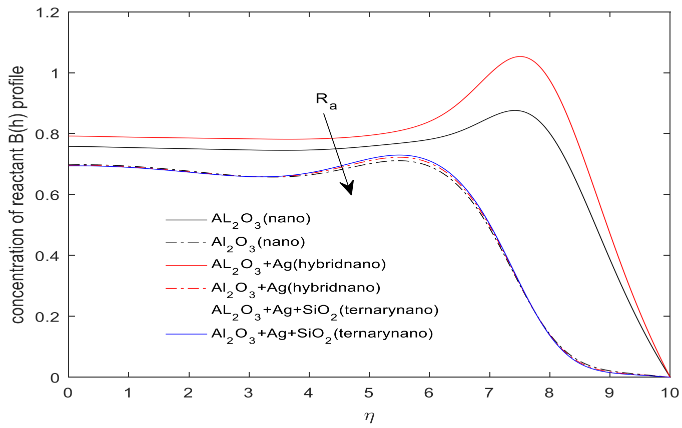

Fig. 3: Variation in Radiation Parameter (R_A) with Concentration of Reactant B.

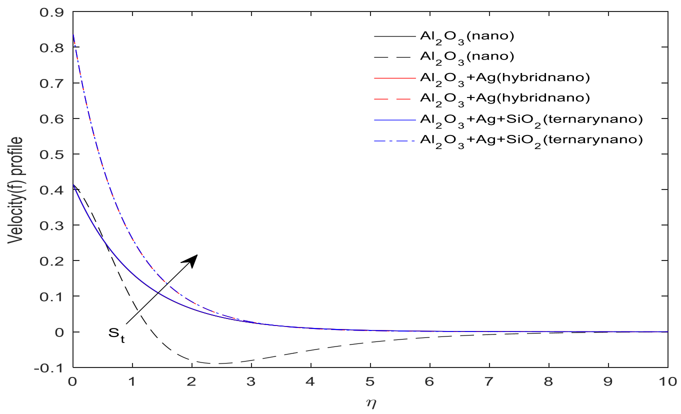

Fig. 4: Variation in Stratification Parameter (S_T) with Velocity.

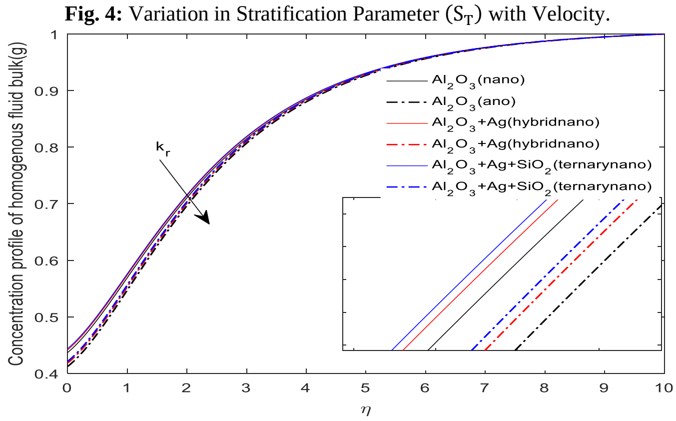

Fig. 5: Variation in Homogenous Fluid Parameter (K_R) with the Concentration of Homogenous Bulk Fluid.

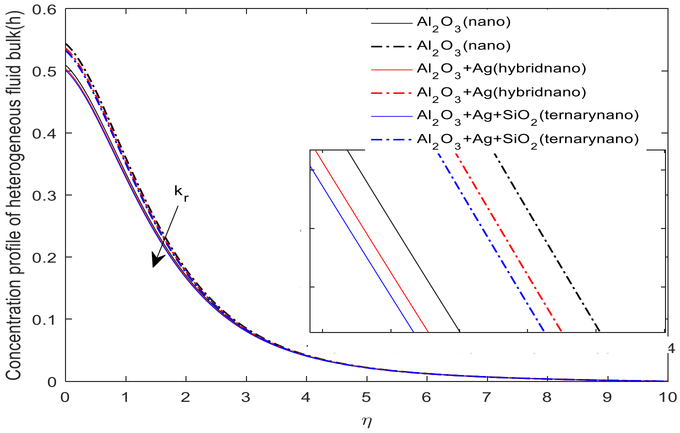

Fig. 6: Variation in Homogenous Parameter (K_R) with Reactant B Concentration Profile.

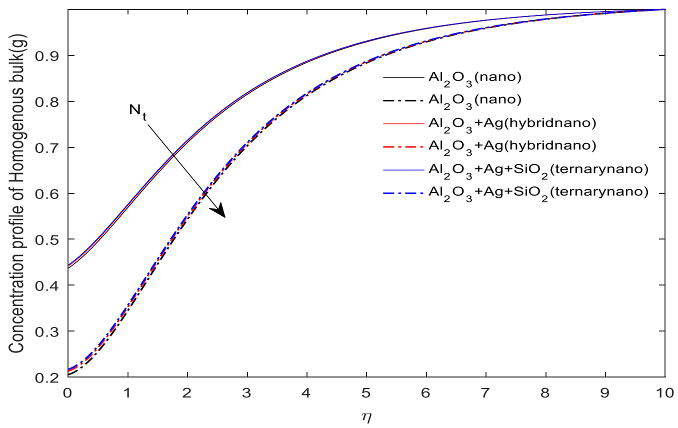

Fig. 7: Variation in Thermophoretic Parameter (N_T) with Homogeneous Bulk Concentration Profile.

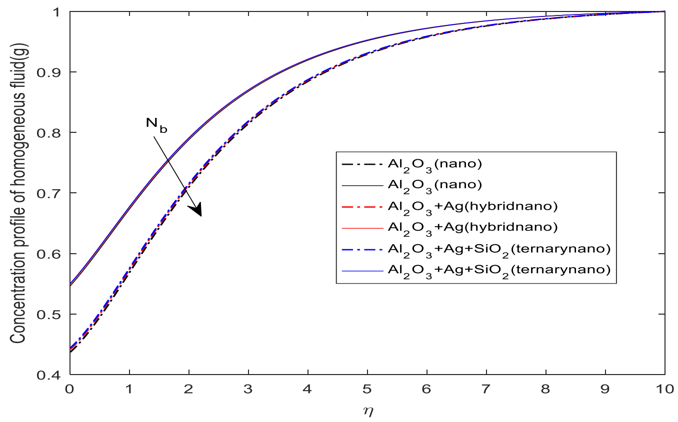

Fig. 8: Variation in Brownian Motion Parameter (N_B) with Homogeneous Fluid Concentration.

5. Conclusion

The quartic autocatalytic reaction of a ternary hybrid nanofluid has been investigated and numerical solution obtained. The result showed the following:

1) That both porosity and radiation are useful tools that can be used to trigger a rise in fluid temperature. Though this behavior of porosity is not always the case but in this scenario it sparks up a rise in temperature. The underlying reason for this behaviour is the presence of the catalytic reaction in which the catalytic reaction causes a rise in kinetic energy and this will bring a sensation of heat which results in the temperature rise.
2) The experiment indicated that radiation and homogeneous parameter causes the concentration of the heterogeneous bulk fluid to reduce.
3) Stratification influences a rise in fluid flow.
4) It was also observed from the result that the ternary hybrid nanofluid rises more than when two or one nano particle(s) is used in instance when a rise in profile results from any change in parameter. This suffices to conclude that the ternary(three) nanofluid is better performing than when two or one nanoparticle are or is used. This was seen in variation involving radiation, porosity and stratification.
5) The present work will be ideal in manufacturing of ceramics as this research involves alumina and silicon nanoparticles which are ideal nanoparticles in making ceramics.

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