

Stress characterization and support measures estimation around a coalmine tunnel passing through jointed rock masses: constraints from BEM simulation

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Abstract

In the present research, the stress characterization around an unsupported coalmine tunnel passing through jointed rock masses was analyzed and effective support system was calculated by BEM numerical simulations. The distribution and magnitudes of major and minor principal stress contours, mean stress, differential stress, total displacement, maximum shear strain, maximum shear stress contours around the tunnel are simulated by using the examine2D software. It is reasonable to mention that examine2D is a plane strain boundary element program for calculation of stresses and displacements around underground and surface excavation in rock. Modeling results reveal that the major principal stress (σ_1) was about 13 MPa at the immediate roof of the tunnel that ultimately increased to 20 MPa toward the left side and right side. Mean stress contour value was 12 MPa at the immediate roof and 15 MPa toward the both rib sides. The distribution contour value of differential stress at the roof and rib sides were 16 MPa and 23 MPa, respectively. The contour values of the strength factor around the tunnel ranged from 0.51 to 1.02, which specify that the loosening zone would be extended up to 1.53 m towards the roof and 1.25 m at the sidewalls. The thickness (1.53 m) of loosening zone can be classified as soft or poor rock mass. In the immediate roof, floor and the both rib sides of the tunnel, the Spalling Criterion values ranged from 2.7 to 8.0 MPa, which indicate no potential for rock-burst around the tunnel. However, flexible support would be required to accommodate the dilatancy deformation during development period. Finally, the stiff support would be required to provide a strong supporting reaction and to maintain the long-term stability of the tunnel.

Keywords: Barapukuria Coalmine; Tunnel; Jointed Rock Mass; Strength Factor; BEM Simulation.

1. Introduction

The development of coalmine tunnel is normally tasked with support systems that should be as economically as possible. For the case of underground excavation, like tunnel development through sparsely jointed rock masses with large joint spacing, tunnel stability is mainly governed by key blocks whose shapes permit free kinematic movement into the opening. For jointed rock masses intersected by discontinuities, the support design philosophy of a tunnel is different between sparsely and moderately jointed rock masses (Hoek et al. 1995, Boon et al. 2015). The choice and extent of support depends largely on the failure mechanism. Failure involves either sliding or falling of key blocks (Boon et al. 2015) as well as raveling or loosening of rock mass material into the excavation opening. The failed material usually consists of numerous rock blocks (Uttili&Crosta 2011ab). In most cases, it is helpful to anticipate the failure pattern of the unsupported opening so that effective support measures can be undertaken. The support design must prevent any rock blocks from loosening and falling into the tunnel. The main support measures employed for jointed rock masses are rock bolts and lining (Hoek et al. 1995). The method of supporting rock masses with steel bars has also been applied in mining works since the late 1800s. Rockbolt can be used to 'lock together' blocks in heavily jointed rock masses to create a 'reinforced arch' around an underground opening/tunnel that is capable

of providing stability to the cavity (Windsor & Thompson 1993, Brown 1999, Lang 1961, Carranza 2009). Systematic rockbolting is nowadays a standard practice in design and construction of tunnels in rock and a key component in technologies used for designing tunnels (Pacher 1964, Rabcewicz 1964, Brown 1981, Carranza 2009).

The Barapukuria coalmine (Fig. 1ab) is the first underground coalmine of Bangladesh, where some permanent tunnels for mine transport system have been developed throughout the unconsolidated and highly jointed rock masses of the Gondwana rock formation (Fig. 1c). During the development period, numerous roof-fall and rib-fall events around the developing tunnel were common phenomenon of mine. In the near future, some new coalmines are going to be developed throughout the same geological formation with identical tectonic discontinuities (Fig. 1d) (Islam and Shinjo 2009ab, Islam et al. 2009, Islam & Hayashi 2008, Islam & Islam 2005, Islam et al. 2015). One of the important issues during the construction of mine tunnels in saturated ground, like Barapukuria coalmine in Bangladesh, is the estimation of the failure pattern associated with stress characterization during unsupported condition. For the case of coalmine tunnel development in Bangladesh, the study of stress characterization of an unsupported tunneling system associated with jointed rock masses and effective support measures have not been carried out yet. Major objective of this paper is to provide a BEM numerical modeling to anticipate the stress characterization of an unsupported tunneling of a coalmine

so that effective support measures can be undertaken. The boundary element method (BEM) has become very popular for tunnel

construction due to its technical feasibility and safety.

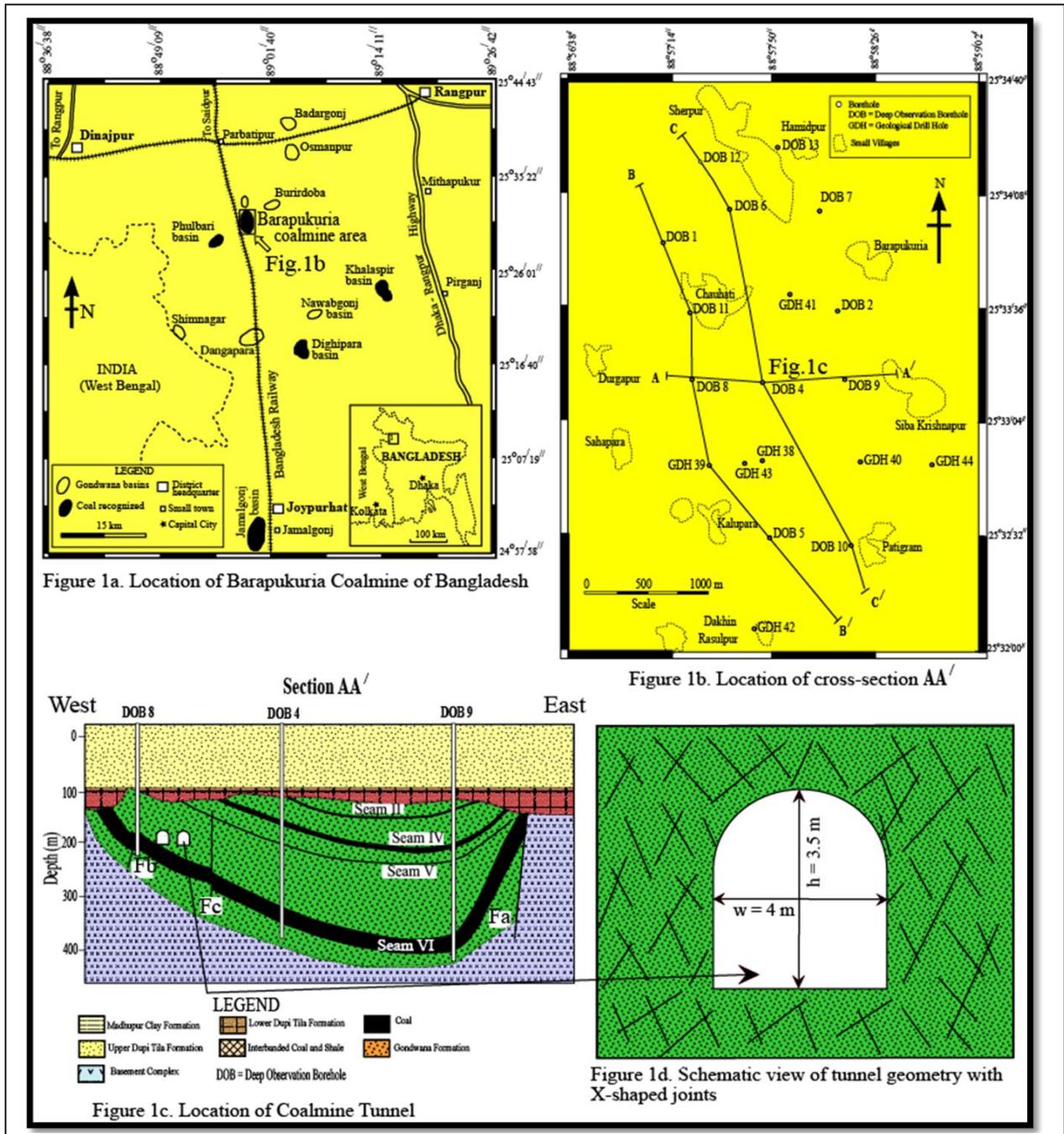


Fig. 1: (A) Location of the Barapukuria Coalmine of NW Bangladesh (after Islam & Hayashi 2008), (B) Location of Cross-Section AA' as Shown in Fig. 1(C), (D) Schematic View of Tunnel Geometry with Geological Discontinuities.

2. Geology and tectonic structures around the tunnel

The surrounding rock masses at the tunnel site consist of fine-grained sandstone, thin layers of siltstone, sandy mudstone with some interlaminated shale. The main rock types along the tunnel alignment include sandstone, siltstone of the Gondwana formation. Joint spacing ranges from 50 to 80 cm and is classified as widely spacing. Highly persistent joints are observed and 1 to 2 mm wide apertures are filled with silty sand and clay (Islam & Islam 2005, Islam & Hayashi 2008, Islam et al. 2009, Islam & Shinjo 2009a, Islam & Shinjo 2009b, Islam et al. 2015).

3. The boundary element method (BEM)

The BEM is a numerical technique for solving initial value problems based on an integral equation formulation (Beskos 1987). The boundary element method has been demonstrated to be a viable alternative to the finite element method due to its features of boundary-only discretization and high accuracy in stress analysis (Mukherjee 1982, Cruse 1988, Banerjee 1994, Cruse 1996). The displacement field is obtained by the integral representation in terms of boundary values and the equation is solved numerically. The method gained its name because of discretising the boundary of the problem into elements (Fig. 2). Boundary values are used to determine displacements and tractions at any interior point of interest (Beskos 1987). This method was applied to various engi-

neering applications such as; foundation engineering, dynamic soil-structure interaction, wave propagation and vibration isolation, and many other purposes. In the BEM, the discretization is only required on the boundary of the problem, which leads to a reduction in the spatial dimension of the problem by one (Manolis & Davies 1993). In other words, the volume integrals are transformed into surface integrals in 3D case and the surface integrals are reduced into line integrals in the 2D case (Hamdan 2013).

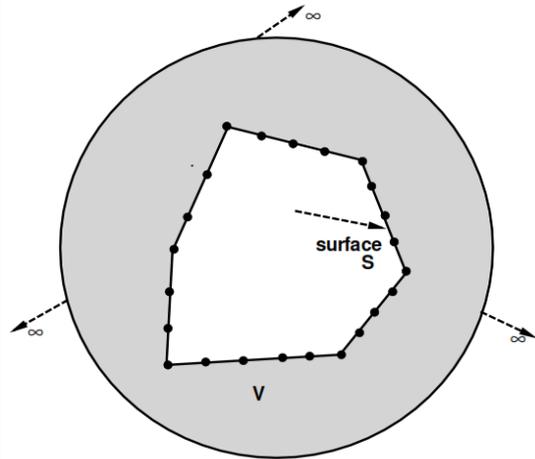


Fig.2: Discretisation in the Boundary Element Method for 3D Unbounded Volume (Hamdan 2013)

4. Model geometry and material properties

The Barapukuria coalmine tunnel cross-section is like a horse-shoe-shaped tunnel, which shows that its final width and height are 4.0 m and 3.5 m, respectively (Fig. 1d). Five rock mechanical parameters for the Gondwana Formation, including unit weight, Poisson's ratio, Young's modulus, cohesion, and angle of internal friction, used in the modeling have been applied from Islam et al. 2015.

5. Governing equations of the present numerical modeling

The modeling theme in the present study is based fundamentally on the governing of (1) of principal stresses (σ_1 and σ_3), mean stress (MPa), differential stress (MPa); (2) Horizontal displacement, vertical displacement, and total displacement around the tunnel; (3) maximum shear strain, Tau XY, strength factor; and (4) Von Mises stress (MPa), and Spalling Criterion. The present BEM modeling is related to plane strain conditions. For the case of plane strain condition, the principal stresses are defined in terms of in plane stress (σ_1 and σ_3) and out of plane stress (σ_2). The σ_1 option will plot contours of the major in-plane principal stress. It is necessary to remind that the in-plane σ_1 is not necessarily the major principal stress in 3D, if the value of σ_2 is greater than σ_1 at a given point, then the in-plane σ_1 will actually be the 3D intermediate principal stress. The σ_3 option will plot contours of the minor in-plane principal stress. It is reasonable to mention that the in-plane σ_3 is not necessarily the minor principal stress in 3D, if the value of σ_2 is less than σ_3 at a given point, then the in-plane σ_3 will actually be the 3D intermediate principal stress. The σ_2 option will plot contours of the out-of-plane principal stress. Remember that σ_2 is not necessarily the intermediate principal stress, depending on the in-plane values of σ_1 and σ_3 at a given point, σ_2 could be the 3D major, intermediate or minor principal stress. In two dimensions there are two principal stresses, which are the major principal stress and the minor principal stress. The major principal stress and the minor principal stresses are expressed by the following equations-

σ_1 = major principal stress

σ_3 = min or principal stress

$$\sigma_1 = \frac{\sigma_y + \sigma_x}{2} + \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2}$$

$$\sigma_3 = \frac{\sigma_y + \sigma_x}{2} - \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2}$$

The mean stress (p) is given by the following equation-

$$p = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

Where the major, intermediate, and minor principal stress correspond to σ_1 , σ_2 and σ_3 .

The differential stress is given by the following equation-

$$q = \sigma_1 - \sigma_3$$

Where σ_1 and σ_3 are the major and minor principal stress.

The Von Mises Stress is given by:

$$\sigma_{VM} = \sqrt{3J}$$

Where J is given by:

$$J = \frac{1}{\sqrt{6}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

The major, intermediate and minor principal stress correspond to σ_1 , σ_2 and σ_3 , but note that σ_2 can be the major, intermediate or minor principal stress, depending on the magnitudes of σ_1 and σ_3 . The Spalling Criterion (Castro et al, 1995, 1997) is given by the following equation-

$$\frac{\sigma_1 - \sigma_3}{UCS}$$

Where UCS indicates unconfined compressive strength. As a general guideline, Spalling Criterion values of 0.4 indicate damage initiation, i.e., beginning of fracturing; values less than 0.4 indicate no potential for burst or failure to develop; and values 0.7 indicate potential for rockburst (in particular strainburst) to occur (<https://www.rocsience.com>).

The strength factor is calculated by dividing the rock strength by the induced stress at every point in the model mesh. All three principal stresses have an influence on the strength factor (σ_1 , σ_3 , and σ_2). In the case of elastic materials, the strength factor can be less than unity, since overstressing is allowed. In the case of plastic materials, the strength factor is always greater than or equal to unity (<https://www.rocsience.com>).

6. Modeling results

BEM numerical modeling results of the study are illustrated in Figs. 3(a-d), 4(a-d), and 5(a-d). The modeling results are presented in the following parameters-

- Distribution contours of major and minor principal stresses (σ_1 and σ_3) (Figs 3ab)
- Distribution contours of mean stress (Fig. 3c)
- Distribution contours of differential stress (Fig. 3d)
- Distribution contours of horizontal displacement (Fig. 4a)
- Distribution contours of vertical displacement (Fig. 4b)
- Distribution contours of total displacement (Fig. 4c)

- Distribution contours of maximum shear strain (Fig. 4d)
- Distribution contours of maximum shear stress τ_{\max} (Fig.5a)
- Distribution contours of strength factor (Fig. 5b)
- Distribution contours of Von Mises Stress (Fig. 5c)

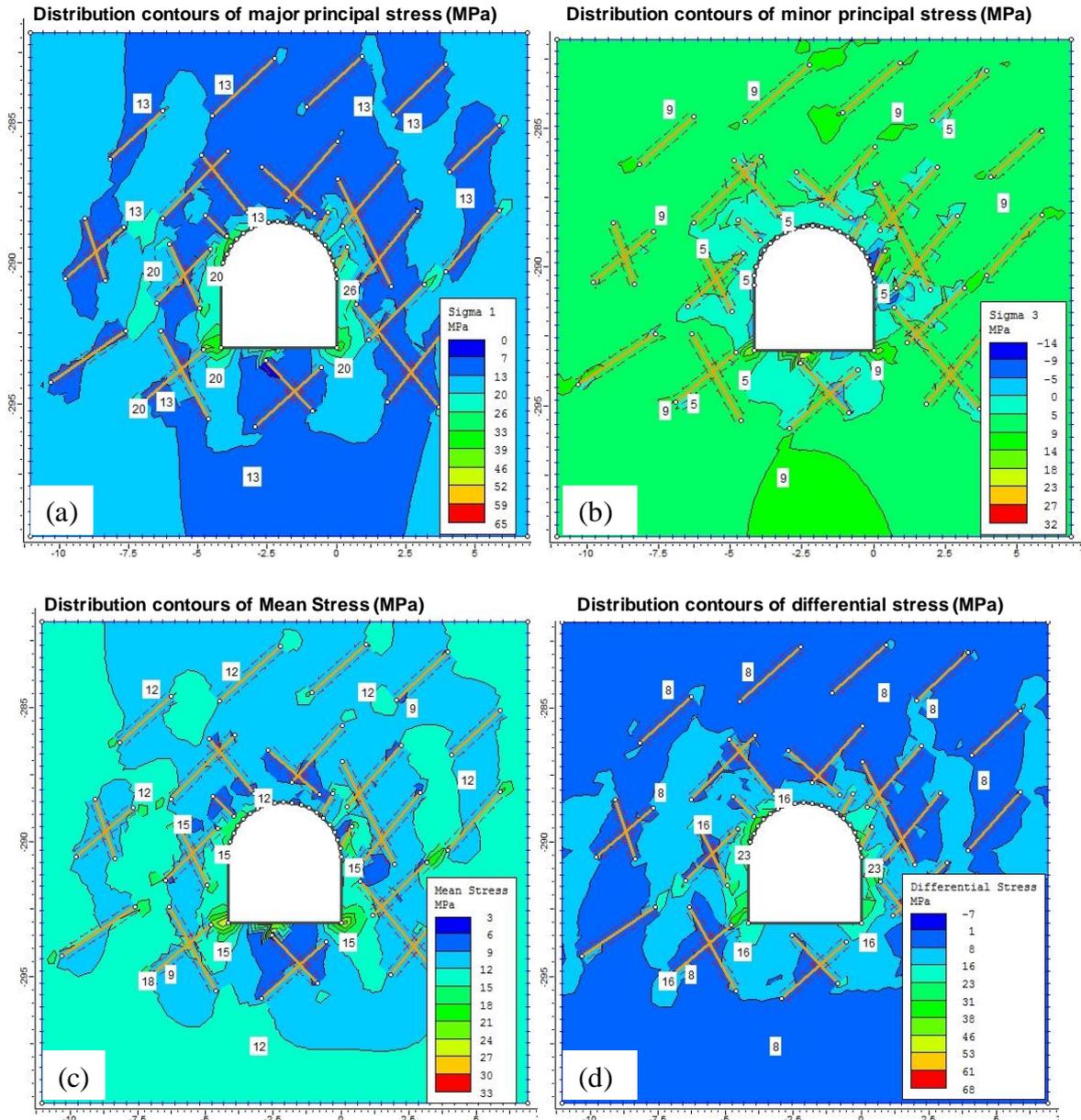


Fig. 3: (A) Distribution Contours of Major Principal Stress (MPa), (B) Minor Principal Stress (MPa), (C) Mean Stress (MPa), and (D) Differential Stress (MPa).

6.1. Distribution contours of σ_1 , σ_3 , mean stress and differential stress values

The contour value of the major principal stress (σ_1) was about 13 MPa at the immediate roof of the model that ultimately increased to 20 MPa toward the left side and 26 MPa toward the right side. The stress value was about 20 MPa at the immediate floor (Fig.3a). For the case of minor principal stress (σ_3), the distribution and magnitudes of contour values were about 5 MPa at the immediate roof, toward the left side and the right side. The stress value was about 9 MPa at the immediate floor (Fig.3b). Mean stress value up to 12 MPa was concentrated in the immediate roof and the value was 15 MPa toward the both rib sides (Fig.3c). The distribution contour value of differential stress at the roof was 16 MPa. In the rib sides, these values were about 23 MPa (Fig. 3d).

6.2. Distribution contours of horizontal, vertical and total displacement values

The absolute horizontal displacement was 0.0011 m at the immediate roof, 0.0027 m toward the rib of left-hand side and 0.0033 m at the right-hand side (Fig. 4a). The absolute vertical displacement at the immediate roof was 0.005 m, 0.0021 m toward the rib of left-hand side and 0.0001 m at the right-hand side (Fig. 4b). Maximum total displacement value of 0.0054 m (Fig. 4c) was simulated at the immediate roof of the tunnel. Displacement value was decreased gradually toward rib sides (Fig. 4c).

6.3. Distribution contours of maximum shear strain and shear stress

In the immediate roof, the maximum shear strain value was 0.0019. In the rib sides, the value was eventually increased to 0.0029. In

the floor, the maximum shear strain value was 0.0019 (Fig. 4d). The maximum shear stress value was 3 MPa at the immediate roof

and left-hand side (Fig.5a), respectively. The shear stress value - 1.0 was concentrated to the right-hand side of the rib side (Fig. 5a).

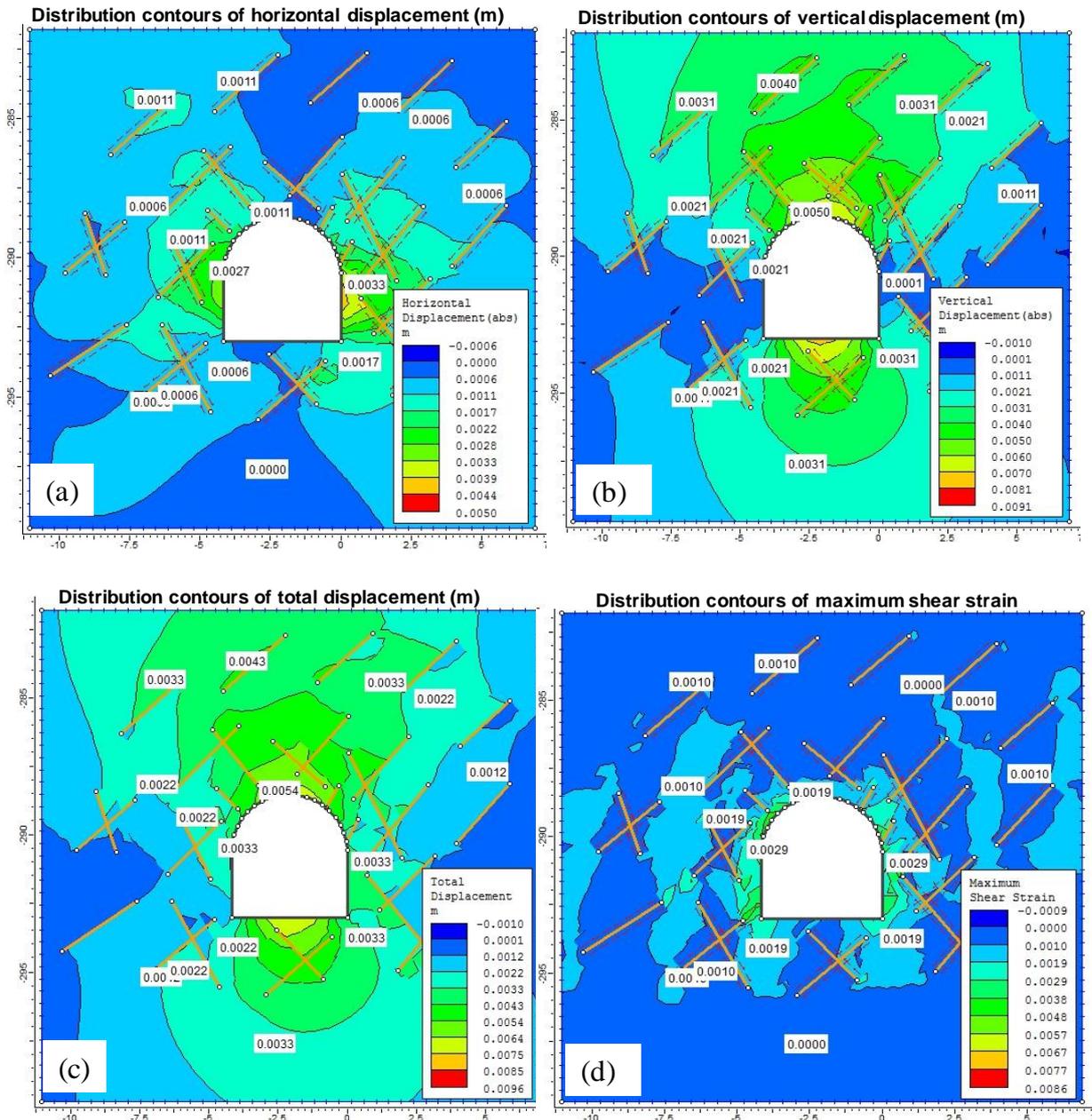


Fig. 4: (A) Distribution Contours of Horizontal Displacement (M), (B) Vertical Displacement (M), (C) Total Displacement (M), and (D) Maximum Shear Strain.

6.4. Distribution contours of strength factor, von mises stress and spalling criterion

As discussed in the previous section 5, all three principal stresses have an influence on the strength factor (σ_1 , σ_3 , and σ_2). For the case of elastic materials, the strength factor can be less than unity, since overstressing is allowed. If the strength factor is greater than 1, this indicates that the material strength is greater than the induced stress. If the strength factor is less than 1, this indicates that the stress in the material exceeds the material strength (i.e. the material would fail) (Islam & Faruque 2012). The magnitudes and distribution contours of the strength factor value around the tunnel with unsupported condition ranged from 0.51 to 1.02. These values specify that the loosening zone would be extended up to 1.53 m towards the roof. The maximum loosening zone at the sidewalls would be extended up to 1.25 m (Fig. 5b). Von Mises stress is considered to be a safe haven for design engineers. Using this

information an engineer can say his design will fail, if the maximum value of Von Mises stress induced in the material is more than strength of the material (<http://www.learnengineering.org/2012/12>). In the immediate roof of the tunnel, the maximum value of Von Mises stress was 7.0 MPa. In the rib sides, the values were 17.5 MPa (Fig. 5c). Spalling Criterion is one of the important parameter to predict rock-burst phenomenon around an underground excavation zone. Spalling Criterion values of 0.4 indicate damage initiation, i.e., beginning of fracturing; values less than 0.4 indicate no potential for rock-burst or failure to develop; and values 0.7 indicate potential for rock-burst (in particular strain-burst) to occur (<https://www.rocscience.com>).

7. Discussions and conclusions

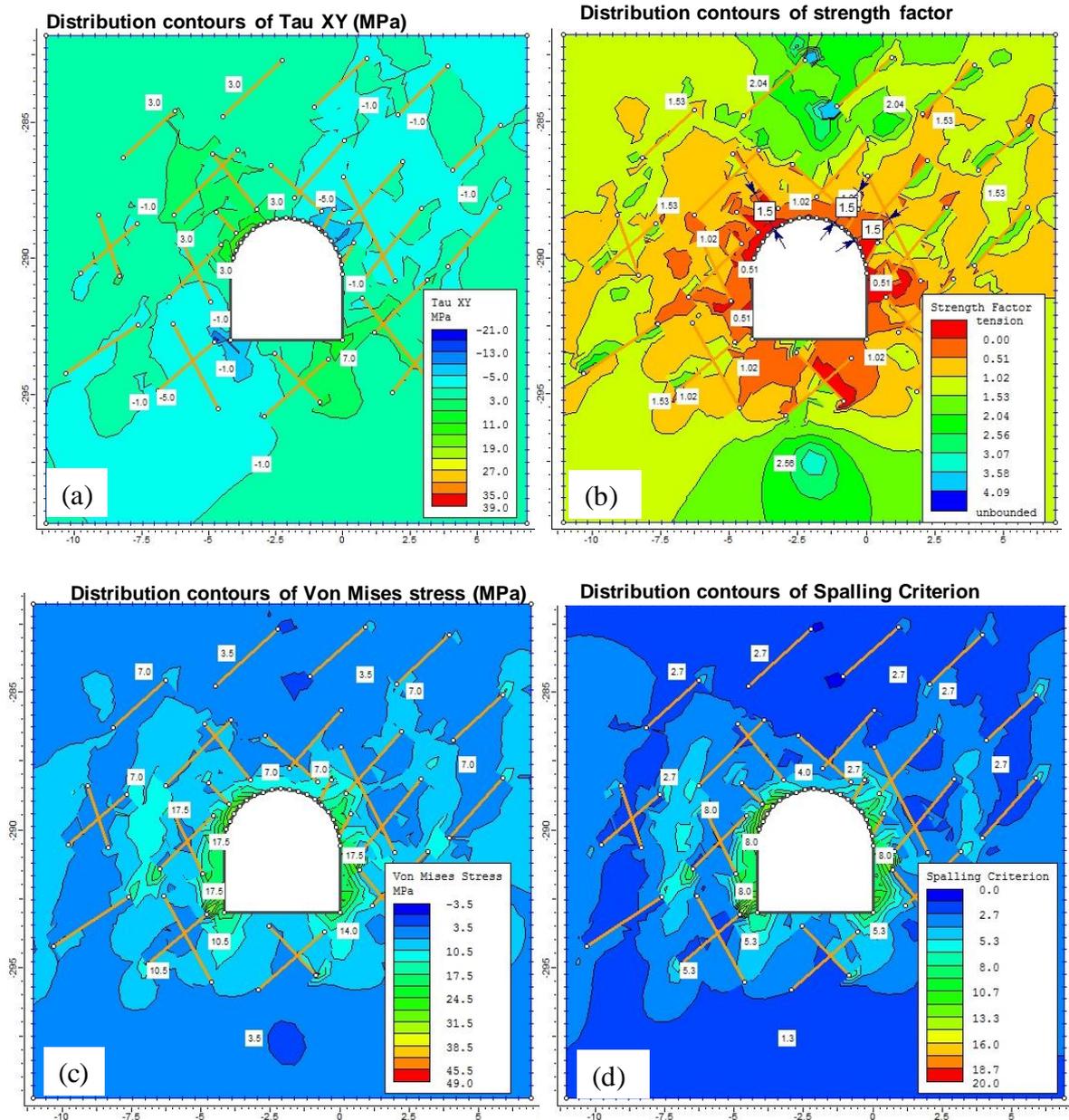


Fig. 5: (A) Distribution Contours of Tau XY (MPa), (B) Strength Factor, (C) Von Mises Stress (MPa), and (D) Spalling Criterion.

The construction of a coalmine tunnel through weak rock masses presents a major challenge to mine geologists and engineers. The variety of geological discontinuities provide significant amount of information regarding to the engineering geological conditions and geotechnical behavior of jointed rock masses. The excavation of an underground tunnel can cause failure of a certain thickness of the rock mass surrounding it, when the rock stress induced by the excavation is beyond the strength of the rock mass (Chen & Zhao 1998, Mikkola & Viitala 2000, Jiang et al. 2000). The rock failure area surrounding the tunnel can be termed the loosening zone, which is measured and defined by the thickness of the zone (Lu & Song 1991, Dong et al. 1994, Muya et al. 2006).

7.1. Support measures evaluation

Literatures review reveal that the loosening zone of the surrounding rock mass is an existing physical state around excavation and corresponds to the post-failure state. When the loosening zone thickness is smaller than 150 cm, normal support measures are sufficient. When the thickness is greater than 150 cm, rigid support measures, such as stone lining and concrete segments, are insufficient to maintain the stability of the tunnel. A surrounding rock mass with a loosening zone thickness of greater than 150 cm

can be classified as soft or poor rock mass, according to the classification system proposed together with the loosening zone theory, or to the rock mass quality classification system (Barton et al. 1974, Barton et al. 1992, Lu & Song 1991).

The loosening zone develops from a small to a large thickness after an opening such as tunnel is excavated. The surrounding rock mass will deform due to rock mass dilatancy within the loosening zone. During this process, the support will be loaded, deformed, and sometimes fails as the loosening zone enlarges. Therefore, the main function of the support is to bear the dilatancy deformations or loads as the loosening zone develops (Muya et al. 2006). Rock support provides resistance against: (1) the self-weight of the loose rock mass within the loosening zone; (2) dilatancy deformation of the broken rock mass in the loosening zone; and (3) elastic and plastic deformations of the rock mass beyond the loosening zone. The second is the major element to be supported (Dong et al. 1996).

For the case of Barapukuria coalmine, the rock masses are usually unconsolidated and moderately hard to soft. In this case, a study result of Song & Lu (2001) would be helpful for necessary support measures of tunnel development in Barapukuria Coalmine. They suggested that supporting the deformed tunnel associated with soft rock can be implemented in two stages. In the first stage, flexible

support is applied to accommodate the dilatancy deformation during development of the loosening zone, immediately after the enlargement of the tunnel cross-section. In the second stage, stiff support is applied to provide a strong supporting reaction and to maintain the long-term stability of the tunnel. Since tunnel enlargement will disturb the rock mass within the existing loosening zone, the tunnel roof may collapse during the enlargement. To avoid this, careful measures should be taken (Song & Lu 2001).

From the literatures review and strength factor value (as shown in Fig. 5) around the coalmine tunnel of the Barapukuria coalmine, it is prominent that the thickness of loosening zone (1.53 m) is greater than 150 cm, which can be classified as soft or poor rock mass. In the immediate roof, floor and the both rib sides of the tunnel, the Spalling Criterion values ranged from 2.7 to 8.0 MPa, which indicate no potential for rock-burst around the tunnel (Fig. 5d).

Although there is no potential for rock-burst around the tunnel, however, flexible support is required to accommodate the dilatancy deformation during development period first and later stiff support would be required to provide a strong supporting reaction and to maintain the long-term stability of the tunnel. During the tunnel development of Barapukuria coalmine, rock bolts, shotcrete and wire mesh were used as the supporting measures. The bolts were applied on the roof and sidewalls to protect roof fall and rib collapse. The bolt spacing was 0.8 m, and the bolt length was 1.8 m. The bolt diameters were approximately 40 mm. The total thickness of shotcrete applied was 100 mm on average. Wire mesh was used together with shotcrete to increase the tensile and bending strengths of the shotcrete.

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