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Research paper

Families of estimators for ratio and product of study characters using mean and proportion of auxiliary character in presence of non-response

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Abstract

In this paper, families of estimators for ratio and product of two population means are suggested using proportion and mean of auxiliary character in presence of non-response. The bias and mean square error (MSE) of the proposed families of estimators are obtained up to the first degree of approximation under two different cases. The specified conditions under which the members of proposed families of estimators attain minimum mean square error have been obtained. Theoretical and empirical comparisons based on real data sets are made to show that the suggested families of estimators are more efficient than the relevant estimators such as usual conventional estimator, (Khare & Sinha 2012 a) estimators and (Sinha 2014) estimators.

Keywords: *Ratio and Product; Auxiliary Variable; Bias; MSE; Efficiency.*

1. Introduction

Estimation of ratio and product of two population means is very prevalent in the field of agriculture, socio-economic, medical, forest surveys etc. and fruitfully utilization of auxiliary information at the estimation stage plays an effective role to increase the efficiency of the estimators. Many authors like (Singh 1965, Shah & Shah 1978, Singh 1982 a, 1982 b, Khare 1987, Srivastativa et al. 1989, Khare 1990, Khare 1991) have considered the problem of estimating ratio and product of two population means using the information of auxiliary character(s) and suggested estimators or classes of estimators.

On the other side, there are many practical situations when auxiliary information is qualitative in nature i.e. auxiliary information is available in the form of attributes and these auxiliary attributes may highly correlated with the study character such as person's height and gender, wool production and breed of sheep, milk production and cow's breed etc. Various authors including (Naik & Gupta 1996, Jhajj et al. 2006, Singh et al. 2008, Shabbir & Gupta 2010, Singh & Solanki 2012, Koyuncu 2012, etc.) have suggested different types of estimators using auxiliary attributes. Further (Khare & Pandey 2000) and (Khare & Sinha 2002, 2004, 2007, 2012 a, 2012 b) have proposed some classes of estimators for the ratio and product of two population means using auxiliary character(s) in the presence of non-response. The aim of this paper is to suggest families of estimators for ratio and product of two population means using proportion and mean of auxiliary character in presence of non-response and study their properties.

2. The proposed estimators

Let $y_i, (i = 1, 2)$ and x_i are the main and auxiliary characters under study having non-negative l^{th} value $(Y_{il}, X_l; l = 1,2...N)$ with the population means $\bar{Y}_i$ $(i = 1,2)$ and $\bar{X}$. Let us consider there is a complete dichotomy on auxiliary character with population proportion $\bar{\emptyset}_N$ in a population with respect to the presence of an attribute $(\emptyset_l$. Let $\emptyset_l$ be the observation of the l^{th} unit of attribute $\emptyset$ which takes two values '1' and '0' if the l^{th} unit possess and does not possesses the attribute respectively. Suppose the whole population is divided into two non overlapping strata of N_1 responding and N_2 non responding units such that $N_1 + N_2 = N$, though they are unknown. Let n be the size of the sample drawn from the population size N by using simple random sampling without replacement (SRSWOR) method and it is has been observed that n_1 units respond and n_2 units do not respond. We have considered here that the responding and non-responding units are same for all the characters and attribute.

Further a subsample of size $r\ (=n_2/k,\ k > 1)$ from n_2 non responding units has been drawn by using SRSWOR method after making extra effort. Using the technique of (Hansen & Hurwitz 1946), the estimators $\bar{y}_i^*(i = 1,2), \bar{x}^*$ and $\bar{\emptyset}^*$ for the population means $\bar{Y}_i$ $(i = 1,2)$, $\bar{X}$ and $\bar{\emptyset}$ on the basis of $(n_1$+r) observations are given by

$$\bar{y}_i^* = \left(\frac{n_1}{n}\right)\bar{y}_{i(1)} + \left(\frac{n_2}{n}\right)\bar{y}'_{i(2)} \tag{1}$$

$$\bar{x}^* = \left(\frac{n_1}{n}\right)\bar{x}_1 + \left(\frac{n_2}{n}\right)\bar{x}'_2 \tag{2}$$

and

$$\bar{\phi}^* = \left(\frac{n_1}{n}\right)\bar{\phi}_1 + \left(\frac{n_2}{n}\right)\bar{\phi}'_2 \tag{3}$$

where $(\bar{y}_{i(1)}, \bar{y}'_{i(2)}),(\bar{x}_1, \bar{x}'_2)$ and $(\bar{\phi}_1, \bar{\phi}'_2)$ are the sample means of the characters $\bar{y}_i$, x and ϕ based on n_1 and r units respectively.

The variances of $\bar{y}_i^*$, $\bar{x}^*$ and $\bar{\phi}^*$ up to the degree of order (n^{-1}) are given by:

$$V(\bar{y}_i^*) = V_{y_i}^* \frac{N-n}{Nn} C_{y_i}^2 + \frac{W_2(k-1)}{n} C_{y_i}'^2, i = 1, 2 \quad (4)$$

$$V(\bar{x}^*) = V_x^* = \frac{N-n}{Nn} C_x^2 + \frac{W_2(k-1)}{n} C_x'^2, i = 1, 2 \quad (5)$$

and

$$V(\bar{\emptyset}^*) = V_{\emptyset}^* = \frac{N-n}{Nn} C_{\emptyset}^2 + \frac{W_2(k-1)}{n} C_{\emptyset}'^2, i = 1, 2 \quad (6)$$

where

$$C_{y_i}^2 = \frac{S_{y_i}^2}{Y_i^2}, C_x^2 = \frac{S_x^2}{X^2}, C_{\emptyset}^2 = \frac{S_{\emptyset}^2}{\bar{\emptyset}^2}, C_{y_i}'^2 = \frac{S_{y_i}'^2}{Y_i^2}, C'^2_x = \frac{S'^2_x}{X^2}, C'^2_{\emptyset} = \frac{S'^2_{\emptyset}}{\bar{\emptyset}^2}$$

Here $(S_{y_i}^2, S_x^2, S_{\emptyset}^2)$ and $(S_{y_i}'^2, S'^2_x, S'^2_{\emptyset})$ are the mean square of $(y_i, x, \emptyset)$ respectively for the entire and non responding group of population.

Let $u_1 (= \bar{y}_1^*/\bar{y}_2^*)$ and $u_2 (= \bar{y}_1^*.\bar{y}_2^*)$ denote the conventional estimators of ratio $\theta_1 (= \bar{Y}_1/\bar{Y}_2)$ and product $\theta_2 (= \bar{Y}_1^*.\bar{Y}_2^*)$ of two population means in the presence of non-response.

Now using the auxiliary character with known population mean, (Khare & Sinha 2012 a) have proposed combined classes of estimators t_{ij} for ratio (θ_1) and product (θ_2) of two population means as follows:

$$t_{ij} = h_{ij}(u_i, z_j), i = 1, 2; j = 1, 2 \quad (7)$$

where $u_1 = (\bar{y}_1^*/\bar{y}_2^*)$, $u_2 = (\bar{y}_1^*.\bar{y}_2^*)$, $z_1 = \bar{x}^*/\bar{X}$ and $z_2 = \bar{x}/\bar{X}$.

Further using the information of auxiliary attribute, (Sinha 2014) suggested classes of estimators for estimating the ratio of the population means (θ_1) and studied their properties. The classes of estimators proposed by (Sinha 2014) are as follows:

$$t_{ij} = h_{ij}(u_1, v_j), i = 3; j = 1, 2 \quad (8)$$

where $v_1 = \bar{\emptyset}_n^*/\bar{\emptyset}_N$ and $v_2 = \bar{\emptyset}_n/\bar{\emptyset}_N$.

Following the lines of (Sinha 2014), we may suggest classes of estimators for estimating the product of the population means (θ_2) as

$$t_{ij} = h_{ij}(u_2, v_j), i = 4; j = 1, 2 \quad (9)$$

and their properties can be studied.

The minimum mean square error of t_{ij} up to the first degree of approximation [i.e. $O(n^{-1})$] under large sample approximation in case of SRSWOR is given by

$$MSE(t_{11})_{min} = (\theta_1)^2 \left[M(u_1) - \frac{[C_{Rx}^*]^2}{V_x^*}\right], \quad (10)$$

$$MSE(t_{12})_{min} = (\theta_1)^2 \left[M(u_1) - \frac{[C_{Rx}]^2}{V_x}\right], \quad (11)$$

$$MSE(t_{21})_{min} = (\theta_2)^2 \left[M(u_2) - \frac{[C_{Px}^*]^2}{V_x^*}\right], \quad (12)$$

$$MSE(t_{22})_{min} = (\theta_2)^2 \left[M(u_2) - \frac{[C_{Px}]^2}{V_x}\right], \quad (13)$$

$$MSE(t_{31})_{min} = (\theta_1)^2 \left[M(u_1) - \frac{[C_{R\emptyset}^*]^2}{V_{\emptyset}^*}\right], \quad (14)$$

$$MSE(t_{32})_{min} = (\theta_1)^2 \left[M(u_1) - \frac{[C_{R\emptyset}]^2}{V_{\emptyset}}\right], \quad (15)$$

$$MSE(t_{41})_{min} = (\theta_2)^2 \left[M(u_2) - \frac{[C_{P\emptyset}^*]^2}{V_{\emptyset}^*}\right] \quad (16)$$

and

$$MSE(t_{42})_{min} = (\theta_2)^2 \left[M(u_2) - \frac{[C_{P\emptyset}]^2}{V_{\emptyset}}\right], \quad (17)$$

where

$$MSE(u_1) = (\theta_1)^2 M(u_1), \quad (18)$$

$$M(u_1) = \left[\frac{N-n}{Nn}\left(C_{y_1}^2 + C_{y_2}^2 - 2\rho C_{y_1} C_{y_2}\right) + \frac{W_2(k-1)}{n}\left(C_{y_1}'^2 + C_{y_2}'^2 - 2\rho_{(2)} C_{y_1}' C_{y_2}'\right)\right],$$

$$MSE(u_2) = (\theta_2)^2 M(u_2) \quad (19)$$

$$M(u_2) = \left[\frac{N-n}{Nn}\left(C_{y_1}^2 + C_{y_2}^2 + 2\rho C_{y_1} C_{y_2}\right) + \frac{W_2(k-1)}{n}\left(C_{y_1}'^2 + C_{y_2}'^2 + 2\rho_{(2)} C_{y_1}' C_{y_2}'\right)\right],$$

$$C_{R\emptyset}^* = \frac{N-n}{Nn}\left(\rho_{10} C_{y_1} - \rho_{20} C_{y_2}\right) C_{\emptyset} + \frac{W_2(k-1)}{n}\left(\rho_{10}' C_{y_1}' - \rho_{20}' C_{y_2}'\right) C_{\emptyset}', \quad (20)$$

$$C_{Rx}^* = \frac{N-n}{Nn}\left(\rho_{13} C_{y_1} - \rho_{23} C_{y_2}\right) C_x + \frac{W_2(k-1)}{n}\left(\rho_{13}' C_{y_1}' - \rho_{23}' C_{y_2}'\right) C_x', \quad (21)$$

$$C_{R\emptyset} = \frac{N-n}{Nn}(\rho_{10} C_{y_1} - \rho_{20} C_{y_2}) C_{\emptyset}, \quad (22)$$

$$C_{Rx} = \frac{N-n}{Nn}(\rho_{10} C_{y_1} - \rho_{20} C_{y_2}) C_{\emptyset}, \quad (23)$$

$$V(\bar{\emptyset}) = V_{\emptyset} = \frac{N-n}{Nn} C_{\emptyset}^2 \quad (24)$$

and

$$V(\bar{x}) = V_x = \frac{N-n}{Nn} C_{\emptyset}^2. \quad (25)$$

Here $(\rho, \rho_{13}, \rho_{23})$ and $(\rho_{(2)}, \rho_{13}', \rho_{23}')$ are the correlation coefficients between $(y_1, y_2), (y_1, x), (y_2, x)$ for the entire and non responding group the population while $(\rho_{10}, \rho_{20}, \rho_{30})$ and $(\rho_{10}', \rho_{20}', \rho_{30}')$ are the biserial correlation coefficients between $(y_1, \emptyset), (y_2, \emptyset), (x, \emptyset)$ for the entire and non responding group of population.

Now following the strategies of (Khare & Sinha 2012 a, Sinha 2014 and Sinha & Kumar 2014), we have suggested two wider families of estimators for estimating the ratio and product of two population means using proportion and mean of auxiliary character for two different cases as follows:

Case I – In this case we assumed that there is incomplete information on study characters y_i(i= 1, 2) as well as auxiliary character x due to non-response, however the population proportion $(\bar{\emptyset}_N)$ and mean $(\bar{X})$ are known in advance. We observe that n_1 units respond and $n_2 (= n - n_1)$ units do not respond for y_i, x and $\emptyset$ in the sample of size n. therefore the proposed families of estimators for estimating ratio and product of two population in presence of non-response is

$$T_{1j} = u_j f_{1j}(v_1, z_1), j = 1, 2 \quad (26)$$

such that

$$f_{1j}(1,1) = 1, \quad (27)$$

where

$u_1 = \bar{y}_1^*/\bar{y}_2^*$, $u_2 = \bar{y}_1^*.\bar{y}_2^*$, $v_1 = \bar{\emptyset}_n^*/\bar{\emptyset}_N$, and $z_1 = \bar{x}^*/\bar{X}$.

Case II – In this case we assumed that there is incomplete information on study characters y_i(i= 1, 2) due to non-response while complete information on auxiliary character x [see (Rao 1986)], however the population proportion ($\bar{\emptyset}_N$) and mean ($\bar{X}$) are known in advance. We scrutinize that $n_1$units respond and $n_2(= n - n_1)$ units do not respond for y_i, x and $\emptyset$ in the sample of size n. therefore the proposed families of estimators for estimating ratio and product of two population means in presence of non-response is

$$T_{2j} = u_j f_{2j}(v_2, z_2),\ j = 1, 2 \tag{28}$$

such that

$$f_{2j}(1,1) = 1, \tag{29}$$

where

$$u_1 = \bar{y}_1^*/\bar{y}_2^*,\ \ u_2 = \bar{y}_1^*.\bar{y}_2^*,\ \ v_2 = \bar{\emptyset}_n/\bar{\emptyset}_N,\ \text{and}\ \ z_2 = \bar{x}/\bar{X}.$$

To study the properties of the proposed families of estimators $T_{1j}$and T_{2j}, we may combine both of them and rewritten as

$$T_{ij} = u_j f_{ij}(v_i, z_i)\ ,\ i = 1, 2; j = 1,2. \tag{30}$$

The function $f_{ij}(v_i, z_i)$is a function of sample values, so for the expansion of $f_{ij}(v_i, z_i)$we assume that it satisfies the following regularity conditions:

i) whatever be the sample chosen, (v_i, z_i) assumes values in a bounded, closed convex subset D of the two dimensional real space containing point (1,1),
ii) the function $f_{ij}(v_i, z_i)$is continuous and bounded in D and
iii) the first and second partial derivative of $f_{ij}(v_i, z_i)$exist and are continuous as well as bounded in D.

3. The bias and mean square error (MSE)

On account of regularity conditions imposed on $f_{ij}(v_i, z_i)$, it may be seen that the bias (B) and the mean square error (MSE) of the estimator T_{ij} will always exist. Expanding the function $f_{ij}(v_i, z_i)$ about the point (1, 1) using Taylor's series up to the second order partial derivatives we have

$$T_{ij} = u_j\Big[f_{ij}(1,1) + (v_i - 1)f_{ij}^{(1)} + (z_i - 1)f_{ij}^{(2)} + \frac{1}{2}\Big\{(v_i - 1)^2 f_{ij}^{(11)}\big(v_i^{\oplus} z_i^{\oplus}\big) + 2(v_i - 1)(z_i - 1)f_{ij}^{(12)}\big(v_i^{\oplus} z_i^{\oplus}\big) + (z_i - 1)^2 f_{ij}^{(22)}\big(v_i^{\oplus} z_i^{\oplus}\big)\Big\}\Big]$$

Using the equations given in equations (27) and (29), we have

$$T_{ij} = u_j\Big[1 + (v_i - 1)f_{ij}^{(1)} + (z_i - 1)f_{ij}^{(2)} + \frac{1}{2}\Big\{(v_i - 1)^2 f_{ij}^{(11)}\big(v_i^{\oplus} z_i^{\oplus}\big) + 2(v_i - 1)(z_i - 1)f_{ij}^{(12)}\big(v_i^{\oplus} z_i^{\oplus}\big) + (z_i - 1)^2 f_{ij}^{(22)}\big(v_i^{\oplus} z_i^{\oplus}\big)\Big\}\Big] \tag{31}$$

where

$$f_{ij}^{(1)} = \left(\frac{\partial f_{ij}}{\partial v_i}\right)_{(1,1)},\ f_{ij}^{(2)} = \left(\frac{\partial f_{ij}}{\partial z_i}\right)_{(1,1)},\ f_{ij}^{(11)} = \frac{\partial^2 f_{ij}}{\partial v_i^2},$$
$$f_{ij}^{(12)} = \frac{\partial^2 f_{ij}}{\partial v_i z_i},\quad f_{ij}^{(22)} = \frac{\partial^2 f_{ij}}{\partial z_i^2},$$

$$v_i^{\oplus} = 1 + \vartheta_1(v_i - 1),\quad z_i^{\oplus} = 1 + \vartheta_2(z_i - 1);$$
$$0 < \vartheta_1, \vartheta_2 < 1\ \forall\, i = 1, 2\,.$$

To obtain the bias and mean square error under large sample approximation up to order (n^{-1}), we assume

$$\epsilon_1 = \frac{(\bar{y}_1^* - \bar{Y}_1)}{\bar{Y}_1},\ \ \epsilon_2 = \frac{(\bar{y}_2^* - \bar{Y}_2)}{\bar{Y}_2},\ \ \epsilon_0 = \frac{(\bar{x}^* - \bar{X})}{\bar{X}},\ \ \epsilon = \frac{(\bar{\emptyset}_n^* - \bar{\emptyset}_N)}{\bar{\emptyset}_N}$$

such that $E(\epsilon_1) = 0, E(\epsilon_2) = 0, E(\epsilon_0) = 0, E(\epsilon) = 0$.

Therefore

$$E(\epsilon_1^2) = \frac{V(\bar{y}_1^*)}{\bar{Y}_1^2},\qquad E(\epsilon_2^2) = \frac{V(\bar{y}_2^*)}{\bar{Y}_2^2},\qquad E(\epsilon_0^2) = \frac{V(\bar{x}^*)}{\bar{X}^2},$$
$$E(\epsilon^2) = \frac{V(\bar{\emptyset}_n^*)}{\bar{\emptyset}_N^2},\qquad E(\epsilon_1\epsilon_2) = \frac{Cov(\bar{y}_1^*, \bar{y}_2^*)}{\bar{Y}_1\bar{Y}_2},$$
$$E(\epsilon_2\epsilon_0) = \frac{Cov(\bar{y}_2^*, \bar{x}^*)}{\bar{Y}_2\bar{X}},\qquad E(\epsilon_1\epsilon_0) = \frac{Cov(\bar{y}_1^*, \bar{x}^*)}{\bar{Y}_1\bar{X}},$$
$$E(\epsilon_2\epsilon) = \frac{Cov(\bar{y}_2^*, \bar{\emptyset}_n^*)}{\bar{Y}_2\bar{\emptyset}_N},\qquad E(\epsilon_1\epsilon) = \frac{Cov(\bar{y}_1^*, \bar{\emptyset}_n^*)}{\bar{Y}_1\bar{\emptyset}_N}$$

and $\quad E(\epsilon_0\epsilon) = \dfrac{Cov(\bar{x}^*, \bar{\emptyset}_n^*)}{\bar{X}\bar{\emptyset}_N}$.

The expressions of the bias (B) and the mean square error (MSE) of T_{ij} up to the first degree of approximation are given by

$$B\,(T_{11}) = B\,(u_1) + \theta_1\Big[C_{Rx}^* f_{11}^{(1)} + C_{R\emptyset}^* f_{11}^{(2)} + \frac{1}{2}\Big(V_x^* f_{11}^{(11)} + 2V_{x\emptyset}^* f_{11}^{(12)} + V_{\emptyset}^* f_{11}^{(22)}\Big)\Big], \tag{32}$$

$$B\,(T_{21}) = B\,(u_1) + R\,\Big[C_{Rx} f_{21}^{(1)} + C_{R\emptyset} f_{21}^{(2)} + \frac{1}{2}\Big(V_x f_{21}^{(11)} + 2V_{x\emptyset} f_{21}^{(12)} + V_{\emptyset} f_{21}^{(22)}\Big)\Big], \tag{33}$$

$$B\,(T_{12}) = B\,(u_2) + P\Big[C_{Px}^* f_{12}^{(1)} + C_{P\emptyset}^* f_{12}^{(2)} + \frac{1}{2}\Big(V_x^* f_{12}^{(11)} + 2V_{x\emptyset}^* f_{12}^{(12)} + V_{\emptyset}^* f_{12}^{(22)}\Big)\Big], \tag{34}$$

$$B\,(T_{22}) = B\,(u_2) + P\Big[C_{Px} f_{22}^{(1)} + C_{P\emptyset} f_{22}^{(2)} + \frac{1}{2}\Big(V_x f_{22}^{(11)} + 2V_{x\emptyset} f_{22}^{(12)} + V_{\emptyset} f_{22}^{(22)}\Big)\Big], \tag{35}$$

$$MSE\,(T_{11}) = (\theta_1)^2\Big[M(u_1) + V_x^*\big\{f_{11}^{(1)}\big\}^2 + V_{\emptyset}^*\big\{f_{11}^{(2)}\big\}^2 + 2C_{x\emptyset}^* f_{11}^{(1)} f_{11}^{(2)} + 2C_{Rx}^* f_{11}^{(1)} + 2C_{R\emptyset}^* f_{11}^{(2)}\Big], \tag{36}$$

$$MSE\,(T_{21}) = (\theta_1)^2\Big[M(u_1) + V_x\big\{f_{21}^{(1)}\big\}^2 + V_{\emptyset}\big\{f_{21}^{(2)}\big\}^2 + 2C_{x\emptyset} f_{21}^{(1)} f_{21}^{(2)} + 2C_{Rx} f_{21}^{(1)} + 2C_{R\emptyset} f_{21}^{(2)}\Big], \tag{37}$$

$$MSE\,(T_{12}) = (\theta_2)^2\Big[M(u_2) + V_x^*\big\{f_{12}^{(1)}\big\}^2 + V_{\emptyset}^*\big\{f_{12}^{(2)}\big\}^2 + 2C_{x\emptyset}^* f_{12}^{(1)} f_{12}^{(2)} + 2C_{Rx}^* f_{12}^{(1)} + 2C_{R\emptyset}^* f_{12}^{(2)}\Big] \tag{38}$$

and

$$MSE\,(T_{22}) = (\theta_2)^2\Big[M(u_2) + V_x\big\{f_{22}^{(1)}\big\}^2 + V_{\emptyset}\big\{f_{22}^{(2)}\big\}^2 + 2C_{x\emptyset} f_{22}^{(1)} f_{22}^{(2)} + 2C_{Rx} f_{22}^{(1)} + 2C_{R\emptyset} f_{22}^{(2)}\Big]. \tag{39}$$

To obtain the minimum mean square error of T_{ij} $(i = 1, 2\ ;\ j = 1, 2)$, we differentiate equation (36) partially with respect to v_1, z_1 and equating them to zero we have

$$\frac{\partial T_{11}}{\partial v_1} = V_x^* f_{11}^{(1)} + C_{x\emptyset}^* f_{11}^{(2)} + C_{Rx}^* = 0 \tag{40}$$

and

$$\frac{\partial T_{11}}{\partial z_1} = V_{\emptyset}^* f_{11}^{(2)} + C_{x\emptyset}^* f_{11}^{(1)} + C_{R\emptyset}^* = 0\,. \tag{41}$$

Now solving the equations (40) and (41), we find

$$f_{11}^{(1)} = \frac{C_{R\emptyset}^{*}C_{x\emptyset}^{*} - V_{\emptyset}^{*}C_{Rx}^{*}}{V_{\emptyset}^{*}V_{x}^{*} - [C_{x\emptyset}^{*}]^{2}} \quad (42)$$

and

$$f_{11}^{(2)} = -\frac{C_{R\emptyset}^{*}}{V_{\emptyset}^{*}} - \frac{C_{x\emptyset}^{*}\left(C_{R\emptyset}^{*}C_{x\emptyset}^{*} - V_{\emptyset}^{*}C_{Rx}^{*}\right)}{V_{\emptyset}^{*}\left(V_{\emptyset}^{*}V_{x}^{*} - [C_{x\emptyset}^{*}]^{2}\right)}. \quad (43)$$

It may be easily verified that $f_{11}^{(1)}$ and $f_{11}^{(2)}$ minimize the value of MSE (T_{11}). So, the mean square error of the proposed families of estimators T_{11} will be minimum if $f_{11}^{(1)}$ and $f_{11}^{(2)}$ are calculated by the equations (42) and (43).
The minimum mean square error of T_{11} using $f_{11}^{(1)}$ and $f_{11}^{(2)}$ respectively from the equations (42) and (43) is as follows

$$\text{MSE }(T_{11})_{\min} = (\theta_1)^2\left[M(u_1) - \frac{[C_{R\emptyset}^{*}]^{2}}{V_{\emptyset}^{*}} - \frac{\left(C_{R\emptyset}^{*}C_{x\emptyset}^{*} - V_{\emptyset}^{*}C_{Rx}^{*}\right)^{2}}{V_{\emptyset}^{*}\left(V_{\emptyset}^{*}V_{x}^{*} - [C_{x\emptyset}^{*}]^{2}\right)}\right] \quad (44)$$

where,

$$C_{x\emptyset}^{*} = \frac{N-n}{Nn}(\rho_{30}C_{x}C_{\emptyset}) + \frac{W_2\,(k-1)}{n}(\rho'_{30}C'_{x}C'_{\emptyset}) \quad (45)$$

and

$$C_{Rx}^{*} = \frac{N-n}{Nn}\left(\rho_{13}C_{y_1} - \rho_{23}C_{y_2}\right)C_x + \frac{W_2\,(k-1)}{n}\left(\rho'_{13}C'_{y_1} - \rho'_{23}C'_{y_2}\right)C'_x. \quad (46)$$

Similarly, the conditions for which the families of estimators T_{21}, T_{12} and T_{22} attain their minimum mean square error can be obtained by differentiating equations (37), (38) and (39) with respect to the corresponding v_i, z_i $(i = 1, 2; j = 1, 2)$ and equating to zero. After solving them we have:

i) Conditions for minimum mean square error of T_{21}

$$f_{21}^{(1)} = \frac{C_{R\emptyset}C_{x\emptyset} - V_{\emptyset}C_{Rx}}{V_{\emptyset}V_{x} - C_{x\emptyset}^{2}} \quad (47)$$

and

$$f_{21}^{(2)} = -\frac{C_{R\emptyset}}{V_{\emptyset}} - \frac{C_{x\emptyset}\left(C_{R\emptyset}C_{x\emptyset} - V_{\emptyset}C_{Rx}\right)}{V_{\emptyset}\left(V_{\emptyset}V_{x} - C_{x\emptyset}^{2}\right)}. \quad (48)$$

ii) Conditions for minimum mean square error of T_{12}

$$f_{12}^{(1)} = \frac{C_{P\emptyset}^{*}C_{x\emptyset}^{*} - V_{\emptyset}^{*}C_{Px}^{*}}{V_{\emptyset}^{*}V_{x}^{*} - [C_{x\emptyset}^{*}]^{2}} \quad (49)$$

and

$$f_{12}^{(2)} = -\frac{C_{P\emptyset}^{*}}{V_{\emptyset}^{*}} - \frac{C_{x\emptyset}^{*}\left(C_{P\emptyset}^{*}C_{x\emptyset}^{*} - V_{\emptyset}^{*}C_{Px}^{*}\right)}{V_{\emptyset}^{*}\left(V_{\emptyset}^{*}V_{x}^{*} - [C_{x\emptyset}^{*}]^{2}\right)}. \quad (50)$$

iii) Conditions for minimum mean square error of T_{22}

$$f_{22}^{(1)} = \frac{C_{P\emptyset}C_{x\emptyset} - V_{\emptyset}C_{Px}}{V_{\emptyset}V_{x} - C_{x\emptyset}^{2}} \quad (51)$$

and

$$f_{22}^{(2)} = -\frac{C_{P\emptyset}}{V_{\emptyset}} - \frac{C_{x\emptyset}\left(C_{P\emptyset}C_{x\emptyset} - V_{\emptyset}C_{Px}\right)}{V_{\emptyset}\left(V_{\emptyset}V_{x} - C_{x\emptyset}^{2}\right)}. \quad (52)$$

Now putting the value of $f_{21}^{(1)}$ and $f_{21}^{(2)}$ in equation (37), $f_{12}^{(1)}$ and $f_{12}^{(2)}$ in equation (38), $f_{21}^{(1)}$ and $f_{21}^{(2)}$ in equation (39), the minimum mean square errors of T_{21}, T_{12} and T_{22} are as follows

$$\text{MSE }(T_{21})_{\min} = (\theta_1)^2\left[M(u_1) - \frac{[C_{R\emptyset}]^{2}}{V_{\emptyset}} - \frac{\left(C_{R\emptyset}C_{x\emptyset} - V_{\emptyset}C_{Rx}\right)^{2}}{V_{\emptyset}\left(V_{\emptyset}V_{x} - C_{x\emptyset}^{2}\right)}\right], \quad (53)$$

$$\text{MSE }(T_{12})_{\min} = (\theta_2)^2\left[M(u_2) - \frac{[C_{P\emptyset}^{*}]^{2}}{V_{\emptyset}^{*}} - \frac{\left(C_{P\emptyset}^{*}C_{x\emptyset}^{*} - V_{\emptyset}^{*}C_{Px}^{*}\right)^{2}}{V_{\emptyset}^{*}\left(V_{\emptyset}^{*}V_{x}^{*} - [C_{x\emptyset}^{*}]^{2}\right)}\right] \quad (54)$$

and

$$\text{MSE }(T_{22})_{\min} = (\theta_2)^2\left[M(u_2) - \frac{[C_{P\emptyset}]^{2}}{V_{\emptyset}} - \frac{\left(C_{P\emptyset}C_{x\emptyset} - V_{\emptyset}C_{Px}\right)^{2}}{V_{\emptyset}\left(V_{\emptyset}V_{x} - C_{x\emptyset}^{2}\right)}\right]. \quad (55)$$

The proposed families of estimators are the function of sample values and many types of functions may be possible under the discussed condition. So some members of these families of estimators T_{ij} may be considered as

$$T_{ij}(ij) = u_j {v_i}^{a_{ij}} {z_i}^{b_{ij}}, \ (i = 1, 2; j = 1, 2) \quad (56)$$

$$T_{ij}(ij) = u_j\left[\omega_1 {v_i}^{a_{ij}} + \omega_2 {z_i}^{b_{ij}}\right]; \quad \omega_1 + \omega_2 = 1, \quad (i = 1, 2; j = 1, 2) \quad (57)$$

$$T_{ij}(ij) = u_j[\omega_1(2 - v_i) + \omega_2(2 - z_i)]; \quad \omega_1 + \omega_2 = 1, \quad (i = 1, 2; j = 1, 2) \quad (58)$$

Sometimes the values of parameters in the optimum values of the constants are not known then one may estimate them on the basis of the sample values or may use past data. (Reddy 1978) has shown that such values are not only stable overtime and region but also don't affect the mean square error of the estimators up to the terms of order n^{-1} (Srivastava & Jhaji 1983).

4. Comparisons of efficiency of proposed class of estimators with relevant estimators

When we compare the efficiency of the proposed estimators $T_{ij} = u_j f_{ij}(v_i, z_i)$, $i = 1, 2; j = 1,2$, with relevant estimators, we observe

i) From equation (10) and (18)

$$\text{MSE}(u_1) - \text{MSE }(t_{11})_{\min} = (\theta_1)^2\frac{[C_{Rx}^{*}]^{2}}{V_{x}^{*}} > 0$$

ii) From equation (11) and (18)

$$\text{MSE}(u_1) - \text{MSE }(t_{12})_{\min} = (\theta_1)^2\frac{[C_{Rx}]^{2}}{V_{x}} > 0$$

iii) From equation (12) and (19)

$$\text{MSE}(u_2) - \text{MSE }(t_{21})_{\min} = (\theta_2)^2\frac{[C_{Px}^{*}]^{2}}{V_{x}^{*}} > 0$$

iv) From equation (13) and (19)

$$\text{MSE}(u_2) - \text{MSE }(t_{22})_{\min} = (\theta_2)^2\frac{[C_{Px}]^{2}}{V_{x}} > 0$$

v) From equation (14) and (18)

$$\text{MSE}(u_1) - \text{MSE }(t_{31})_{\min} = (\theta_1)^2\frac{[C_{R\emptyset}^{*}]^{2}}{V_{\emptyset}^{*}} > 0$$

vi) From equation (15) and (18)

$$\text{MSE}(u_1) - \text{MSE }(t_{32})_{\min} = (\theta_1)^2\frac{[C_{R\emptyset}]^{2}}{V_{\emptyset}} > 0$$

vii) From equation (16) and (19)

$$\mathrm{MSE}(\mathrm{u}_2) - \mathrm{MSE}\ (\mathrm{t}_{41})_{\min} = (\theta_2)^2 \frac{[\mathrm{C}^*_{\mathrm{P}\emptyset}]^2}{\mathrm{V}^*_{\emptyset}} > 0$$

viii) From equation (17) and (19)

$$\mathrm{MSE}(\mathrm{u}_2) - \mathrm{MSE}\ (\mathrm{t}_{42})_{\min} = (\theta_2)^2 \frac{[\mathrm{C}_{\mathrm{P}\emptyset}]^2}{\mathrm{V}_{\emptyset}} > 0$$

ix) From equation (18) and (44)

$$\mathrm{MSE}(\mathrm{u}_1) - \mathrm{MSE}(\mathrm{T}_{11})_{\min} = (\theta_1)^2 \left[\frac{[\mathrm{C}^*_{\mathrm{R}\emptyset}]^2}{\mathrm{V}^*_{\emptyset}} + \frac{\left(\mathrm{C}^*_{\mathrm{R}\emptyset}\mathrm{C}^*_{\mathrm{x}\emptyset} - \mathrm{V}^*_{\emptyset}\mathrm{C}^*_{\mathrm{Rx}}\right)^2}{\mathrm{V}^*_{\emptyset}\left(\mathrm{V}^*_{\emptyset}\mathrm{V}^*_{\mathrm{x}} - \left[\mathrm{C}^*_{\mathrm{x}\emptyset}\right]^2\right)}\right] > 0$$

x) From equation (18) and (53)

$$\mathrm{MSE}(\mathrm{u}_1) - \mathrm{MSE}(\mathrm{T}_{21})_{\min} = (\theta_1)^2 \left[\frac{[\mathrm{C}_{\mathrm{R}\emptyset}]^2}{\mathrm{V}_{\emptyset}} + \frac{(\mathrm{C}_{\mathrm{R}\emptyset}\mathrm{C}_{\mathrm{x}\emptyset} - \mathrm{V}_{\emptyset}\mathrm{C}_{\mathrm{Rx}})^2}{\mathrm{V}_{\emptyset}(\mathrm{V}_{\emptyset}\mathrm{V}_{\mathrm{x}} - \mathrm{C}^2_{\mathrm{x}\emptyset})}\right] > 0$$

xi) From equation (19) and (54)

$$\mathrm{MSE}(\mathrm{u}_2) - \mathrm{MSE}\ (\mathrm{T}_{12})_{\min} = (\theta_2)^2 \left[\frac{[\mathrm{C}^*_{\mathrm{P}\emptyset}]^2}{\mathrm{V}^*_{\emptyset}} + \frac{\left(\mathrm{C}^*_{\mathrm{P}\emptyset}\mathrm{C}^*_{\mathrm{x}\emptyset} - \mathrm{V}^*_{\emptyset}\mathrm{C}^*_{\mathrm{Px}}\right)^2}{\mathrm{V}^*_{\emptyset}\left(\mathrm{V}^*_{\emptyset}\mathrm{V}^*_{\mathrm{x}} - \left[\mathrm{C}^*_{\mathrm{x}\emptyset}\right]^2\right)}\right] > 0$$

xii) From equation (19) and (55)

$$\mathrm{MSE}(\mathrm{u}_2) - \mathrm{MSE}\ (\mathrm{T}_{22})_{\min} = (\theta_2)^2 \left[\frac{[\mathrm{C}_{\mathrm{P}\emptyset}]^2}{\mathrm{V}_{\emptyset}} + \frac{(\mathrm{C}_{\mathrm{P}\emptyset}\mathrm{C}_{\mathrm{x}\emptyset} - \mathrm{V}_{\emptyset}\mathrm{C}_{\mathrm{Px}})^2}{\mathrm{V}_{\emptyset}(\mathrm{V}_{\emptyset}\mathrm{V}_{\mathrm{x}} - \mathrm{C}^2_{\mathrm{x}\emptyset})}\right] > 0$$

xiii) From equation (14), (18) and (44)

$$\mathrm{MSE}(\mathrm{T}_{11})_{\min} < MSE(\mathrm{t}_{31})_{\min} < MSE(\mathrm{u}_1)$$

xiv) From equation (15), (18) and (53)

$$\mathrm{MSE}(\mathrm{T}_{21})_{\min} < MSE(\mathrm{t}_{32})_{\min} < MSE(\mathrm{u}_1)$$

xv) From equation (16), (19) and (54)

$$\mathrm{MSE}(\mathrm{T}_{12})_{\min} < MSE(\mathrm{t}_{41})_{\min} < MSE(\mathrm{u}_2)$$

xvi) From equation (17), (19) and (55)

$$\mathrm{MSE}(\mathrm{T}_{22})_{\min} < MSE(\mathrm{t}_{42})_{\min} < MSE(\mathrm{u}_2)$$

5. An empirical study

Data Set-I

96 village wise population of rural area under Police-station - Singur, District – Hooghly, West Bengal has been taken under the study from District Census Handbook 1981 published by Govt. of India. The last 25% villages (i.e. 24 villages) have been considered as non-responding group of the population. Here we have taken the study characters and auxiliary characters as follows:

y_1 : Number of agricultural laborers in the village.
y_2 : Total area of the village.
x : Number of occupied houses in the village.
$\emptyset$: Number of occupied houses less than or equal to 250.

The parameters of this data set are as follows:

N = 96	$\bar{Y}_1 = 162.67$	$\bar{Y}_2 = 164.01$	$\bar{X} = 303.10$	$\bar{\phi} = 0.6146$
$C_{y_1} = 1.431$	$C_{y_2} = 0.800$	$C_x = 1.163$	$C_\phi = 0.796$	$\rho = 0.779$
$\rho_{13} = 0.915$	$\rho_{10} = 0.502$	$\rho_{23} = 0.886$	$\rho_{20} = 0.643$	$\rho_{30} = 0.629$
$N_2 = 24$	$\bar{Y}_{1(2)} = 216.62$	$\bar{Y}_{2(2)} = 183.32$	$\bar{X}_{(2)} = 361.96$	$\bar{\phi}_{(2)} = 0.5833$
$C'_{y_1} = 1.859$	$C'_{y_2} = 0.857$	$C'_x = 1.402$	$C'_\phi = 0.982025$	$\rho_{(2)} = 0.791$
$\rho'_{13} = 0.926$	$\rho'_{10} = 0.489$	$\rho'_{23} = 0.883$	$\rho'_{20} = 0.660$	$\rho'_{30} = 0.608$

We have considered Data set I to estimate θ_1 and the tors $T_{11}(11) = u_1 v_1^{a_{11}} z_1^{b_{11}}$ and $T_{21}(21) = u_1 v_2^{a_{21}} z_2^{b_{21}}$ have been considered for comparing the efficiency of the proposed families of estimators T_{11} and T_{21} with other relevant estimators which is shown in Table 1.

Data Set II

109 Village/Town/ward wise population of urban area under Police Station –Baria, Tahasil-Champua, Orissa, India has been taken under the study from District Census Handbook, 1981, Orissa, published by Govt. of India. The first 25% villages (i.e.27 villages) have been considered as non-response group of the population. Here we have taken the study characters and auxiliary characters as follows:

y_1 : Number of occupied houses in the village.
y_2 : Average number of persons in the village.
x : Total area of the village.
$\emptyset$: Area greater than equal to 150.

The parameters of the population are as follows

N = 109	$\bar{Y}_1 = 88.8624$	$\bar{Y}_2 = 5.5547$	$\bar{X} = 255.97$	$\bar{\phi} = 0.7248$
$C_{y_1} = 0.6639$	$C_{y_2} = 0.1263$	$C_x = 0.6065$	$C_\phi = 0.6191$	$C'_{y_1} = 0.7660$
$C'_{y_2} = 0.1052$	$C'_x = 0.7029$	$C'_\phi = 0.4994$	$\rho = -0.188$	$\rho_{13} = 0.851$
$\rho_{10} = 0.492$	$\rho_{23} = -0.082$	$\rho_{20} = -0.068$	$\rho_{30} = 0.592$	$\rho_{(2)} = -0.074$
$\rho'_{13} = 0.806$	$\rho'_{10} = 0.316$	$\rho'_{23} = 0.078$	$\rho'_{20} = 0.028$	$\rho'_{30} = 0.028$

Data Set II is considered to estimate θ_2 and to show the efficiency of the proposed families of estimators over the relevant estimators, $T_{12}(12) = u_2 v_1^{a_{12}} z_1^{b_{12}}$ and $T_{22}(22) = u_2 v_2^{a_{22}} z_2^{b_{22}}$ have been considered to calculate the relative efficiency of the proposed families of estimators T_{12} and T_{22} with respect to other relevant estimators, which is shown in Table 2.
The relative efficiency (R.E.) of the estimators in percentage is calculated by the formula

$$\mathrm{R.E.} = \frac{\mathrm{MSE}(\hat{\mathrm{R}})}{\mathrm{MSE}(.)} \times 100.$$

Table 1: R. E. (·) (in %) of estimator with respect to u_1 for different values of k

Estimators	(N = 96, n = 30) 1/k		
	1/4	1/3	1/2
u_1	100.00 (61394.20)*	100.00 (46618.38)	100.00 (33485.89)
t_{31}	106.28 (57762.15)	106.08 (44258.22)	105.52 (31724.33)
T_{11}	172.00 (35687.18)	158.00 (29471.33)	141.60 (23636.92)
t_{32}	101.54 (60457.75)	102.04 (45681.93)	102.87 (32549.44)
T_{21}	105.80 (58024.20)	107.79 (43248.40)	111.19 (30115.90)

* Figures in parenthesis give MSE(·) in 10^{-6}.

Table 2: R. E. (·) (in %) of estimators with respect to u_2 for different values of k

Estimators	(N = 109, n = 30) 1/k		
	1/4	1/3	1/2
u_2	100.00 (6091.12)*	100.00 (4872.90)	100.00 (3703.40)
t_{41}	118.62 (5134.78)	120.85 (4032.14)	124.36 (2978.07)
T_{12}	324.12 (1871.87)	327.17 (1489.65)	339.35 (1091.32)
t_{42}	110.82 (5496.25)	113.91 (4278.02)	119.14 (3108.53)
T_{22}	134.89 (4515.79)	147.77 (3297.56)	174.03 (2128.07)

* Figures in parenthesis give MSE(·)

6. Conclusion

From Table 1, it has been observed that the estimators T_{11} andT_{21} are more efficient than the corresponding estimators t_{31}, t_{32} and conventional estimator u_1 at different level of sub sampling fraction $(1/k)$. The mean square error of T_{11} and T_{21} decreases as the sub-sampling fraction increases but on comparing the relative efficiency $(R.E.)$ of T_{11} and T_{21} with respect to u_1, we observe that the $R.E.$ of (T_{11}) with respect to u_1 decreases but the $R.E.$ of (T_{21}) with respect to u_1 increases as sub-sampling fraction increases. This is due to the fact that $MSE(u_1)$ decreases at a faster rate than $MSE(T_{11})$ as sub-sampling fraction increases.
Similarly, Table 2 shows that the estimators T_{12} and T_{22} are more efficient than t_{41}, t_{42} and conventional estimator u_2 at different level of sub sampling fraction $(1/k)$. On comparing the relative efficiency $(R.E.)$, we observe that $R.E.$ of all the estimators t_{41}, t_{42}, T_{12} and T_{22} with respect to u_2 increases while their mean square errors decrease as sub-sampling fraction increases.

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