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Research paper

Existence and stability of collinear points in elliptic restricted three body problem with radiating and oblate primaries

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Abstract

The location of the collinear points in elliptical restricted three body problem, taking into account the effect of oblateness and radiation pressure of both primaries, has been obtained in this paper. Vinti's method has been exploited and the x-coordinates are obtained in the form of series solution. The linear stability has been investigated and it is found that the points are unstable in the Lyapunov's sense. The problem is also numerically explored taking into account two binary systems: Luyten-726 and Kruger-60.

Keywords:*ERTBP; Collinear Points; Generalized Photogravitational System;Oblateness; Linear Stability.*

1. Introduction

Elliptic Restricted Three Body problem (ERTBP) is being vastly used as a model for the problem of finding the equilibrium points, since this model is found to be better equipped than Classical Restricted three body problem (CRTBP) in studying the long-time behavior of important dynamical systems. Though the position of the primaries are assumed to be fixed in the ERTBP, the Hamiltonian of the system depends explicitly on time; for which a pulsating coordinate system has been introduced by using the variable distance between primaries as a unit of length ([1], [28], [29] etc). Poynting [21] explained that small meteors or cosmic dust are affected by not only gravitational force but also the radiation force as they come near a luminous body, however the CRTBP neglects the effect of radiation force acting on the infinitesimal mass, if one or both primaries are intense emitter of radiation. The relativistic form of this problem considering the radiation force was given by Robertson [22]. Several studies ([2], [3], [5], [11], [12], [25]) of the restricted problem have since been analyzing the effect of radiation pressure.

The model of three body problem in its classical form was based on the assumption that the two primaries and infinitesimal mass are formed of homogeneous layers and is spherically symmetrical. It is however; found that although meteors and meteoroids have regular shapes [20], there are massive celestial bodies such as Saturn and Jupiter which are sufficiently oblate. The oblateness of a body can produce perturbation deviation from two-body motion. Various authors have studied the effect of this perturbation in ERTBP by taking one or both primaries as a source of radiation or oblate spheroid or both ([9], [26], [18] and [19]). A number of communications ([16], [14], [7], [8], [13], [23], [24], [4] etc) had taken into account the drag forces in CRTBP.

Table 1: Data Related To The Binary Systems.

Binary systems	Mass ratio	Radiation factor for first primary q_1	Radiation factor for second primary q_2
Luyten-726	0.499	0.999998	0.999999
Kruger-60	0:394	0.99992	0.999996

We have studied the position and stability of collinear points in ERTBP, when both primaries are oblate spheroid and are source of radiation. We explored the possibilities of existence and stability of the collinear points around the two binary systems Luyten-726 and Kruger-60. The binary system Luyten-726-8(AB) is the sixth closest system from earth discovered by Luyten [15]. Both the stars of this system are red dwarfs that are moving in highly eccentric orbits. The binary system Kruger-60 was first observed by Henry et.al. [6], this binary system also consists of red dwarfs. The relevant data of the two binary systems is given in Table 1.

2. Equation of motion

Consider two radiating oblate spheroid S_1 and S_2 with masses m_1 and m_2 respectively moving in a plane about their common center of mass O in Keplerian elliptical orbit having eccentricity e. The infinitesimal mass is moving in the plane of motion of S_1 and S_2. Assume that A_1 and A_2 denote the oblateness coefficients of the bigger and smaller primaries respectively such that $0 < A_i << 1$; $(i = 1, 2)$. Furthermore, the radiation factor of the primaries is denoted by $q_i = 1 - \beta_i = 1 - \dfrac{F_{pi}}{F_{gi}}, i = 1, 2$, where, F_{pi} and F_{gi} are the radiation pressure forces and gravitational forces respectively. Then the equations of motion of a particle of infinitesimal mass in a non-dimensional rotating-pulsating barycentric coordinate system (x; y; z) is given as [17]:

$$x'' - 2y' = \frac{1}{\left(1 + e\cos\nu\right)}\frac{\partial\Omega}{\partial x};$$

$$y'' + 2x' = \frac{1}{\left(1 + e\cos\nu\right)}\frac{\partial\Omega}{\partial y}. \qquad (1)$$

where,

$$\Omega = \left[\frac{x^2+y^2}{2} + \frac{1}{n^2}\left(\frac{(1-\mu)q_1}{r_1} + \frac{\mu q_2}{r_2} + \frac{(1-\mu)q_1A_1}{2r_1^3} + \frac{\mu A_2 q_2}{2r_2^3}\right)\right] \qquad (2)$$

$$r_1^2 = (x+\mu)^2 + y^2 + z^2 \;;$$

$$r_2^2 = (x-1+\mu)^2 + y^2 + z^2; \qquad (3)$$

and

$$n^2 = 1 + \frac{3A_1}{2} + \frac{3A_2}{2}. \qquad (4)$$

In the differential equations dash denotes the differentiation with respect to true anomaly υ. Also e and n are the eccentricity and mean motion respectively.

3. Location of the Lagrangian equilibrium points L_1, L_2 and L_3

The position of the collinear equilibrium points are obtained by solving the equation $\frac{\partial\Omega}{\partial x} = 0$, taking $x' = y' = y = 0$. Consequently, we get the equation:

$$n^2 x - \frac{q_1(1-\mu)(x+\mu)}{\left|x+\mu\right|^3}\left(1 + \frac{3A_1}{2\left|x+\mu\right|^2}\right) - \frac{\mu q_2(x+\mu-1)}{\left|x+\mu-1\right|^3}\left(1 + \frac{3A_2}{2\left|x+\mu-1\right|^2}\right) = 0. \qquad (5)$$

The collinear equilibrium points L_1, L_2 and L_3 are defined as follows:

1) L_1 lies between the bigger and smaller primary: $-\mu < x < 1-\mu$,
2) L_2 is to the right of the smaller primary: $x > 1-\mu$,
3) L_3 lies to the left of the bigger primary: $x < -\mu$.

3.1. Location of collinear point L_1

The position of the collinear point L_1 is given by $-\mu < x < 1-\mu$; then $|\mu + x| = \mu + x$ and $|\mu + x - 1| = -(\mu + x - 1)$. Assuming $x + \mu - 1 = -\rho$, and substituting this relation in equation (5) and re-arranging the terms of the equation, we obtain:

$$\frac{\mu}{3(1-\mu)} = \frac{\rho^3 N_1}{(1-\rho)^2 D_1}; \qquad (6)$$

where,

$$N_1 = 1 - 2A_2 - \left(\frac{4}{3} + 2A_1\right)\beta_1 + \left(-1 + 2A_1 + 9A_2 + (2+7A_1)\beta_1\right)\rho$$
$$+\left(\frac{1}{3} - 3A_1 - 17A_2 - (\frac{4}{3} + 14A_1)\beta_1\right)\rho^2$$
$$+\left(-4A_1 + \frac{35A_2}{2} + (\frac{1}{3} + \frac{43A_1}{2})\beta_1\right)\rho^3;$$

$$D_1 = 1 + \frac{45A_2}{2}(1-\beta_2) - 60A_2(1-\beta_2)\rho + \frac{135A_2}{2}(1-\beta_2)\rho^2$$
$$-\left(1 + \frac{3A_1}{2} + \frac{75A_2}{2} - 36A_2\beta_2\right)\rho^3 + \frac{15A_2}{2}(1-\beta_2)\rho^4.$$

Assuming $\lambda = \left(\dfrac{\mu(1 + \frac{45A_2}{2}(1-\beta_2))}{3(1-\mu)(1 - 2A_2 - (\frac{4}{3} + 2A_1)\beta_1}\right)^{\frac{1}{3}}$.

We have $\rho \approx \lambda$, for small ρ, hence a series expansion of ρ can be represented as follows:

$$\rho = \lambda\left(1 + c_1\lambda + c_2\lambda^2 + ...\right) \qquad (7)$$

The value of ρ in series form is substituted from equation (7) into equation (6) and comparing the coefficients, we obtained the value of the coefficients as follows:

$$c_1 = -\frac{1}{3}.\frac{1 + 2A_1 + \frac{175A_2}{2} - \left(\frac{2}{3} - 3A_1 + 95A_2\right)\beta_1 - \frac{165}{2}A_2\beta_2}{\left(1 - 2A_2 - \left(\frac{4}{3} + 2A_1\right)\beta_1\right)\left(1 + \frac{45A_2}{2}(1-\beta_2)\right)}$$

$$c_2 = -\frac{1}{9}.\frac{1 - 9A_1 - \frac{781A_2}{2} + \left(-\frac{16}{3} + 708A_2 - 40A_1\right)\beta_1 + \frac{675}{2}A_2\beta_2}{\left(1 - 2A_2 - \left(\frac{4}{3} + 2A_1\right)\beta_1\right)^2\left(1 + \frac{45A_2}{2}(1-\beta_2)\right)^2} \qquad (8)$$

Thus, we obtain the coordinate for the collinear point L_1. The graphical behavior of the x-coordinate of the collinear point L_1 as a function of the radiation factors β_1 and β_2, taking A_1 = 0.001, A_2 = 0.001, around the binary system Luyten 726 and Kruger 60 are shown in Figure 1.

Also, the shift in the x-coordinate of the point as a function of the oblateness factor is shown in Figure 2 around both the binary systems, where the values of the radiation factors of the two primaries are taken according to Table 1.

a) Binary System Luyten 726-8 b) Binary System Kruger-60

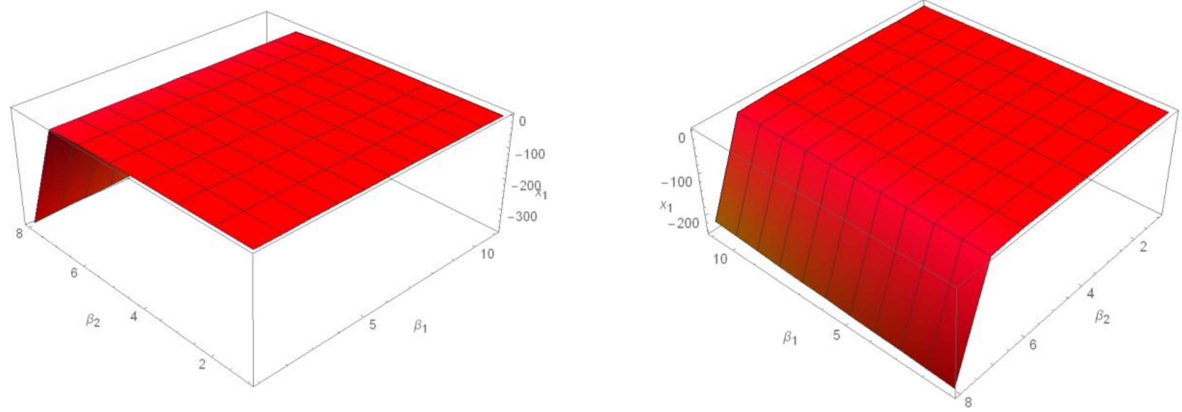


Fig.1: Variation in the X-Coordinate of the L_1 Point with Respect to Radiation Factors.

a) Binary System Luyten 726-8 b) Binary System Kruger-60

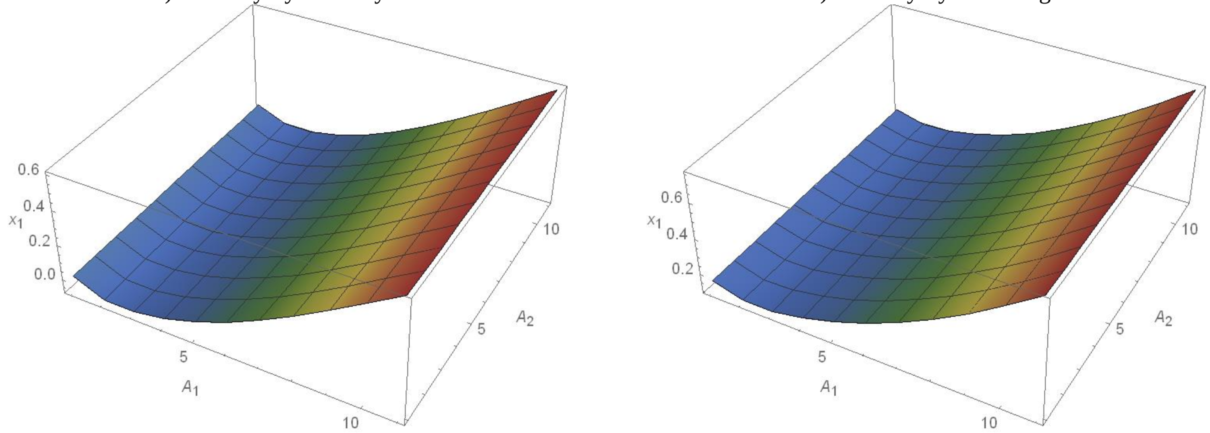


Figure 2: Variation in the X-Coordinate of the L_1 Point with Respect to Oblateness Factor.

3.2. Location of collinear point L2

The position of the collinear point L_2 is given by $x > 1-\mu$; then $|\mu+x| = \mu+x$ and $|\mu+x-1| = -(\mu+x-1)$. Assuming $x+\mu-1=\rho$, substituting this relation in equation (5) and re-arranging the terms of the equation, we obtain:

$$\frac{\mu}{3(1-\mu)} = \frac{\rho^3 N_2}{(1-\rho)^2 D_1}, \tag{9}$$

where,

$$N_2 = 1+2A_2+\left(\frac{4}{3}+2A_1\right)\beta_1 + (1+10A_1+3A_2-(2+7A_1)\beta_1)\rho \\ +\left(\frac{1}{3}-15A_1-A_2+(\frac{4}{3}+14A_1)\beta_1\right)\rho^2+\left(20A_1-\frac{3A_2}{2}-(\frac{1}{3}+\frac{43A_1}{2})\beta_1\right)\rho^3.$$

Assuming $\lambda = \left(\dfrac{\mu(1+\frac{45A_2}{2}(1-\beta_2))}{3(1-\mu)(1+2A_2+(\frac{4}{3}+2A_1)\beta_1}\right)^{\frac{1}{3}}$.

The value of ρ in series form is substituted from equation (7) into (10) and comparing the coefficients, we obtained the value of the coefficients as:

$$c_1 = \frac{1}{3}.\frac{1-10A_1-\frac{73A_2}{2}+\left(\frac{14}{3}+11A_1+25A_2\right)\beta_1+\frac{75}{2}A_2\beta_2}{\left(1+2A_2+\left(\frac{4}{3}+2A_1\right)\beta_1\right)\left(1+\frac{45A_2}{2}(1-\beta_2)\right)};$$

$$c_2 = -\frac{1}{9}.\frac{1-45A_1+\frac{43A_2}{2}+\left(\frac{16}{3}+184A_1+804A_2\right)\beta_1-\frac{45}{2}A_2\beta_2}{\left(1+2A_2+\left(\frac{4}{3}+2A_1\right)\beta_1\right)^2\left(1+\frac{45A_2}{2}(1-\beta_2)\right)^2}. \tag{10}$$

The graphical behavior of the system, concerning the shift in x-coordinate of the collinear point L_2 as a function of the radiation factors β_1 and β_2; taking A_1 = 0.001 and A_2 = 0.001, around the binary system Luyten-726 and Kruger 60 are shown in Figure 3. Also, the shifts in the x-coordinate of the point as a function of the oblateness factor around both the binary systems are shown in Figure 4, where the values of the radiation factors of the two primaries are taken according to Table 1.

3.3. Location of collinear point L3

The position of the collinear point L_3 is given by $x < -\mu$; then $|\mu+x| = \mu+x$ and $|\mu+x-1| = -(\mu+x-1)$. Assuming $x+\mu = -\rho$, substituting this relation in equation (5) and re-arranging the terms of the equation, we obtain:

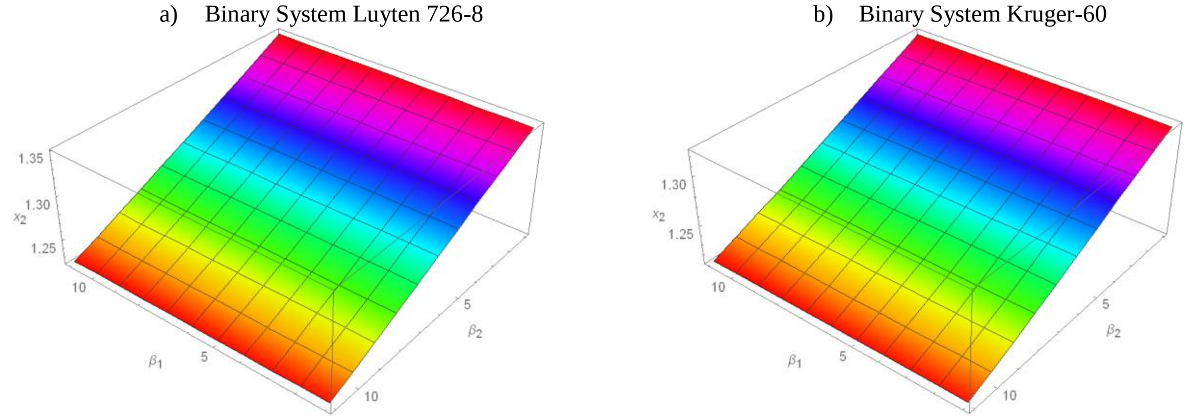


Fig.3: Variation in the X-Coordinate of the L_2Point with Respect to Radiation Factor.

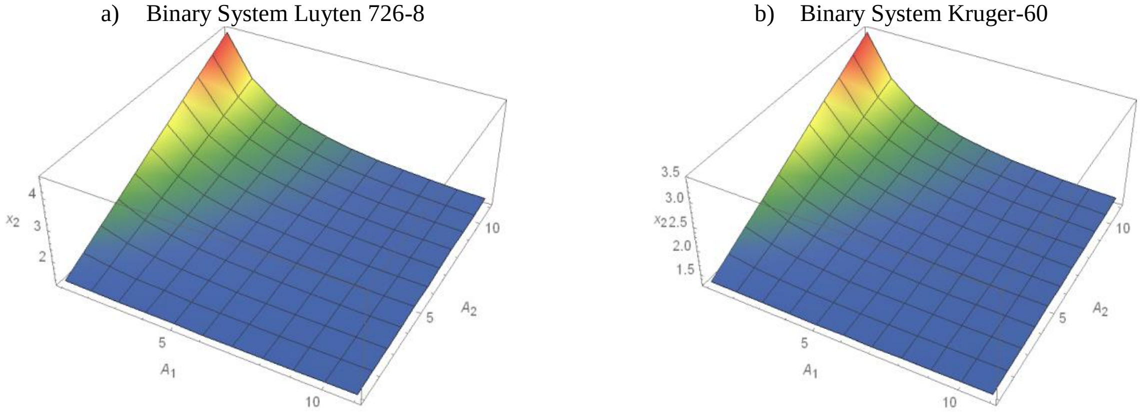


Fig.4: Variation in the X-Coordinate of the L_2Point with Respect to Oblateness Factor.

$$\frac{\mu}{1-\mu}=\frac{(1+\rho)^2\left(1-\rho^3-\frac{3(A_1+A_2)\rho^3}{2}+\frac{3A_1\rho^{-2}}{2}-\left(1+\frac{3}{2}A_1\rho^{-2}\right)\beta_1\right)}{\rho^3\left(\begin{array}{l}3+3\rho+\rho^2-\frac{3}{2}(A_1+A_2)(1+\rho)^3\rho^{-1}\\-\frac{3}{2}A_2(1+\rho)^{-2}\rho^{-1}+\left(\frac{3}{2}A_2(1+\rho)^{-2}+1\right)\rho^{-1}\beta_2\end{array}\right)} \tag{11}$$

Now replacing $\rho=1+\gamma$ and expanding $1/\rho$ up to $o\left[\rho^3\right]$. Thus from equation (11), we get

$$\begin{aligned}\frac{\mu}{(1-\mu)}&=-\frac{4}{7}\beta_1-\frac{6}{7}A_2+\left(\frac{93A_2}{98}+\frac{6A_1}{49}\right)\beta_1+\frac{6}{49}A_2\beta_2\\&+\left(1+\frac{11}{14}A_1-\frac{85}{56}A_2+\left(\frac{81A_2}{49}-\frac{67A_1}{98}-\frac{19}{21}\right)\beta_1+\left(\frac{5A_2}{49}-\frac{5A_1}{14}-\frac{1}{7}\right)\beta_2\right)u\\&+\left(1+\frac{341}{168}A_1-\frac{37}{21}A_2+\left(\frac{8401A_2}{4704}-\frac{3085A_1}{1568}-\frac{989}{1008}\right)\beta_1\right.\\&\qquad\left.+\left(\frac{1685A_1}{1176}+\frac{2137A_2}{4704}+\frac{2}{7}\right)\beta_2\right)u^2\\&+\left(\frac{1567}{1728}+\frac{25987}{8064}A_1-\frac{13705}{8064}A_2\right.\\&+\left(\frac{16063A_2}{9408}-\frac{14993A_1}{4704}-\frac{5465}{6048}\right)\beta_1\\&\left.+\left(-\frac{57017A_1}{18816}-\frac{165115A_2}{112896}-\frac{4519}{12096}\right)\beta_2\right)u^3,\end{aligned}$$

where, $u=-\frac{12\gamma}{7}$.

Using Lagrange's inversion integral formula, γ is obtained in terms of $\frac{\mu}{1-\mu}$ as

$$\begin{aligned}\gamma=&-\frac{A_2}{2}-\frac{\beta_1}{3}\left(1-A_1-\frac{A_2}{2}\right)-\frac{7}{12}\left(\frac{\mu}{1-\mu}\right)\left(1-\frac{11}{14}A_1-\frac{11}{56}A_2\right.\\&\left.-\frac{5\beta}{21}\left(1-\frac{383A_1}{70}+\frac{309A_2}{28}\right)+\frac{\beta_2}{7}\left(1+\frac{5A_1}{2}-\frac{5A_2}{7}\right)\right)\\&+\frac{7}{12}\left(\frac{\mu}{1-\mu}\right)^2\left(1-\frac{55}{168}A_1-\frac{13}{672}A_2-\frac{71\beta_1}{504}\left(1-\frac{14929A_1}{497}+\frac{432209A_2}{3976}\right)\right.\\&\left.+\frac{\beta_2}{7}\left(1+\frac{299A_1}{84}-\frac{6947A_2}{672}\right)\right)\\&-\frac{7}{12}\left(\frac{\mu}{1-\mu}\right)^3\left(\frac{1889}{1728}-\frac{2672}{24192}A_1+\frac{1031}{378}A_2+\frac{30481\beta_1}{18144}\times\right.\\&\left.\left(1+\frac{7576A_1}{1141}-\frac{1873826A_2}{213367}\right)+\frac{569\beta_2}{4032}\left(1+\frac{213425A_1}{23893}-\frac{977987A_2}{47796}\right)\right)\\&+o\left[\frac{\mu}{1-\mu}\right]^4\end{aligned}$$

The graphical behavior of the system, concerning the shift in x-coordinate of the collinear point L_3 as a function of the radiation factors β_1 and β_2 taking A_1 = 0:001 and A_2 = 0:001, around the binary system Luyten-726 and Kruger 60 are shown in Figure 5. Also the shifts in the x-coordinate as a function of the oblateness factor around both the binary systems are shown in Figure 6, where the values of the radiation factors of the two primaries are taken according to Table 1.

4. Linear stability of collinear points

The position of the equilibrium points is denoted by (a_0, b_0). Assuming (u, v) as the displacement of the infinitesimal mass from the equilibrium point such that $x = a_0 + u$ and $y = b_0 + v$ and substituting these values in (1), we obtain the system of equations, taking only linear terms of u and v as follows

$$\begin{aligned} u'' - 2v' &= \frac{1}{1+e\cos v}\left(\left(\Omega_{xx}\right)^0 u + \left(\Omega_{xy}\right)^0 v\right), \\ v'' + 2u' &= \frac{1}{1+e\cos v}\left(\left(\Omega_{yz}\right)^0 u + \left(\Omega_{yy}\right)^0 v\right). \end{aligned} \tag{13}$$

Here, the superscript 0 indicates that the derivatives are to be evaluated at the equilibrium point (a_0, b_0). The values of the second order partial derivatives for the collinear points are given as follows:

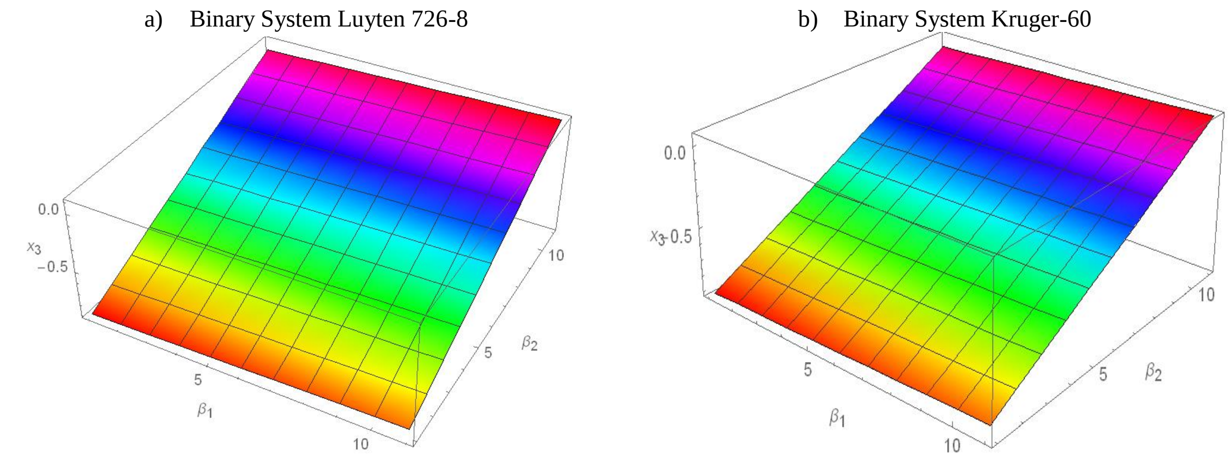


Fig.5: Variation in the X-Coordinate of the L_3 Point with Respect to Radiation Factor.

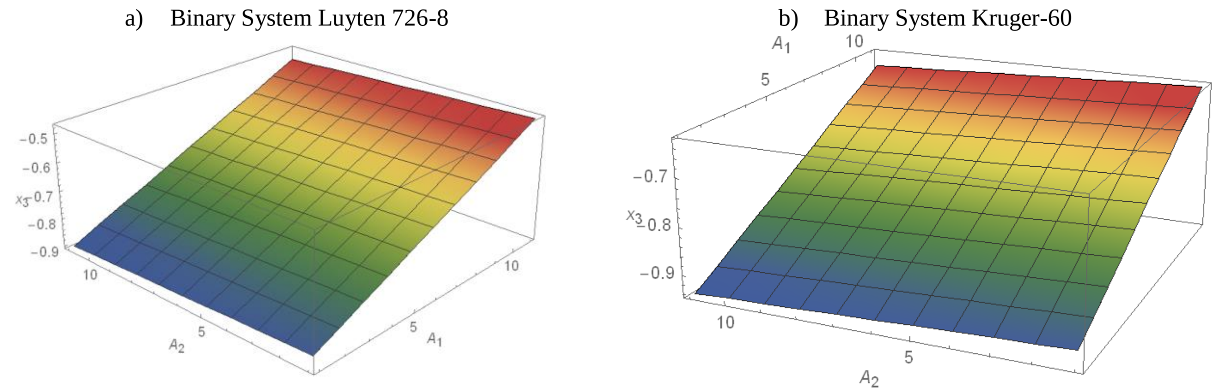


Fig.6: Variation in the X-Coordinate of the L_3 Point with Respect to Oblateness Factor.

$$\begin{aligned} \Omega_{xx}^0 &= 1 + 2(\delta_1 + 2\delta_2), \\ \Omega_{yy}^0 &= 1 - (\delta_1 + \delta_2), \\ \Omega_{xy}^0 &= \Omega_{yx}^0 = 0; \end{aligned}$$

where,

$$\delta_1 = \frac{1}{n^2}\left(\frac{(1-\mu)q_1}{r_1^3} + \frac{\mu q_2}{r_2^3}\right),$$

$$\delta_2 = \frac{2}{3n^2}\left(\frac{(1-\mu)q_1 A_1}{r_1^3} + \frac{\mu q_2 A_2}{r_2^5}\right).$$

The solution to the above system of equation can be expressed as follows:

$$u = A\exp(\alpha v); \qquad v = B\exp(\alpha v).$$

Substituting in equation (13), we get

$$\begin{aligned} A\left(\alpha^2 - \Phi_{xx}^0\right) + B\left(-2\alpha - \Phi_{xy}^0\right) &= 0; \\ A\left(2\alpha - \Phi_{yx}^0\right) + B\left(\alpha^2 - \Phi_{yy}^0\right) &= 0; \end{aligned}$$

where,

$$\begin{aligned} \Phi_{xx} &= \frac{1}{1+e\cos v}\Omega_{xx}, & \Phi_{xy} &= \frac{1}{1+e\cos v}\Omega_{xy}, \\ \Phi_{yx} &= \frac{1}{1+e\cos v}\Omega_{yx}, & \Phi_{yy} &= \frac{1}{1+e\cos v}\Omega_{yy}. \end{aligned}$$

The characteristic equation of system (13), is given as follows:

$$\alpha^4 + \left(4 - \Phi_{xx}^0 - \Phi_{yy}^0\right)\alpha^2 + \left(\Phi_{xx}^0 \Phi_{yy}^0\right) = 0$$

$$\text{ie. } \alpha^4 + \left(4 - \frac{1}{1+e\cos v}(2 + \delta_1 + 3\delta_2)\right)\alpha^2 + \frac{1}{(1+e\cos v)^2}(1 + 2\delta_1 + 4\delta_2)(1 - \delta_1 - \delta_2) = 0 \tag{15}$$

The two square roots of the biquadratic equation are given by (15) be α_1^2 and α_2^2. Then using relation between roots and coefficient we have,

$$\alpha_1^2\alpha_2^2 = \frac{1}{(1+e\cos\nu)^2}(1+2\delta_1+4\delta_2)(1-\delta_1-\delta_2), \tag{16}$$

And

$$\alpha_1^2+\alpha_2^2 = -\left(4-\frac{1}{(1+e\cos\nu)^2}(1+2\delta_1+3\delta_2)\right). \tag{17}$$

The system is stable around the collinear point if the roots of characteristic equation are purely imaginary, that is the roots α_1^2 and α_2^2 are negative, and thus we get the condition as represented by the following inequalities:

$$4-\frac{1}{(1+e\cos\nu)^2}(1+2\delta_1+3\delta_2)>0 \Rightarrow \delta_1+3\delta_2<2+4e\cos\nu<2+4e, \tag{18}$$

and

$$(1+2\delta_1+4\delta_2)(1-\delta_1-\delta_2)>0. \tag{19}$$

Taking both the factors of the inequality (19) negative yields contradictory condition. Therefore taking both the factors positive, we get the condition for stability of collinear point as:

$$-\frac{1}{2}<\delta_1+\delta_2<1. \tag{20}$$

Taking $x=(1-\mu)(\mu+x)+\mu(x+\mu-1)$, the equation (5) can be re-written as

$$\frac{(1-\mu)(x+\mu)}{r_1}\left(r_1-\frac{q_1}{n^2}\left(\frac{1}{r_1^2}+\frac{3A_1}{2r_1^4}\right)\right) + \frac{\mu(x+\mu-1)}{r_2}\left(r_2-\frac{q_2}{n^2}\left(\frac{1}{r_2^2}+\frac{3A_1}{2r_2^4}\right)\right)=0 \tag{21}$$

We have analyzed the stability condition of each of the collinear point separately in the subsequent discussions as given below:

4.1. The stability of collinear point L_1

For the collinear point L_1 which is expressed as $-\mu<x<1-\mu$, $r_1=x+\mu$ and $r_2=-(x+\mu-1)$. Assuming $r_1=1-\rho$ and $r_2=1-r_1$, the equation (21) become:

$$(1-\mu)\left(r_1-\frac{q_1}{n^2}\left(\frac{1}{r_1^2}+\frac{3A_1}{2r_1^4}\right)\right)-\mu\left(r_2-\frac{q_2}{n^2}\left(\frac{1}{r_2^2}+\frac{3A_1}{2r_2^4}\right)\right)=0$$

Simplifying the above relation, we get the expression in the following form:

$$\begin{aligned}\delta_1+\delta_2 = 1+(1-\mu)\Bigg[&3-\frac{9A_1}{2}-\frac{151A_2}{2} \\ &+\left(\frac{\mu}{1-\mu}\right)^{1/3}(4.16017-2.08008A_1+66.9094A_2) \\ &+\left(\frac{\mu}{1-\mu}\right)^{2/3}(1.44225-4.56712A_1-157.365A_2) \\ &+\left(\frac{\mu}{1-\mu}\right)\left(-\frac{10}{9}-\frac{19A_1}{6}-\frac{3793A_2}{27}\right) \\ &+\beta_1\Bigg\{-4+102A_2+\left(\frac{\mu}{1-\mu}\right)^{1/3}\times \\ &(-2.3112+10.1693A_1-59.2439A_2) \\ &+\left(\frac{\mu}{1-\mu}\right)^{2/3}(-1.282-19.4437A_1-16.6482A_2) \\ &+\left(\frac{\mu}{1-\mu}\right)\left(-\frac{28}{27}-\frac{160A_1}{9}\right)\Bigg\} \\ &+\beta_2\Bigg\{3-\frac{9A_1}{2}+138A_2+\left(\frac{\mu}{1-\mu}\right)^{1/3}\times \\ &(-1.38672+2.08008A_1-132.779A_2) \\ &+\left(\frac{\mu}{1-\mu}\right)^{2/3}(-2.40375+14.6629A_1+719.362A_2) \\ &+\left(\frac{\mu}{1-\mu}\right)\left(-\frac{1}{3}+\frac{47A_1}{6}+\frac{12374A_2}{27}\right)\Bigg\}\Bigg]\end{aligned} \tag{22}$$

4.2. The stability of collinear point L_2

For the collinear point L_2 which is expressed as $x>1-\mu$, $r_1=x+\mu=1+\rho$ and $r_2=-(x+\mu-1)$. Assuming $r_1=1-\rho$ and $r_2=r_1-1$, the equation (21) becomes:

$$(1-\mu)\left(r_1-\frac{q_1}{n^2}\left(\frac{1}{r_1^2}+\frac{3A_1}{2r_1^4}\right)\right)+\mu\left(r_2-\frac{q_2}{n^2}\left(\frac{1}{r_2^2}+\frac{3A_1}{2r_2^4}\right)\right)=0$$

Simplifying the above relation, we get the expression in the following form:

$$\begin{aligned}\delta_1+\delta_2 = 1+(1-\mu)\Bigg[&3-\frac{9A_1}{2}-\frac{137A_2}{2} \\ &+\left(\frac{\mu}{1-\mu}\right)^{1/3}(-4.16017+27.0411A_1+191.714A_2) \\ &+\left(\frac{\mu}{1-\mu}\right)^{2/3}(1.44225-33.4121A_1-83.4902A_2) \\ &+\left(\frac{\mu}{1-\mu}\right)\left(-\frac{8}{9}-\frac{19A_1}{6}-\frac{940A_2}{27}\right) \\ &+\beta_1\Bigg\{4-90A_2+\left(\frac{\mu}{1-\mu}\right)^{1/3}(-10.6315-17.5652A_1+283.893A_2) \\ &+\left(\frac{\mu}{1-\mu}\right)^{2/3}(5.128+145.08A_1+222.765A_2) \\ &+\left(\frac{\mu}{1-\mu}\right)\left(\frac{44}{27}-\frac{688A_1}{9}-\frac{22690A_2}{81}\right)\Bigg\}\end{aligned}$$

$$+\beta_2\left\{3-\frac{9A_1}{2}+136A_2+\left(\frac{\mu}{1-\mu}\right)^{1/3}(4.16017-27.0411A_1-378.922A_2)\right.$$
$$+\left(\frac{\mu}{1-\mu}\right)^{2/3}(-1.44225+33.4121A_1+162.814A_2)$$
$$\left.\left.+\left(\frac{\mu}{1-\mu}\right)\left(-\frac{1}{9}-\frac{19A_1}{6}+\frac{1885A_2}{27}\right)\right\}\right] \qquad (23)$$

where, $M=\frac{\mu}{1-\mu}$.

4.3. The stability of collinear point L_3

For the collinear point L_3 which is expressed as $x<-\mu$, $r_1=-(x+\mu)=\rho$ and $r_2=-(x+\mu-1)$. Assuming $r_1=1-\rho$, and $r_2=r_1+1$, the equation (21) become

$$(1-\mu)\left(r_1-\frac{q_1}{n^2}\left(\frac{1}{r_1^2}+\frac{3A_1}{2r_1^4}\right)\right)+\mu\left(r_2-\frac{q_2}{n^2}\left(\frac{1}{r_2^2}+\frac{3A_1}{2r_2^4}\right)\right)=0$$

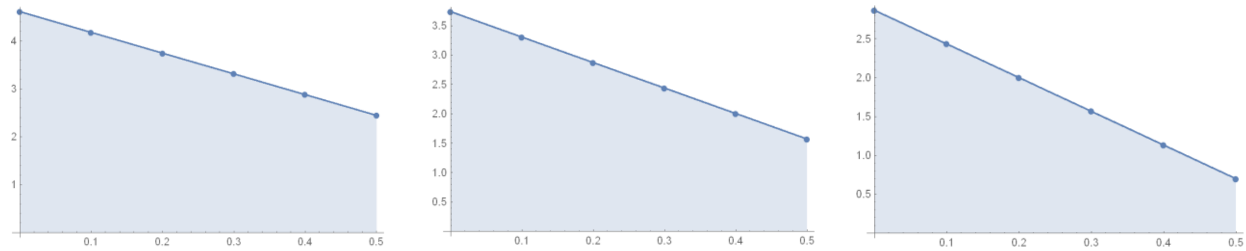
Simplifying, we get

$$\delta_1+\delta_2=1+(1-\mu)\left[+M\left\{3-3A_1+3A_2\left(\frac{A_1}{2}-A_2\right)\beta_1+\left(3A_1-\frac{5A_2}{2}-2\right)\beta_2\right\}\right.$$
$$+M^2\left\{\frac{7A_1}{8}-\frac{7A_2}{4}\left(\frac{29A_1}{8}+\frac{A_2}{16}-\frac{7}{6}\right)\beta_1+\left(-\frac{29A_1}{24}+\frac{133A_2}{96}+\frac{7}{12}\right)\beta_2\right\}$$
$$+M^3\left\{-\frac{49}{48}+\frac{217A_1}{96}+\frac{217A_2}{192}+\left(-\frac{4001A_1}{576}+\frac{437A_2}{72}+\frac{119}{72}\right)\beta_1\right.$$
$$\left.\left.+\left(-\frac{325A_1}{144}-\frac{1031A_2}{1152}+\frac{7}{48}\right)\beta_2\right\}\right].$$

Analyzing the value of $\delta_1+\delta_2$ in the case of each of the collinear point, it is observed that the value of $\delta_1+\delta_2$ is greater than 1, when the oblateness factors A_1 and A_2 are assumed to be zero. However, for very large value of the oblateness factor the stability condition might change. The stability factor $\delta_1+\delta_2=\delta$(say) of collinear point L_1 after varying the values of radiation factor β_1 are shown in Figures 7 and 8 around the binary system Luyten-726 and Kruger-60 respectively. For the Figure 7 and 8, β_2 is assumed to be equal to 0, 0.2 and 0.4.

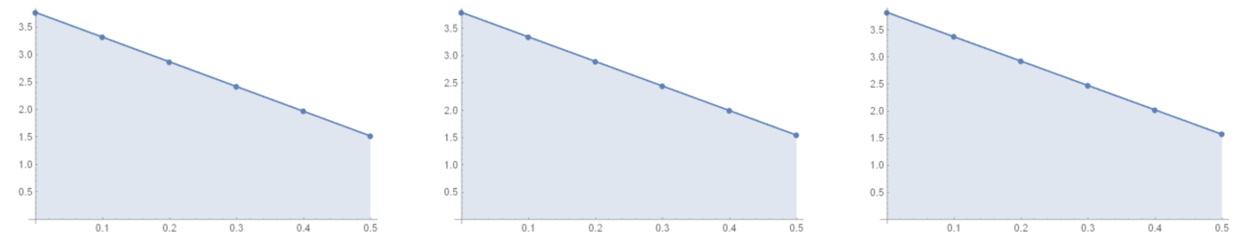
Similarly, in the Figures 9 and 10, the variation in the value of δ are shown for the collinear point L_2 with respect to β_1, when β_2 is assumed to be equal to 0, 0.2 and 0.4, around the binary system Luyten-726 and Kruger-60 respectively.In the Figures 11 and 12, the variation in the value of δ are shown for collinear point L_3 with respect to β_1, when β_2 is assumed to be equal to 0, 0.2 and 0.4, around the binary system Luyten-726 and Kruger-60 respectively.

In the Figures 13, 15 and 17 the variation in the value of δ for L_1, L_2 and L_3 with respect to A_1 are shown, when A_2 is assumed to be equal to 0, 0.06, 0.12 and 0.18 around the binary system Luyten-726. And in Figures 14, 16 and 18, the variation in the value of δ with respect to A_1 are shown for L_1, L_2 and L_3 around the binary system Kruger-60.



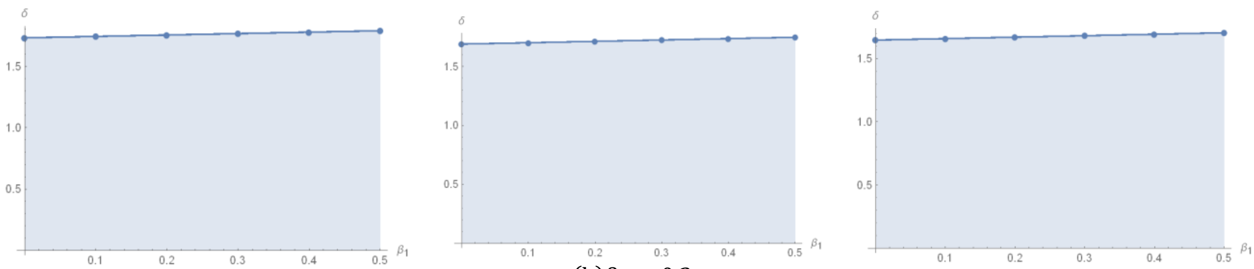
(a)$\beta_2=0.0$ (b)$\beta_2=0.2$ (c)$\beta_2=0.4$

Fig.7: Δ versus B_1 for Collinear Point L_1around Luyten 726-8.



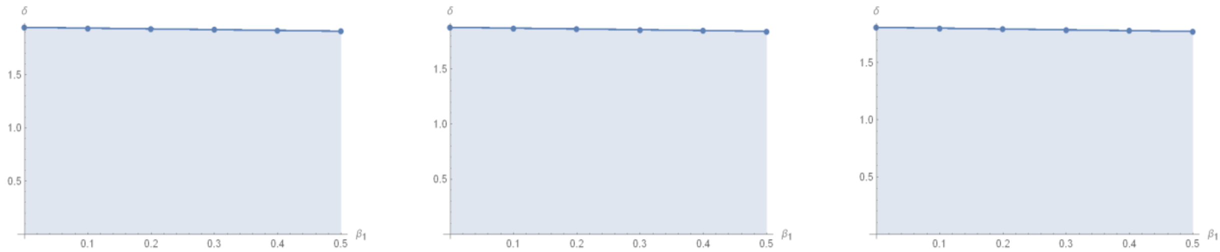
(a)$\beta_2=0.0$ (b)$\beta_2=0.2$ (c)$\beta_2=0.4$

Fig.8: Δ versus B_1 for Collinear Point L_1around Kruger-60.



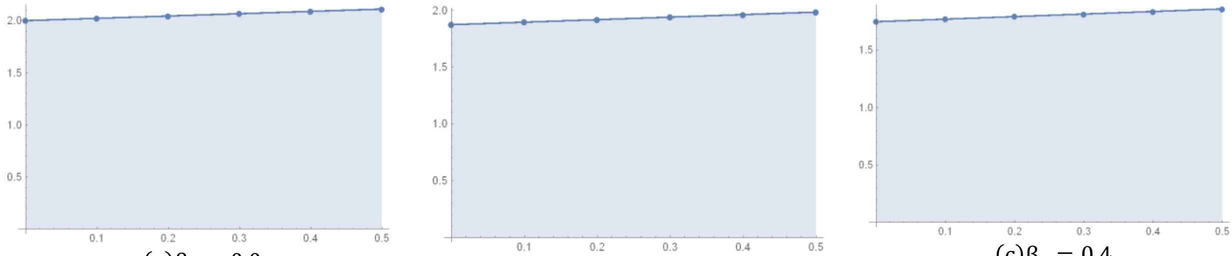
(a)$\beta_2=0.0$ (b)$\beta_2=0.2$ (c)$\beta_2=0.4$

Fig.9: Δ versus B_1 for Collinear Point L_2around Luyten 726-8.



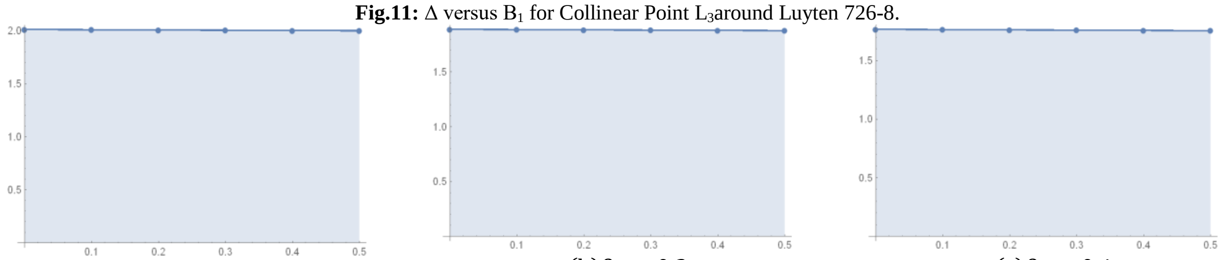
(a)$\beta_2 = 0.0$ (b)$\beta_2 = 0.2$ (c)$\beta_2 = 0.4$

Fig.10: Δ versus B_1 for Collinear Point L_2around Kruger-60.



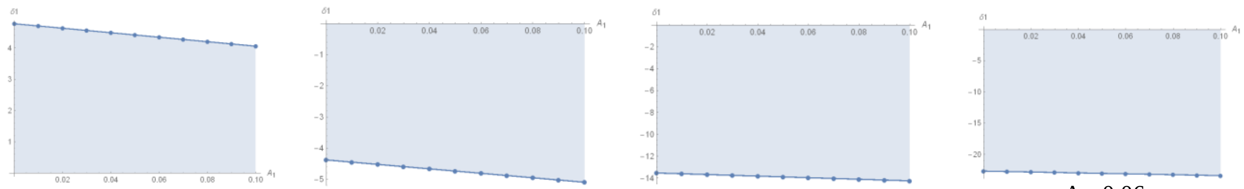
(a)$\beta_2 = 0.0$ (b)$\beta_2 = 0.2$ (c)$\beta_2 = 0.4$

Fig.11: Δ versus B_1 for Collinear Point L_3around Luyten 726-8.



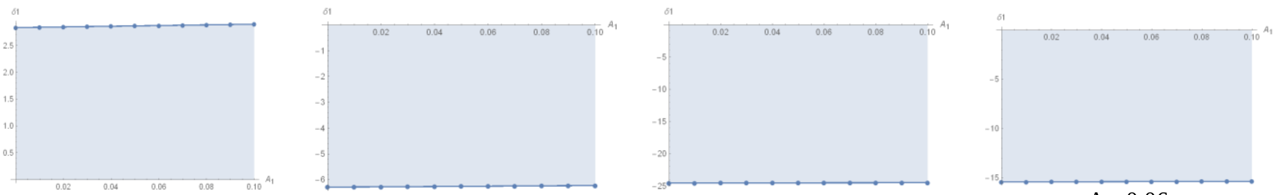
(a)$\beta_2 = 0.0$ (b)$\beta_2 = 0.2$ (c)$\beta_2 = 0.4$

Fig.12: Δ versus B_1 for Collinear Point L_3around Kruger-60.



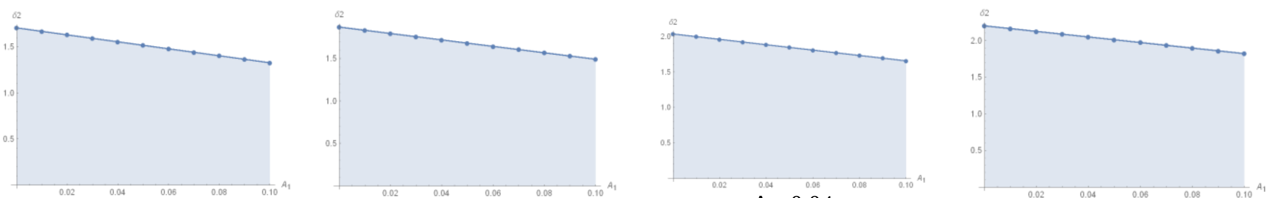
A_1=0.00 A_1=0.02 A_1=0.04 A_1=0.06

Fig. 13: Δ versus $A_1$for Collinear Point L_1AroundLuyten 726-8.



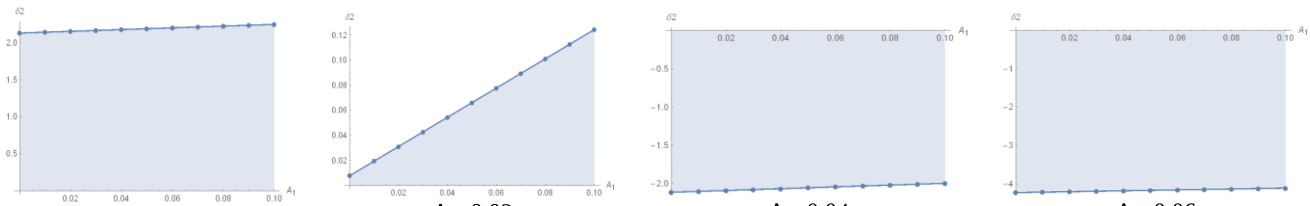
A_1=0.00 A_1=0.02 A_1=0.04 A_1=0.06

Fig. 14: Δ versus $A_1$for Collinear Point L_1around Kruger-60.



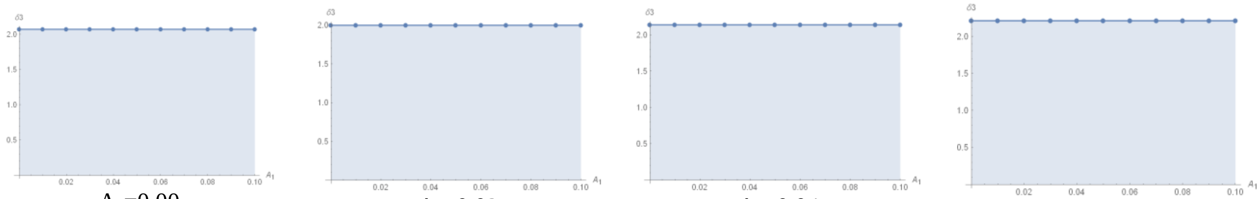
A_1=0.00 A_1=0.02 A_1=0.04 A_1=0.06

Fig. 15: Δ versus $A_1$for Collinear Point L_2around Luyten 726-8.



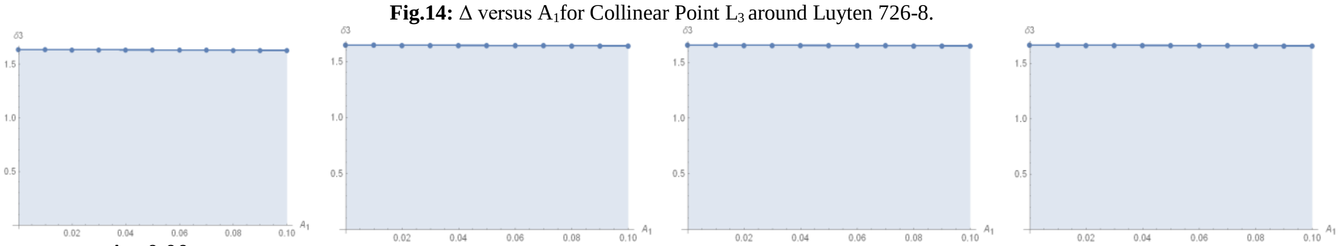
A_1=0.00 A_1=0.02 A_1=0.04 A_1=0.06

Fig.13: Δ versus $A_1$for Collinear Point L_2around Kruger-60.



A_1=0.00 A_1=0.02 A_1=0.04 A_1=0.06

Fig.14: Δ versus $A_1$for Collinear Point L_3 around Luyten 726-8.



A_1=0.00 A_1=0.02 A_1=0.04 A_1=0.06

Fig.15: Δ versus $A_1$for Collinear Point L_3around Kruger-60.

5. Discussion and Conclusion

The elliptical restricted three body problem is studied when both the primaries are radiating and oblate. The location of the collinear points is obtained in the form of series, exploiting Vinti's method [30]. The linear stability of the collinear points has been studied and it is found that all the three points are unstable.

Kumar & Ishwar [10] obtained the expression for the collinear point L_1 in generalized photogravitational model and Singh & Umar [27] studied the solution for all three equilibrium points and investigated their stability. However, the novelty of this paper lies in the application of Vinti's method to obtain the value of the x-coordinate of the collinear equilibrium points in the form of an infinite series and investigation of the stability by establishing a parameter defining the possible range of stability.

The two binary systems Luyten-726 and Kruger-60 have been considered for the purpose of numerical exploration of the model. The effect of radiation pressure and oblateness on the x-coordinate around the binary systems has been plotted and shown in Figures 1-6. The variation in the stability factor with radiation has been studied and it is found that value remained > 1 throughout our calculation for all three collinear points. But the study of the variation in the value of with respect to oblateness factor A_1 and A_2, indicated that the value is not always greater than 1 for the collinear point L_1, however the collinear point is still unstable as the values came out to be << -1/2.

References

[1] Astakhov SA &Farrelly D (2004). Capture and escape in the elliptic restricted three-body problem. Monthly Notices of the Royal Astronomical Society, 354(4), 971-979.http://dx.doi.org/10.1111/j.1365-2966.2004.08280.x.

[2] Brown JM & Petersen JD (2014). Applying Dynamical Systems Theory to Optimize Libration Point Orbit Station keeping Maneuvers for WIND. In AIAA/AAS Astrodynamics Specialist Conference.http://dx.doi.org/10.2514/6.2014-4304.

[3] Chernikov YA (1970). The photo-gravitational restricted three body problem. Soviet Astronomy. AJ,14, 176-181.

[4] Gonczi R, Froeschle C, &Froeschle, C (2012). Poynting-Robertson drag and orbital resonance. ICARUS, 51, 633-654.http://dx.doi.org/10.1016/0019-1035(82)90152-X.

[5] Hamilton DP & Burns JA (1992). Orbital stability zones about asteroids II. The destabilizing effects of eccentric orbits and of solar radiation. ICARUS, 96, 43-64.http://dx.doi.org/10.1016/0019-1035(92)90005-R.

[6] Henry TJ, Kirkpatrick JD & Simons DA (1994). The solar neighborhood, 1: Standard spectral types (K5-M8) for northern dwarfs within eight parsecs. Astronomical Journals, 108, 1437-1444. http://dx.doi.org/10.1086/117167.

[7] Ishwar B &Kushvah BS (2006). Linear stability of Triangular equilibrium points in the generalized photo-gravitational restricted three body problem with Poynting-Robertson drag. Journal of Dynamical Systems and Geometric theories, 4, 1-5.

[8] Kumar A, Sharma JP, & Ishwar B (2007). Linear Stability of Triangular Equilibrium Points in the Photogravitational Restricted Three Body Problem with Poynting-Robertson Drag when both Primaries are Oblate Spheroid. Journal of Dynamical Systems and Geometric Theories, 5, 193-202. http://dx.doi.org/10.1080/1726037X.2007.10698538.

[9] Kumar S & Ishwar B (2009). Solutions of generalized photogravitational elliptic restricted three body problem. In A. H. Siddiqi, A. K. Gupta, & M. Brokate (Eds.), Modeling of Engineering and Technological Problems (p.456). Melville, NY: AIP.http://dx.doi.org/10.1063/1.3183564.

[10] Kumar S & Ishwar B(2011). Location of collinear equilibrium points in the generalised photo-gravitational elliptic restricted three body problem. International Journal of Engineering, Science and technology, 3(2), 157-162.http://dx.doi.org/10.4314/ijest.v3i2.68143.

[11] Kunitsyn AL (2000). The stability of triangular liberation points in the photo-gravitational three-body problem. Prikl. Mat. Mekh, 65, 788-794.

[12] Kunitsyn AL (2001). The stability of collinear liberation points in the photo-gravitational three-body problem. Journal of Applied Mathematics and Mechanics, 65, 703-706.http://dx.doi.org/10.1016/S0021-8928(01)00075-2.

[13] Kushvah BS (2008). The e ect of radiation pressure on the equilibrium points in the generalized photogravitational restricted three body problem. Astrophysics Space Sci-ences, 315, 231-241.

[14] Kushvah BS & Ishwar B (2004). Triangular equilibrium points in the generalized pho-togravitational restricted three body problem with Poynting Robertson drag. Review Bulletin of Calcutta Mathematical Society, 12, 109-114.

[15] Luyten WJ (1949). New stars with proper motions exceeding 0.5" annually. Astronomical Journal, 55, 15.http://dx.doi.org/10.1086/106322.
[16] Mishra V & Ishwar B (2015). Normalization of Hamiltonian in photo-gravitational elliptic restricted three body problem with Poynting Robertson drag. International Journal of Advanced Astronomy, 3, 42-45. http://dx.doi.org/10.14419/ijaa.v3i1.4534.
[17] Narayan A &Shrivastav A (2012). Effects of oblateness and radiation of primaries on the equilibrium points in the elliptical restricted three body problem. International Journal of Mathematical Sciences, 32(10), pg: 330-342.
[18] Narayan A & Singh N (2014a). Effects of radiation on stability of triangular equilib-rium points in elliptic restricted three body problem. International Journal of Applied Mathematical Research, 3, 45-53.
[19] Narayan A & Singh N (2014b). Motion and stability of triangular equilibrium points in elliptic restricted three body problem under the radiating primaries. Astrophsics and Space Sciences, 352, 57-70.http://dx.doi.org/10.1007/s10509-014-1903-1.
[20] Norton OR &Chitwood L (2008). Field guide to Meteors and Meteorites. (1st ed.) Springer (ebook).http://dx.doi.org/10.1007/978-1-84800-157-2.
[21] Poynting JH (1903). The pressure of light. The Inquirer, (pp. 195-196).
[22] Robertson HP (1937). Dynamical e ects of radiation in the solar system. Monthly Notices of the Royal Astronomical Society, 97, 423-438.http://dx.doi.org/10.1093/mnras/97.6.423.
[23] Singh J & Aminu A (2014). Instability of triangular liberation points in the perturbed photogravitational R3BP with Poynting Robertson drag. Astrophsics and Space Sci-ences, 351, 473-482.http://dx.doi.org/10.1007/s10509-014-1862-6.
[24] Singh J &Amuda T (2014). Poynting-Robertson (P-R) drag and oblateness e ects on mo-tion around triangular equilibrium points in the photogravitational RTBP. Astrophsics and Space Sciences 350, 119-126.http://dx.doi.org/10.1007/s10509-013-1707-8.
[25] Singh J, Leke O, &Aishetu U(2010). Analysis on the stability of triangular points in the perturbed photogravitational restricted three-body problem withvariable masses. Astrophysics and Space Sciences, 32, 299-308.http://dx.doi.org/10.1007/s10509-010-0339-5.
[26] Singh J & Umar A (2012). On the stability of triangular points in the elliptic R3BP under radiating and oblate primaries. Astrophsics and Space Sciences, 341(2), 349-358.http://dx.doi.org/10.1007/s10509-012-1109-3.
[27] Singh J & Umar A (2013). Collinear equilibrium points in the elliptic R3BP with oblateness and radiation. Advances in Space Research, 52, 1489-1496.http://dx.doi.org/10.1016/j.asr.2013.07.027.
[28] Szebehely V (1967). Theory of Orbits: The Restricted problem of three bodies. Academic Press,New York and London
[29] Szebehely V &Giacaglia GEO (1964). On the Elliptic Restricted Problem of Three Bodies. The Astronomical Journal, 69, 230-235.http://dx.doi.org/10.1086/109261.
[30] Vinti J, Der G, &Bonavito N (1998). Orbital and Celestial Mechanics. American Institute of Aeronautics and Astronautics, Reston (ebook).