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Research paper

Fractional action cosmology with an effective Λ -term

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Abstract

We continue studying the cosmological models derived from the fractional variational principle applied to the gravitational sector of the action functional. Within the frame of these models, the effective cosmological term could arise as a result of the non-zero Hubble parameter. At the same time, the continuity equation for the matter remains unchanged in its standard form. In this work, we are going to obtain some exact solutions for our model originating from the several kinematic assumptions. At that, we find the main cosmography parameters of the model and the corresponding equations of state of matter that fills the universe se. First, we proceed from an initially given law of evolution of the universe in some standard scenarios. Then, several exact solutions are obtained from the proposed earlier evolutionary laws for the effective cosmological term.

Keywords: *Cosmological Models; Effective Cosmological Term; Exact Solutions; Fractional Einstein-Hilbert Action.*

1. Introduction

The first direct evidence of the accelerated expansion of our Universe was provided by SNeIa absolute magnitude versus redshift data [1], [2]. Furthermore, according to the recent cosmological observations [3] - [7], we should be assured that the present epoch of accelerated expansion is a strictly proven fact. After the discovery of the accelerated expansion, many models have been proposed to explain this mysterious behavior of the Universe, both in the context of general relativity (GR) and of alternative gravity theories.

First of all, the GR models with rather exotic forms of matter called commonly Dark Energy (DE) have been proposed, but none of them are fully convincing. There are numerous models available in the literature to explain the nature of Dark Energy. The simplest and the most natural candidate among them are the cosmological constant [8] with equation of state (EoS) parameter $w=-1$. Unfortunately, this model suffers from two serious problems such as fine-tuning and cosmic coincidence. Later, various kinds of DE models were proposed to explain the nature of DE. We could mention a few of them such as quintessence [9], phantom [10], tachyon [11], Chaplygin gas [12], Quinton [13], holographic dark energy [14] and Yang--Mills fields [15], [16].

Another approach in an attempt to explain the accelerated expansion could be considered as some modifications of the gravitational theory itself. It is appropriate to mention among these modifications such as the multidimensional theory, Branford models, teleparallel theory, and so forth. Moreover, the variety of modified gravity theories derived from $f(R)$, $f(G)$, $f(R,G)$ and $F(R,T)$ could be considered as the gravitational alternative for DE (see, e.g., [17] - [19] and references therein).

Recently, a class of phenomenological models based on the concept of fractional calculus of variations, which is called the fractional action cosmology (FAC) has been proposed by El-Nabulsi [20]-[23]. Later, we significantly improved and developed these models in our works [24] - [26]. Our models are built on the Lebesgue-Satieties measure $d\rho(x)$ generalizing the standard 4-dimensional measure d^4x and applying to the gravity action functional. The action in FAC is written as a fractional Riemann-Satieties integral:

$$S_L^{\alpha}[q_i]=\Gamma^{-1}(\alpha)\int_{t_0}^{t} L(t',q_i(t'),\dot{q}_i(t'))(t-t')^{\alpha-1}dt'$$

With the integrating function $g_t(t')=\Gamma^{-1}(\alpha)[t^{\alpha}-(t-t')^{\alpha}]$.

Meanwhile, the continuity equation in the framework of our version of FAC has been retained in its standard form [26]. We have shown there that the effective Λ - term could be treated as a kinematically induced by the Hubble parameter cosmological term. As a result, we have found several interesting properties of the FAC models which could be applied to the problem of accelerated expansion.

Based on the motivations mentioned above, we would like to study the FAC models with the different rates of evolution, and with some phenomenological laws for $\Lambda_{eff}(t)$. That is why we are going to obtain several exact solutions to the model equations, and the corresponding cosmography and EoS parameters.

2. Basic equations of FAC

Cosmological models for the modified fractional Einstein-Hilbert action with a varying cosmological term Λ are derived in our article [26]. There we consider a variational principle for the gravity action functional of a fractional order,

$$S_{EH}^{\alpha}=M_P^2\int\left(\frac{R}{2}-\Lambda\right)\sqrt{-g}\,g_{t'}(t)\,d^4x,$$

where $M_P^{-2} = 8\pi G$ is the reduced Planck mass. Then, we assume that the matter content of the universe is minimally coupled to gravity, that is the total action of the system can be expressed as $S^{\alpha}_{total} = S^{\alpha}_{EH} + S_m$, where the matter action is of the standard measure $S_m = \int L\sqrt{-g}d^4x$. Applying the fractional variational procedure in a spatially flat Friedmann-Robertson-Walker metric,

$$ds^2 = dt^2 - a^2(t)\delta_{ik}dx^i dx^k,$$

Where $a(t)$ is a scale factor, one can obtain the following dynamical equations:

$$3H^2 + 3\frac{(1-\alpha)}{t}H = t^{1-\alpha}\rho + \Lambda, \tag{1}$$

$$2\dot{H} + 3H^2 + \frac{2(1-\alpha)}{t}H + \frac{(1-\alpha)(2-\alpha)}{t^2} = -t^{1-\alpha}p + \Lambda, \tag{2}$$

Where $H(t) = \dot{a}/a$ is the Hubble parameter, and we set $M_P^{-2}\Gamma(\alpha) = 1$ for the sake of simplicity. After that, one can derive the continuity equation as follows

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{3}$$

The latter expresses the standard energy-momentum conservation law for a perfect fluid, exactly as it does in General Relativity.
It can be readily verified that the continuity equation (3) in the case $\alpha \in (0,1)$ can be derived from Eqs. (1) and (2), only if the time-varying cosmological term $\Lambda(t)$ satisfies

$$\frac{d}{dt}\left(t^{\alpha-1}\Lambda\right) = \frac{3(1-\alpha)}{t^{2-\alpha}}\left[\dot{H} - 2\frac{(2-\alpha)}{t}H\right].$$

The formal integration of this equation yields

$$\Lambda(t) = \Lambda_0 t^{1-\alpha} + 3(1-\alpha)\left[\frac{H(t)}{t} - (2-\alpha)t^{1-\alpha}\int t^{\alpha-3}H(t)dt\right], \tag{4}$$

where Λ_0 is a constant of integration. Substituting Eq. (4) into the main equations of our model, (1) and (2), we obtain the following set of equations:

$$3H^2 = t^{1-\alpha}\rho_{eff}, \tag{5}$$

$$2\dot{H} + 3H^2 - \frac{1-\alpha}{t}H + \frac{(1-\alpha)(2-\alpha)}{t^2} = -t^{1-\alpha}p_{eff}, \tag{6}$$

Where the effective energy density and pressure are represented by

$$\rho_{eff} = \rho + \Lambda_{eff}, \quad p_{eff} = p - \Lambda_{eff}, \tag{7}$$

$$\Lambda_{eff} = \Lambda_0 - 3(1-\alpha)(2-\alpha)\int t^{\alpha-3}H(t)dt. \tag{8}$$

The last equation means that the effective cosmological term is the sum of the cosmological constant Λ_0 and induced cosmological term

$$\Lambda_{ind} = -3(1-\alpha)(2-\alpha)\int t^{\alpha-3}H(t)dt.$$

It should be noted also that the deceleration parameter is defined just as in the standard cosmology,

$$q = -\frac{a^2\ddot{a}}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2}, \tag{9}$$

being a kinematic parameter of the model [27].
Thus, the set of dynamical equations (5), (6) consists of two independent equations, and fully determines the dynamics of our model. However, to determine three parameters, say H, ρ and p, one more condition should be set. For example, an effective EoS can play the role of such additional equation. Indeed, if we consider the effective barotropic fluid, then the effective EoS follows from Eqs. (5) and (6) in the form

$$w_{eff} = \frac{p_{eff}}{\rho_{eff}} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2} + \frac{1-\alpha}{3(tH)}\left[1 - \frac{2-\alpha}{(tH)}\right]. \tag{10}$$

Assuming the matter also obeys a barotropic EoS $p = w_m\rho$, we obtain the following equations between the barotropic indexes of the effective fluid and matter:

$$w_m = \frac{p}{\rho} = -1 + \frac{1+w_{eff}}{1 - \frac{\Lambda_{eff}}{3H^2}t^{1-\alpha}}, \tag{11}$$

$$w_{eff} = w_m - (1+w_m)\frac{\Lambda_{eff}}{3H^2}t^{1-\alpha}. \tag{12}$$

Combining equations (5), (6), one can derive the continuity equation for the effective values of the energy density and pressure as follows

$$\dot{\rho}_{eff} + 3H(\rho_{eff} + p_{eff}) = -3(1-\alpha)(2-\alpha)t^{\alpha-3}H, \tag{13}$$

which reduces to the continuity equation for matter (3) due to (7) and (8). Thus, the set of equations (5), (6) consists of two independent equations, and can determine the dynamics of our model.
However, this approach requires some specification of the matter content of the universe or some hypotheses concerning the behavior of the effective EoS. Consideration of hypothetical forms of the dark matter and energy causes the most problems in modern cosmology. Therefore, we are going to consider an alternative approach to our model. In this approach, to make the system of equations (5), (6) closed i.e. to make the number of unknowns and number of equations equal, we suppose that one can add to these equations some kinematic equation for the model parameters.
First, we suppose that the rate of expansion is given as a function of time in accordance to some well-known scenarios. Then, to determine the evolution of our model, we want to supplement the system of equations (5), (6) by some effective cosmological term given by a function of time and/or other parameters of the model.

3. Exact models from a given H(t)

In this section, we would like to construct some cosmological models in FAC using different scenarios of evolution of the universe, which have been studied by the researchers (see, e.g., [28]).
3.1. The power-law scenario
In this case, we have

$$a(t) = a_0 t^n \Rightarrow H(t) = \frac{n}{t}. \tag{14}$$

Substituting $H(t)$ from the last equation into (8), we get

$$\Lambda_{eff}(t) = \tilde{\Lambda}_0 + \frac{3(1-\alpha)(2-\alpha)n}{3-\alpha}\frac{1}{t^{3-\alpha}}. \tag{15}$$

The effective EoS can be obtained from (14) and (10) as follows

$$w_{eff} = -1 + \frac{C_{n,\,\alpha}}{3n^2}, \tag{16}$$

where $C_{n,\,\alpha} = (3-\alpha)n - (1-\alpha)(2-\alpha)$. Thus, w_{eff} is a constant. Due to (14), the deceleration parameter (9) is a constant $q = -1 + 1/n$ as well. Nevertheless, the EoS of matter is of a time-varying magnitude, and according to equation (11) equals

$$w_m = -1 + \frac{3-\alpha}{3n - \Lambda_0 t^{3-\alpha}}, \tag{17}$$

Where $\Lambda_0 = \tilde{\Lambda}_0(3-\alpha)/C_{n,\,\alpha}$. It is easy to see from (10) that for a given effective EoS $w_{eff}(t)$ one can obtain the following equation for the Hubble parameter

$$2\dot{H} + 3(1+w_{eff})H^2 - \frac{1-\alpha}{t}H + \frac{(1-\alpha)(2-\alpha)}{t^2} = 0. \tag{18}$$

This equation can be solved with any $w_{eff} = \text{constant}$. Substituting H from (14) and w_{eff} from (16) into equation (18), we can conclude that this equation is satisfied identically. The plots of the main parameters of the model in this case can be viewed in Fig.1.

It is interesting to study the possibility to realize this model by some, for example, scalar field. Let us undergo such a scalar-field reconstruction. We assume now that the matter content is a single self-interacting scalar field φ which couples minimally to gravity. The effective energy density ρ_{eff} and pressure p_{eff} for a scalar field with potential $V(\varphi)$ are given by

$$\rho_{eff} = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \quad p_{eff} = \frac{1}{2}\dot{\varphi}^2 - V(\varphi), \tag{19}$$

respectively.

Combining equations (5) and (6) and taking into account (14) and (19), one can obtain the following equations

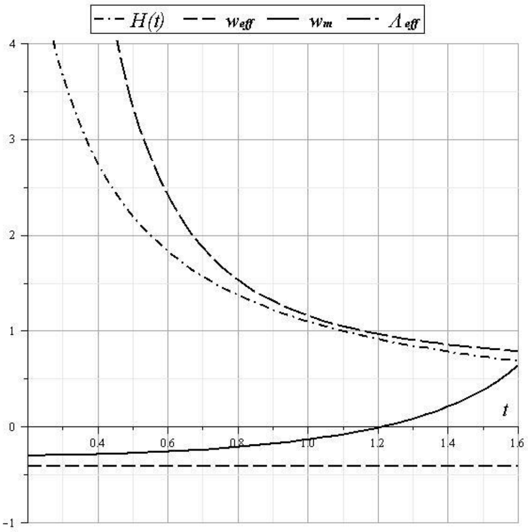

Fig. 1: Shows the Main Parameters in the Power-Law Scenario Versus Time. We employed $n = 1.1$, $\alpha = 0.7$ and $\Lambda_0 = 0.6$.

$$\dot{\varphi}^2 = \frac{C_{n,\,\alpha}}{t^{3-\alpha}}, \tag{20}$$

And

$$V(t) = \frac{3n^2 - C_{n,\,\alpha}}{t^{3-\alpha}}. \tag{21}$$

Integrating equation (20), we can readily get

$$\varphi(t) = \pm\sqrt{C_{n,\,\alpha}}\, t^{(\alpha-1)/2} + \varphi_0,$$

Where φ_0 is a constant of integration. Substituting t from this equation into (21), one can reconstruct the scalar-field potential in the following form

$$V(\varphi) = V_{n,\,\alpha}(\varphi - \varphi_0)^{2(3-\alpha)/(1-\alpha)}, \tag{22}$$

Where $V_{n,\,\alpha} = [(1-\alpha)/2C_{n,\,\alpha}]^{2(3-\alpha)/(1-\alpha)}(3n^2 - C_{n,\,\alpha})$.

3.2. Emergent scenario

In this scenario [28], one has the following scale factor and the Hubble parameter

$$a(t) = \tilde{a}_0\left(\lambda + e^{\mu t}\right)^n \Rightarrow H(t) = \frac{n\mu e^{\mu t}}{\lambda + e^{\mu t}}, \tag{23}$$

where n, μ and λ are constants. First of all, let us introduce more adequate parameters instead of these constants. Investigating $H(t)$ in (17) at the limits $t \to 0$ and $t \to \infty$, one can establish that $\mu = H_\infty / n$ and $\lambda = (H_\infty - H_0)/H_0$, where $H_0 = H_{|t=0}$ and $H_\infty = H_{|t=\infty}$. Therefore, we can rewrite equation (17) for $H(t)$ as follows

$$H(t) = H_\infty \frac{H_0}{H_0 - (H_0 - H_\infty)e^{-(H_\infty/n)t}}, \tag{24}$$

And the scale factor as

$$a(t) = a_0\left\{1 + \frac{H_0}{H_\infty}\left[e^{(H_\infty/n)t} - 1\right]\right\}^n, \tag{25}$$

Where $a_0 = a_{|t=0}$. The deceleration parameter (9) for the rate of expansion (23) is as follows

$$q(t) = -1 + \frac{H_0 - H_\infty}{n H_0} e^{-(H_\infty/n)t}, \tag{26}$$

From equations (8), (10) and (17), one can obtain the induced cosmological term,

$$\Lambda_{ind} = -\int \frac{3(1-\alpha)(2-\alpha)H_0 H_\infty t^{\alpha-3}}{H_0 - (H_0 - H_\infty)e^{-(H_\infty/n)t}}\,dt, \tag{27}$$

and the effective EoS,

$$\begin{aligned} w_{eff} &= -1 + \frac{2}{3}\frac{H_0 - H_\infty}{n H_0} e^{-(H_\infty/n)t} \\ &+ \frac{1-\alpha}{3tH_0H_\infty}\left(H_0 - (H_0 - H_\infty)e^{-(H_\infty/n)t}\right) \\ &\times\left[1 - \frac{2-\alpha}{tH_0H_\infty}\left(H_0 - (H_0 - H_\infty)e^{-(H_\infty/n)t}\right)\right]. \end{aligned} \tag{28}$$

The evolution of this model according to equations (24) - (26) and (28) for the specific values of parameters is shown in Fig. 2.

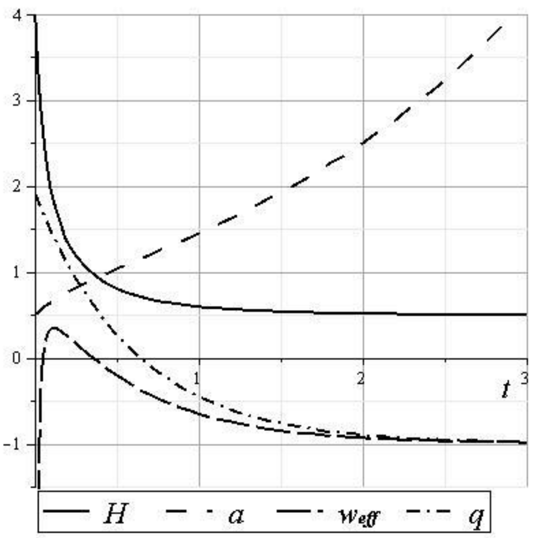

Fig. 2: Shows the Main Parameters in the Emergent Scenario versus Time with $n = 0.3$, $\alpha = 0.96$, $a_0 = 0.5$, $H_0 = 4$ and $H_\infty = 0.5$.

It demonstrates that the effective EoS w_{eff} starts increasing from $w_{eff}(0) = -\infty$ at $t = 0$, crosses the phantom line $w_{eff} = -1$, and then asymptotically approaches to the same value $w_{eff}(t \to \infty) = -1$. However, for a certain period, $0 < w_{eff} < 1$.

3.3. Intermediate scenario

In this scenario, the scale factor and the Hubble parameter are given by

$$a(t) = a_0 e^{Bt^\beta} \Rightarrow H(t) = B\beta t^{\beta-1}, \tag{29}$$

where B and β are constants. Using this $H(t)$ in (8), we obtain the following effective cosmological term

$$\Lambda_{eff}(t) = \Lambda_0 + \frac{3(1-\alpha)(2-\alpha)}{3-\alpha-\beta} \cdot \frac{1}{t^{3-\alpha-\beta}}. \tag{30}$$

Besides that, we could substitute (8) into equations (9) and (12) to obtain the deceleration parameter and the effective EoS as follows

$$q(t) = -1 + \frac{\beta-1}{\beta B} \cdot \frac{1}{t^\beta}, \tag{31}$$

$$w_{eff} = -1 + \frac{2(\beta-1)}{3\beta B t^\beta} + \frac{1-\alpha}{3\beta B t^\beta}\left(1 - \frac{2-\alpha}{\beta B t^\beta}\right). \tag{32}$$

The EoS of matter can be readily found from equation (11) with the help of equations (29), (30) and (32).
In this scenario, the main parameters of the model demonstrate some similarities to the corresponding parameters of emergent scenario in the case $\beta < 1$ but substantially differs from them when $\beta > 1$. In Fig.3, we compare graphically two parameters of the model calculated with $\beta = 0.6$ and $\beta = 1.1$.

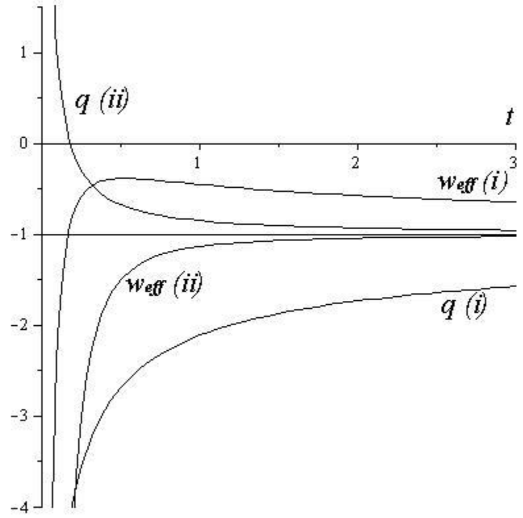

Fig. 3: Shows the Deceleration Parameter q and Effective Eos w_{eff} versus Time with $\alpha = 0.9$ and $B = 0.6$ in the Case :(i) $\beta = 0.6$, and (ii) $\beta = 1.1$.

4. Exact models from a given Λeff

Now, we would like to note an interesting feature of our models in FAC. From equation (8) one could conclude that a number of exact solutions for our model can be obtained by the phenomenological laws of evolution of the effective cosmological term. For this end, let us substitute the effective cosmological term $\Lambda_{eff} = \Lambda(t, a, H)$ into equation (8), and differentiate it with respect to time. As a result, we obtain the following equation

$$\frac{\partial\Lambda}{\partial t} + \frac{\partial\Lambda}{\partial a}\dot{a} + \frac{\partial\Lambda}{\partial H}\dot{H} + \frac{3(1-\alpha)(2-\alpha)}{t^{3-\alpha}}H = 0. \tag{33}$$

We can consider this equation as the main one for finding $a(t) \Rightarrow H(t)$ or $H(t) \Rightarrow a(t)$. After that, we can restore the rest parameters of our model by applying the equations mentioned above. By means of this method in our work [26], the unusual solution with some useful properties has been obtained from the law

$$\Lambda_{eff} = \beta H^2,$$

which yields the following equations for the Hubble parameter

$$H(t) = \frac{3(1-\alpha)}{2\beta} \cdot \frac{1}{t^{2-\alpha}} + H_0,$$

And the scale factor

$$a(t) = a_0 \exp\left(H_0 t - \frac{3}{2\beta} t^{\alpha-1}\right).$$

Later, this model has been tested via the kinematic parameters and the observational data [29].

4.1. Examples of the standard scenarios

Let us consider some examples, making use of the phenomenological laws often discussed in the literature (see, e.g., [30], and bibliography therein).
Let us suppose now that

$$\Lambda_{eff}(a) = C + \beta a^{-m}, \tag{34}$$

where C , β and m are constants. Substituting (34) into equation (33), one can obtain

$$a(t)=\left[\frac{\beta m}{3(1-\alpha)(2-\alpha)}\right]^{1/m} t^{(3-\alpha)/m},$$

This equation immediately yields the Hubble parameters, $H=(3-\alpha)/mt$, and the deceleration parameter, $q=-1+m/(3-\alpha)=$ constant . As one can see, we obtain here an example of the power-law scenario (14).
Then, let us assume the following law

$$\Lambda_{eff}(t)=\frac{C}{t^2}, \tag{35}$$

where C is a constant. Applying equation (8) to the law (35), one can get

$$H=\frac{2C}{3(1-\alpha)(2-\alpha)}\cdot\frac{1}{t^{\alpha}},$$

and, hence,

$$a(t)=a_0 e^{B_\alpha t^{1-\alpha}}, \tag{36}$$

where $B_\alpha = 2C/3(1-\alpha)^2(2-\alpha)$. Comparing this equation with (29), we can conclude that the law (35) leads to the intermediate scenario with $\beta=1-\alpha$.

4.2. A new scenario

In this case, we are going to proceed from the following law

$$\Lambda_{eff}(t)=\Lambda_0+e^{-\beta t}, \tag{37}$$

where Λ_0 and β are constants. By means of equation (33), the effective Λ - term (37) yields the Hubble parameter as

$$H(t)=H_{\alpha,\beta}\, t^{3-\alpha}e^{-\beta t}, \tag{38}$$

where $H_{\alpha,\beta}=\beta/3(1-\alpha)(2-\alpha)$.
Using the Maple package in solving the equation $H(t)=\dot{a}/a$ for the scale factor, we obtain rather complicated expression which includes Whittaker's function $M(\mu,\nu,z)$ as follows

$$a=a_0\exp\left\{H_{\alpha,\beta}\beta^{\frac{\alpha}{2}-4}t^{\frac{\alpha}{2}}e^{\frac{\beta t}{2}}\left[(6-5\alpha+\alpha^2)M\left(-\frac{\alpha}{2},\frac{1-\alpha}{2},\beta t\right)\right.\right.$$

$$\left.\left.-(6-5\alpha+\alpha^2+3\beta t-\alpha\beta+\beta^2t^2)M\left(\frac{2-\alpha}{2},\frac{1-\alpha}{2},\beta t\right)\right]\right\}.$$

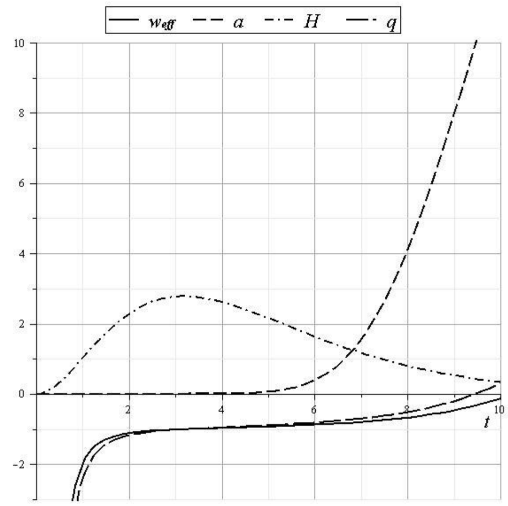

Fig. 4: Shows the Scale Factor a , Hubble Parameter H , Deceleration Parameter q and Effective Eos w_{eff} Versus Time with $\alpha=0.9$, $\beta=0.66$ and $a_0=3\cdot10^{-6}$.

Let us recall that the Whittaker functions $M(\mu,\nu,z)$ can be defined in terms of Kummer's confluent hypergeometric functions $M(a,b,;z)$ by [31]

$$M(\mu,\nu,z)=\exp(-z/2)t^{\nu+1/2}M(\nu-\mu,1+2\nu;z).$$

In Fig. 4, we have depicted the result of numerical solution for the scale factor along with some other parameters of this model.
Using equation (38), we can obtain the explicit expressions for the deceleration parameter,

$$q=-1-\frac{3-\alpha-\beta t}{H_{\alpha,\beta}}t^{\alpha-4}e^{\beta t}, \tag{39}$$

and the effective EoS,

$$w_{eff}=-1-\frac{t^{\alpha-4}e^{\beta t}}{3H_{\alpha,\beta}}\left[5-\alpha-2\beta t+\frac{(1-\alpha)(2-\alpha)}{H_{\alpha,\beta}}t^{\alpha-4}e^{\beta t}\right].$$

So far, the problem of a scalar-field reconstruction for this model remains unsolved. In our view, the most probable way to solve this problem consists of involving the compound of different physical fields and matter.

5. Conclusion

Thus, in this work we have considered a simple method of solving the dynamic equations in FAC equipped with an effective cosmological term. A part of results has been obtained from the given rate of expansion of the universe corresponding to some well-known scenarios. For such standard scenarios, we have found not only the effective cosmological term, but also the effective EoS and deceleration parameter. One could note the great similarity in the behavior of the effective cosmological terms of the different scenarios expressed by equations (15) and (30). It is interesting that in an inflationary scenario, the effective cosmological term behaves similarly [26]. In the case of the power-law scenario, we have reconstructed it by means of the scalar field potential (22). We want to emphasize the obvious attractiveness of this approach owing to its simplicity.
In the rest of our work, we considered some examples of an alternative approach to solving the model equation. The key idea of this approach consists in applying the phenomenological laws assumed earlier by some authors for the evolution of a cosmologi-

cal term. First of all, we have demonstrated that the known scenarios can arise from some phenomenological cosmological terms. Then, we have developed a peculiar cosmological model in the framework of FAC proceeding from the certain law (37). Besides, the analytic results have been accompanied by the graphic illustrations that could allow us to visualize and analyze the behavior of these models. Finally, we would like to point out that the efficiency of approaches considered in this work to the study of FAC is not restricted by the examples mentioned above.

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