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Research paper

Effectiveness of KS elements in satellite orbit prediction using earth's gravity, drag and solar radiation pressure

T. R. Saritha Kumari [1] , M. Xavier James Raj [1*]

[1] *Applied Mathematics Division, Vikram Sarabhai Space Centre, Thiruvananthapuram-695022*
**Corresponding author E-mail: xavierjamesraj@gmail.com*

Abstract

Satellite moving under the gravitational field of Earth deviates from its two-body elliptic orbit, due to the combined effects of the gravitational field of Earth, atmospheric drag, solar radiation pressure, third-body gravitational effects, etc. This paper utilizes the KS regular element equations to solve Newtonian equations of motion to obtain numerical solution with respect to perturbing forces, like, Earth's gravity (includes zonal, sectorial and tesseral harmonics terms), atmospheric drag and solar radiation pressure. Effectiveness of the theory is illustrated by comparing the results with some of the existing theories in literature.

Keywords: *Orbit Prediction; Solar Radiation Pressure; Semi-Major Axis; KS Elements.*

1. Introduction

The effect of various perturbing forces like the shape of the Earth, atmospheric drag, the Sun's radiation, attraction due to Sun and Moon, the Earth's magnetic field, etc. causes the geocentric space object to deviate from its two-body elliptic orbit. For near Earth's satellite orbit, the perturbations due to asphericity of the Earth and atmospheric drag plays a major role, but for high altitude orbits, solar radiation pressure is more important than atmospheric drag. Hence to predict the motion of the satellite precisely, a mathematical model for these forces must be selected properly for integrating the resulting differential equations of motion. The classical Newtonian equations of motion, which are nonlinear, are not suitable for long-term integration for computing accurate orbit.

The KS transformation by Kustaanheimo and Stiefel [1] is used to regularize the nonlinear Kepler equation of motion and reduced it into linear differential equations of a harmonic oscillator of constant frequency. The method of KS elements [2] has been found to be a very powerful method for obtaining numerical solution with respect to any type of perturbing forces, as the equations are less sensitive to round off and truncation errors [3]. The equations are everywhere regular comparing to the classical Newtonian equations, which are singular at the collision of two bodies.

In this paper a detailed study is carried out for orbit prediction using KS differential equations by including the non-spherical gravitational potential (zonal, sectorial and tesseral harmonic terms) of the Earth, atmospheric drag and solar radiation pressure as perturbing forces. Higher order Earth's gravity (zonal, sectorial and tesseral) terms are included by utilizing the recurrence relations of associated Legendre polynomial and its derivatives. To know the effectiveness of the theory, the results are compared with some of the existing theories in literature.

2. Equations of motion

The Newtonian equations of motion are given by,

$$\ddot{\vec{x}} + \frac{K^2}{r^3}\vec{x} = \vec{P} - \frac{\partial V}{\partial x}, \; K^2 = k^2(M+m), \tag{1}$$

where $\vec{x}$ denotes the position vector of the particle with respect to a coordinate system centred at the mass M and perturbed by a perturbed force $\vec{P}$ and a perturbing potential V, dot represent differentiation with respect to the time t, r is the distance between the masses, k^2 is the universal gravitational constant, and $\frac{\partial V}{\partial \vec{x}}$ is the gradient of the scalar function $V(\vec{x}, t)$.

KS element equations of motion [2], [4] are given by,

$$\frac{d\omega}{dE} = -\frac{r}{8\omega^2}\frac{\partial V}{\partial t} - \frac{1}{2\omega}\left(\frac{d\vec{u}}{dE}, L^T\vec{P}\right)$$

$$\frac{d\vec{\alpha}}{dE} = \left\{\frac{1}{2\omega^2}\left[\frac{V}{2}\vec{u} + \frac{r}{4}\left(\frac{\partial V}{\partial \vec{u}} - 2L^T\vec{P}\right)\right] + \frac{2}{\omega}\frac{d\omega}{dE}\frac{d\vec{u}}{dE}\right\}\sin\frac{E}{2}$$

$$\frac{d\vec{\beta}}{dE} = \left\{\frac{-1}{2\omega^2}\left[\frac{V}{2}\vec{u} + \frac{r}{4}\left(\frac{\partial V}{\partial \vec{u}} - 2L^T\vec{P}\right)\right] + \frac{2}{\omega}\frac{d\omega}{dE}\frac{d\vec{u}}{dE}\right\}\cos\frac{E}{2}$$

$$\frac{d\tau}{dE} = \frac{1}{8\omega^3}(K^2 - 2rV) - \frac{r}{16\omega^3}\left(\vec{u}, \frac{\partial V}{\partial \vec{u}} - 2L^T\vec{P}\right) - \frac{2}{\omega^2}\frac{d\omega}{dE}\left(\vec{u}, \frac{d\vec{u}}{dE}\right)$$

with

$$\vec{u} = \vec{\alpha}(E)\cos\frac{E}{2} + \vec{\beta}(E)\sin\frac{E}{2},$$

$$\frac{d\vec{u}}{dE} = -\frac{1}{2}\vec{\alpha}(E)\sin\frac{E}{2} + \frac{1}{2}\vec{\beta}(E)\cos\frac{E}{2},$$

$$\tau = t + \frac{1}{2\omega}(\vec{u}, \vec{u}^*),$$

$$2\omega^2 = \frac{K^2}{r} - \frac{1}{2}\left|\dot{\vec{x}}\right|^2 - V,$$

where E, ω , t are, respectively, generalized eccentric anomaly, angular frequency and physical time.

The components of the position vector $\vec{x}$ of the particle are computed as, $\vec{x} = L(\vec{u})\vec{u}$,

where,

$$L(\vec{u}) = \begin{bmatrix} u_1 & -u_2 & -u_3 & u_4 \\ u_2 & u_1 & -u_4 & -u_3 \\ u_3 & u_4 & u_1 & u_2 \\ u_4 & -u_3 & u_2 & -u_1 \end{bmatrix}$$

The radial distance of the particle is computed as, $r = {u_1}^2 + {u_2}^2 + {u_3}^2 + {u_4}^2 = \sqrt{{x_1}^2 + {x_2}^2 + {x_3}^2}$

Velocity vector of the particle are as follows:

$$\dot{x}_1 = \frac{4\omega}{r}(u_1{u_1}^* - u_2{u_2}^* - u_3{u_3}^* + u_4{u_4}^*)$$

$$\dot{x}_2 = \frac{4\omega}{r}(u_2{u_1}^* + u_1{u_2}^* - u_4{u_3}^* + u_3{u_4}^*)$$

$$\dot{x}_3 = \frac{4\omega}{r}(u_3{u_1}^* + u_4{u_2}^* + u_1{u_3}^* + u_2{u_4}^*).$$

3. Geo-potential

To model for the acceleration caused by the Earth's coefficients are taken from WGS84_EGM96 [5]. The forces acting on an artificial satellite due to the Earth's gravity harmonics (zonal, sectorial and tesseral) is modeled using [6], [7] as below,

$$V = \frac{K^2}{r}\sum_{n=2}^{\infty}\left(\frac{R}{r}\right)^n\sum_{m=0}^{n}\left(C_{nm}\cos(m\lambda) + S_{nm}\sin(m\lambda)\right)P_{nm}\sin(\phi),$$

where R is mean equatorial radius of Earth, ϕ is the geocentric latitude, λ is the longitude, C_{nm} and S_{nm} are dimensionless constants known as gravity coefficients for zonal, sectorial and tesseral harmonics and P_{nm} represent the set of associated Legendre polynomials [7].

4. Atmospheric drag

The acceleration vector on a space object due to atmospheric drag $\vec{a}_{aero}$ is calculated from [7], [8]

$$\vec{a}_{aero} = -\frac{1}{2}\rho {v_{rel}}^2 BC \frac{\vec{v}_{rel}}{|\vec{v}_{rel}|}, \quad (2)$$

where ρ is the local atmospheric density, $\vec{v}_{rel}$ is the space objects velocity vector relative to the atmosphere, v_{rel} is the space object's scalar velocity and BC is the ballistic coefficient. The NRLMSISE00 [9] atmospheric model was used to compute atmospheric densities.

The velocity vector relative to the rotating atmosphere is given by

$$\vec{v}_{rel} = \left[\frac{dx}{dt} + \omega_{\oplus} y \quad \frac{dy}{dt} - \omega_{\oplus} x \ \frac{dz}{dt}\right]^T,$$

where $\omega_{\oplus} = 7.29211 \text{x } 10^{-5}$ rad/sec is the rotational rate of Earth.

Usually, $C = \frac{c_{DA}}{M}$, where M is the mass of the object, c_D is the drag coefficient and A is the projected cross-sectional area of the space object perpendicular to the velocity vector.

5. Solar radiation pressure (SRP)

The acceleration vector on a space object due to solar radiation pressure $\vec{a}_{srp}$ is calculated from [10],

$$\vec{a}_{srp} = c_r \frac{A}{m} k\varphi \frac{\vec{r}_{sat} - \vec{r}_{sun}}{\vec{r}_{sun}}, \tag{3}$$

where, c_r is the constant of reflectivity of the satellite, A is the area of the transverse section of the satellite perpendicular to the disturbing force, m is the satellite mass, k is the ratio of the solar constant and the speed of the light, φ is the shadow function, which has value '1', if the satellite is fully illuminated and '0', if it is in the Earth's shadow, $\vec{r}_{sat}$ is the geocentric vector of the satellite and $\vec{r}_{sun}$ is the the geocentric vector of the Sun.

SIGHT algorithm [7] available in Vallado is used to determine, if the satellite has a direct line of sight with the Sun, and hence whether it is in Earth's shadow. SIGHT algorithm assumes the light from the Sun acts as a point source [7]. Let $\vec{r_1}$ be the geocentric position vector of the satellite and $\vec{r_2}$ be the geocentric position vector of the Sun. The value of $\tau_{min,}$ which minimizes the distance to the central body is given by,

$$\tau_{min} = \frac{|\vec{r_1}|^2 - \vec{r_1}.\vec{r_2}}{|\vec{r_1}|^2 + |\vec{r_2}|^2 - 2\vec{r_1}.\vec{r_2}}$$

A parametric representation of a line between the two position vectors, $\vec{r_1}$ and $\vec{r_2}$, is given by

$$|\vec{c}(\tau_{min})|^2 = (1 - \tau_{min})|\vec{r_1}|^2 + (\vec{r_1}.\vec{r_2})\tau_{min}$$

If $\tau_{min} < 0.0$ or $\tau_{min} > 1.0$, then the satellite is illuminated. ie. if, $|\vec{c}(\tau_{min})|^2 \geq 1.0$, then the satellite is illuminated. else, the satellite is not illuminated
If the satellite is illuminated, the perturbation due to solar radiation pressure is computed using Equation (03). If the satellite is not illuminated, the perturbation due to solar radiation pressure is set to zero.

6. Results and conclusion

The numerical integration of the above differential equations of motion are carried out with Earth's gravity harmonics, air drag and solar radiation pressure as perturbing forces. The numerical integration of the KS differential equations of motion has been carried out using a fixed step size of fourth order Runge-Kutta method with respect to the initial conditions. The constants used for Earth's equatorial radius (R), and Earth's Gravitational constant ($\boldsymbol{k^2}$) are 6378.145 km, and 398600.4418 km^3/s^2, respectively. In our analysis we assumed, C_D = 2.2 and C_R = 1.5.

As a first step, the artificial satellite Explorer-19, which is at 750 Km height is chosen for the analysis. The initial position and velocity components of satellite are provided in Table 1 [10]. In this study perturbations due to the gravity harmonics terms up to J_4, air drag and solar radiation pressure are considered.

Table 1: Initial Condition

x_1 (km)	3538.646
x_2 (km)	-2902.799
x_3 (km)	-5483.478
$\dot{x}_1$ (km/sec)	5.842408
$\dot{x}_2$ (km/sec)	-1.772259
$\dot{x}_3$ (km/sec)	4.707377

Orbital epoch is 14 Feb 1976 00:00:00 UTC, with area to mass ratio as 13.04 x 10^{-07} km^2/kg. The accuracy check in the solution at any fictitious time is obtained using the bilinear relation,

$$BI = u_4 u_1' - u_3 u_2' + u_2 u_3' - u_1 u_4'$$

If this BI is equal to zero, implies the stupendous accuracy in the solution.

The above initial values are used to compute the position and velocity components with respect to various perturbing forces and the accuracy of the solution is determined by computing the bilinear relation.

Table 2 provides the bilinear relation under the perturbing forces at any time (days). In this Table 2, column 1 contains the time in days, column, 2 contains the approach used for orbit prediction, The 3rd column gives the results generated with gravity alone, 4th column is the results with gravity and drag and without SRP and 5th column includes all the perturbations. The results obtained using the present KS theory is compared with Hany's results ([10]). Comparison shows that the KS results are more accurate than that of Hany's results.

Table 2: Comparison of the Values of Bilinear Relation Corresponding to Their Perturbations Forces

Time (Days)	Approach	The value of bilinear relation (BI)		
		Only gravity	With pert. and without SRP	With pert. and with SRP
0.768722	Hany	-2.1191E-10	-2.4374E-10	-2.0372E-10
	KS	1.3195E-10	1.6456E-10	1.7277E-10
1.537444	Hany	-4.1473E-10	-4.1745E-10	-4.1109E-10
	KS	2.9755E-10	3.5485E-10	3.2546E-10
2.306164	Hany	-6.0754E-10	-6.5119E-10	-6.1845E-10
	KS	4.3987E-10	5.0647E-10	4.7337E-10
3.843645	Hany	-9.8134E-10	-1.1559E-10	-1.0486E-10
	KS	7.6846E-10	8.3704E-10	7.8162E-10
4.612355	Hany	-1.1905E-09	-1.2632E-10	-1.2187E-10
	KS	9.3108E-10	9.9336E-10	9.4341E-10
5.381111	Hany	-1.3960E-09	-1.4697E-09	-1.4297E-09
	KS	1.0723E-09	1.1715E-09	1.1095E-09
6.14981	Hany	-1.6516E-09	-1.7016E-09	-1.9654E-09
	KS	1.2550E-09	1.3583E-09	1.2766E-09
6.918555	Hany	-1.8762E-09	-1.9726E-09	-1.9654E-09
	KS	1.4257E-09	1.5355E-09	1.4610E-09
7.687293	Hany	-2.0972E-09	-2.1937E-09	-2.1836E-09
	KS	1.6060E-09	1.7365E-09	1.6374E-09
69.108767	Hany	-1.3006E-08	-1.2827E-08	-1.2816E-08
	KS	1.0113E-08	1.0696E-08	1.0453E-08
69.877513	Hany	-1.2918E-08	-1.2700E-08	-1.2776E-08
	KS	1.0077E-08	1.0612E-08	1.0370E-08
70.646255	Hany	-1.2781E-08	-1.2550E-08	-1.2756E-08
	KS	1.0005E-08	1.0553E-08	1.0318E-08

As a second step, four test cases have been chosen for detailed numerical study with varying eccentricity, whose initial conditions (osculating orbital elements) are given in Table-3.

Table 3: Initial Conditions for Different Test Cases

Variables	Case A	Case B	Case C	Case D
Semi major Axis (a) (km)	46478.6	46478.6	46478.6	46478.6
Eccentricity (e)	0.001	0.5	0.6	0.7
Inclination (i) (deg)	51.6	51.6	51.6	51.6
RAAN (Ω) (deg)	349.213	349.213	349.213	349.213
Arg. of Perigee (**ω**) (deg)	25.24	25.24	25.24	25.24
True Anomaly (f) (deg)	311.234	311.234	311.234	311.234
Mean Anomaly (deg)	311.32	344.703	349.282	353.178
Apogee (km)	40146.942	63339.76	67987.62	72635.48
Perigee (km)	40053.984	16861.16	12213.3	7565.443

The Earth's gravity harmonics terms up to $J_{6,6}$, air drag and solar radiation pressure are included as the perturbing forces in the equations of motion. Orbital epoch considered is 02 June 2015 00:00:00 UTC, with area to mass ratio as 10.454 x 10^{-03} km^2/kg.

The initial conditions provided in Table 3 are used for orbit propagation. Osculating orbital elements obtained for 30 days duration with a step size of 1degree using the present KS theory is compared with High Precision Orbit propagator (HPOP). High Precision Orbit propagator uses numerical integration of the differential equations of motion to generate ephemeris. HPOP, available in STK 9.2 is used for comparing the results with the present KS theory.

Table 4 to 7 gives the comparison between KS and HPOP values of orbital elements (osculating elements) for 30 days. In these tables, column 1 contains the time in days, column 2 contains the approach used for orbit prediction, columns 3 to 9 contain the information regarding osculating orbital elements semi-major axis (a), eccentricity (e), orbital inclination (i), right ascension of ascending node (Ω), argument of perigee (ω) plus mean anomaly (M), apogee altitude (Ha) and perigee altitude (Hp). From these tables, it is clear that the results obtained from the present KS regular equations match very well with HPOP results.

Table 4: Comparison Between KS and HPOP Values of Orbital Elements (Case A)

Days	Approach	a (km)	e	i (deg)	Ω (deg)	ω + M (deg)	Ha (km)	Hp (km)
1	HPOP	46477.2938	0.000989	51.5990	349.2070	288.4800	40145.1180	40053.1956
	KS	46477.3006	0.000989	51.5994	349.2072	288.4793	40145.1070	40053.2202
5	HPOP	46477.0159	0.000970	51.5990	349.1840	96.1570	40143.9701	40053.7877
	KS	46477.0127	0.000969	51.5993	349.1837	96.1567	40143.9286	40053.8229
10	HPOP	46478.2539	0.000945	51.6000	349.1550	215.7530	40144.0239	40056.2099
	KS	46478.2674	0.000941	51.5999	349.1547	215.7540	40143.8881	40056.3727
15	HPOP	46478.5823	0.000932	51.6000	349.1250	335.3480	40143.7663	40057.1243
	KS	46478.5863	0.000926	51.6000	349.1241	335.3486	40143.4966	40057.4020
20	HPOP	46476.9937	0.000899	51.5990	349.0950	94.9440	40140.6571	40057.0563
	KS	46476.9983	0.000891	51.5993	349.0948	94.9456	40140.2507	40057.4720
25	HPOP	46478.2936	0.000871	51.6000	349.0660	214.5410	40140.6485	40059.6647
	KS	46478.2876	0.000860	51.5999	349.0658	214.5431	40140.1088	40060.1924
30	HPOP	46478.5498	0.000856	51.6000	349.0360	334.1350	40140.2041	40060.6215
	KS	46478.5245	0.000844	51.6000	349.0352	334.1385	40139.6181	40061.1570

Table 5: Comparison Between KS and HPOP Values of Orbital Elements (Case B)

Days	Approach	a (km)	e	i (deg)	Ω (deg)	ω + M (deg)	Ha (km)	Hp (km)
1	HPOP	46474.3036	0.499972	51.5980	349.2040	321.8950	63332.0053	16860.3278
	KS	46474.2650	0.499971	51.5985	349.2041	321.8958	63331.9117	16860.3442
5	HPOP	46475.3509	0.499948	51.5990	349.1570	129.7060	63332.4777	16861.9501
	KS	46475.3026	0.499947	51.5990	349.1569	129.7104	63332.3366	16861.9945
10	HPOP	46475.0006	0.499934	51.5980	349.1080	249.4640	63331.2750	16862.4523
	KS	46474.9532	0.499930	51.5982	349.1077	249.4722	63331.0527	16862.5796
15	HPOP	46478.2470	0.499937	51.5990	349.0550	369.2220	63336.3059	16863.9142
	KS	46478.2936	0.499932	51.5987	349.0542	369.2339	63336.1613	16864.1519
20	HPOP	46475.3205	0.499887	51.5980	348.9990	128.9860	63329.5692	16864.7977
	KS	46475.2428	0.499879	51.5978	348.9981	129.0026	63329.0827	16865.1290
25	HPOP	46475.0065	0.499870	51.5970	348.9500	248.7430	63328.3118	16865.4272
	KS	46474.9625	0.499860	51.5971	348.9491	248.7658	63327.8056	16865.8453
30	HPOP	46477.9257	0.499867	51.5970	348.8960	368.5010	63332.5850	16866.9924
	KS	46478.0107	0.499859	51.5977	348.8957	368.5281	63332.3078	16867.4396

Table 6: Comparison Between KS and HPOP Values of Orbital elements (Case C)

Days	Approach	a (km)	e	i (deg)	Ω (deg)	ω + M (deg)	Ha (km)	Hp (km)
1	HPOP	46471.2600	0.599956	51.5980	349.2010	326.5090	67973.8371	12212.4086
	KS	46471.1700	0.599955	51.5979	349.2011	326.5106	67973.6287	12212.4275
5	HPOP	46472.1300	0.599932	51.5980	349.1360	134.4610	67974.1026	12213.8743
	KS	46472.0200	0.599930	51.5984	349.1356	134.4687	67973.8452	12213.9208
10	HPOP	46471.8600	0.599919	51.5980	349.0690	254.3900	67973.0708	12214.3715
	KS	46471.7400	0.599915	51.5975	349.0683	254.4061	67972.7242	12214.4896
15	HPOP	46478.2000	0.599942	51.5980	348.9950	374.3200	67984.2874	12215.8322
	KS	46478.2700	0.599938	51.5983	348.9950	374.3440	67984.2271	12216.0474
20	HPOP	46472.0900	0.599875	51.5970	348.9180	134.2600	67971.4096	12216.5058
	KS	46471.9600	0.599867	51.5969	348.9177	134.2919	67970.8281	12216.8204
25	HPOP	46471.8600	0.599860	51.5960	348.8510	254.1890	67970.3410	12217.1092
	KS	46471.7500	0.599851	51.5960	348.8504	254.2308	67969.7396	12217.4952
30	HPOP	46477.8500	0.599878	51.5970	348.7780	374.1190	67980.7481	12218.6680
	KS	46478.0200	0.599871	51.5970	348.7773	374.1694	67980.6867	12219.0816

Table 7: Comparison Between KS and HPOP Values of Orbital Elements (Case D)

Days	Approach	a (km)	e	i (deg)	Ω (deg)	ω + M (deg)	Ha (km)	Hp (km)
1	HPOP	46462.7958	0.699919	51.5970	349.1950	330.4980	72604.8708	7564.4469
	KS	46462.5248	0.699917	51.5968	349.1947	330.5023	72604.3089	7564.4667
5	HPOP	46463.3951	0.699897	51.5970	349.0910	138.8280	72604.8369	7565.6793
	KS	46463.1153	0.699894	51.5973	349.0906	138.8461	72604.2307	7565.7259
10	HPOP	46463.1981	0.699885	51.5960	348.9850	259.2220	72603.9530	7566.1692
	KS	46462.893	0.699881	51.5962	348.9849	259.2584	72603.2366	7566.2754
15	HPOP	46483.3502	0.699984	51.5980	348.8700	379.6180	72642.8093	7567.6171
	KS	46483.6881	0.699982	51.5982	348.8698	379.6727	72643.3046	7567.7975
20	HPOP	46463.3612	0.699846	51.5950	348.7480	140.0280	72602.4284	7568.0200
	KS	46463.0497	0.699838	51.5953	348.7477	140.1022	72601.5112	7568.3141
25	HPOP	46463.2053	0.699833	51.5940	348.6430	260.4220	72601.5387	7568.5979
	KS	46462.9154	0.699823	51.5943	348.6421	260.5161	72600.6028	7568.9541
30	HPOP	46488.0426	0.699962	51.5960	348.5280	380.8190	72649.7865	7570.0247
	KS	46488.8807	0.699960	51.5968	348.5270	380.9316	72651.1039	7570.3834

The following figures 1 to 12, shows the differences between the KS and HPOP values of the important orbital parameters semi-major axis, eccentricity and inclination, which define size, shape and orientation of the orbit for Cases A, B, C and D.

CASE A

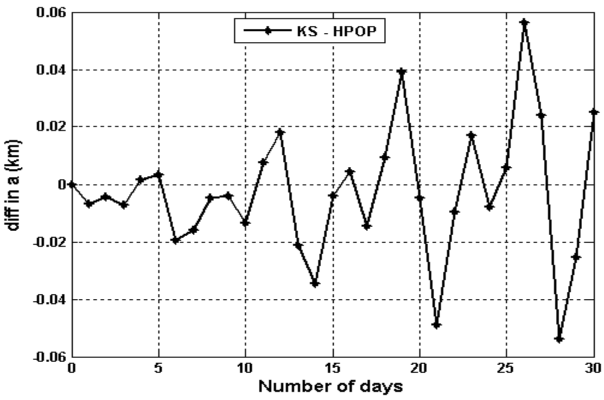

Fig. 1: Difference Between KS and HPOP in Semi-Major Axis.

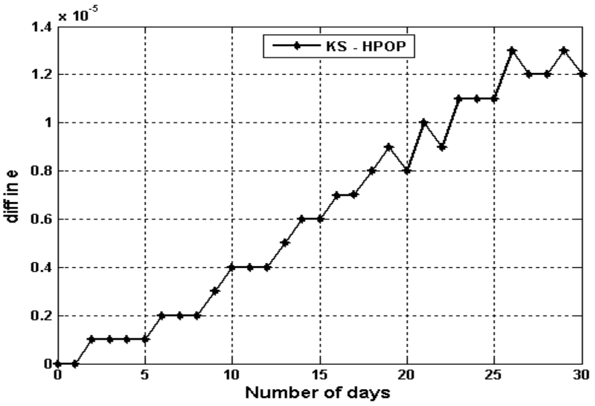

Fig. 2: Difference Between KS and HPOP in Eccentricity.

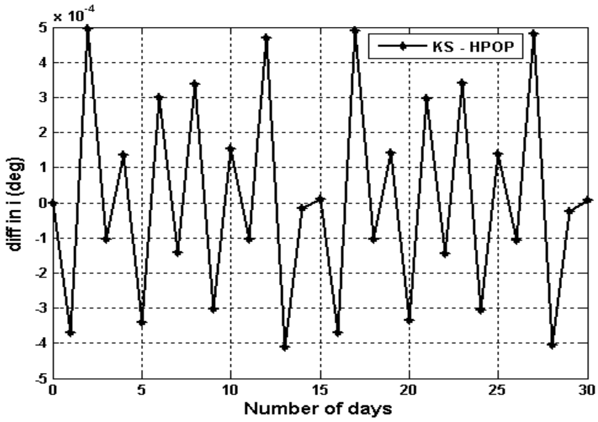

Fig. 3: Difference Between KS and HPOP in Inclination.

CASE B

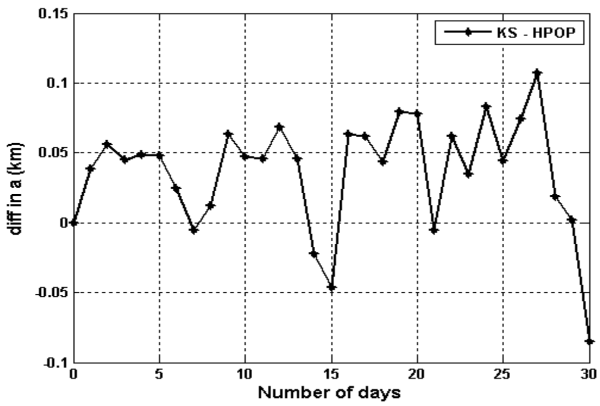

Fig. 4: Difference Between KS and HPOP in Semi-Major Axis.

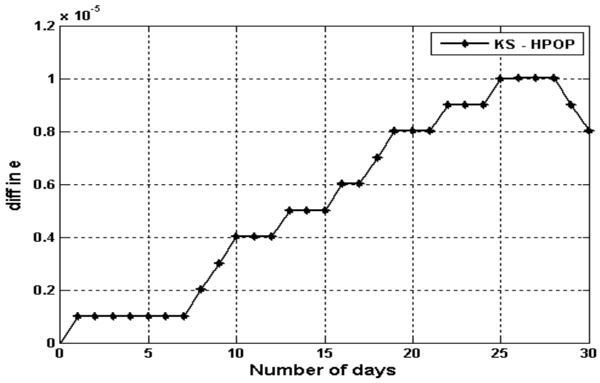

Fig. 5: Difference between KS and HPOP in Eccentricity.

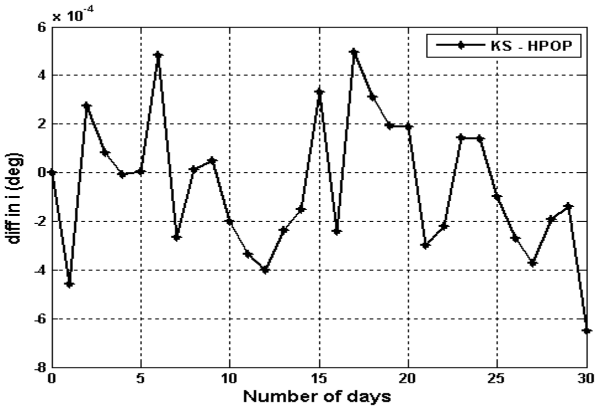

Fig. 6: Difference between KS and HPOP in Inclination.

CASE C

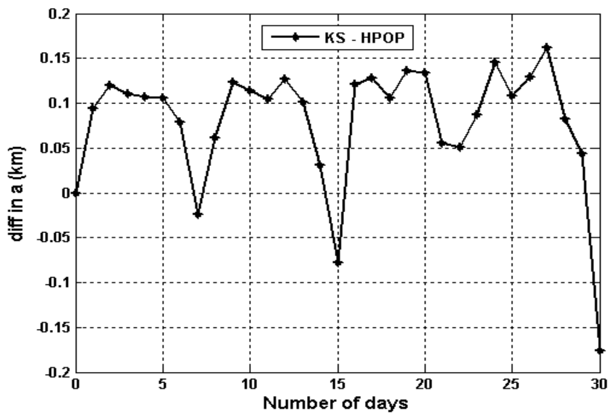

Fig. 7: Difference Between KS and HPOP in Semi-Major Axis.

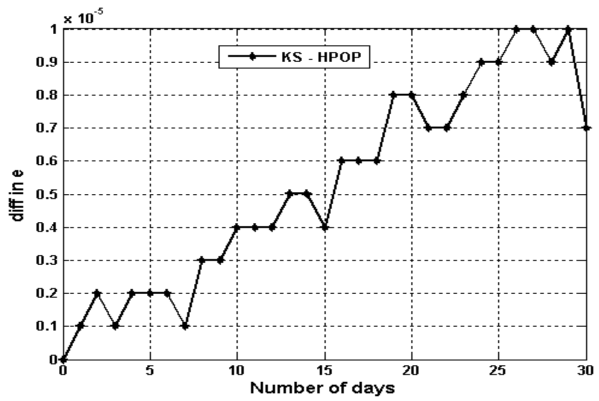

Fig. 8: Difference Between KS and HPOP in Eccentricity.

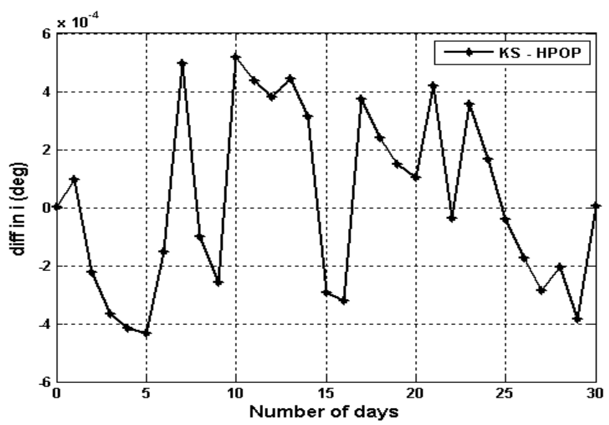

Fig. 9: Difference between KS and HPOP in Inclination.

CASE D

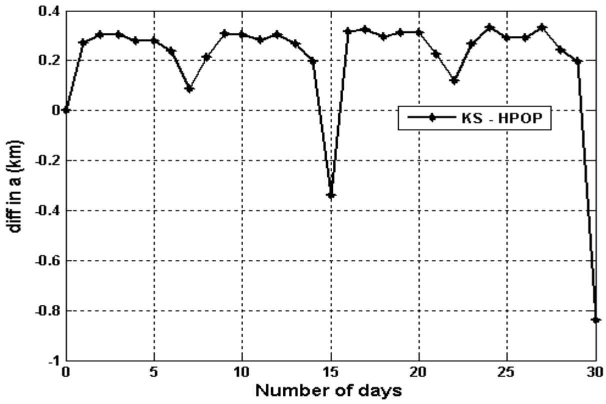

Fig. 10: Difference between KS and HPOP in Semi-Major Axis.

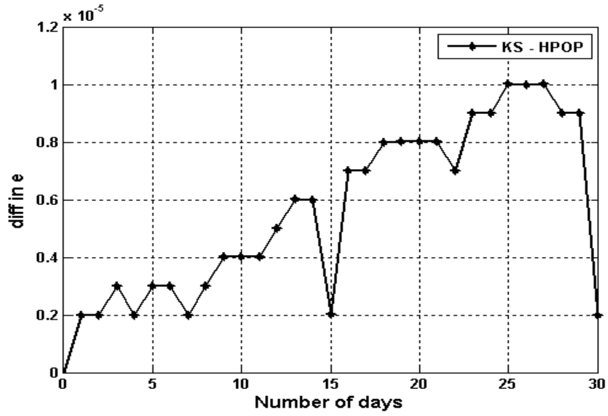

Fig. 11: Difference between KS and HPOP in Eccentricity.

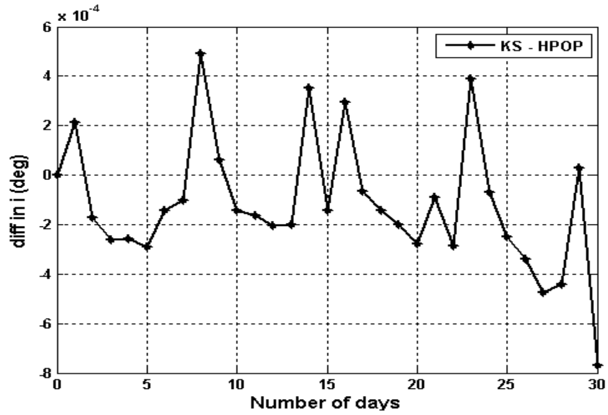

Fig. 12: Difference between KS and HPOP in Inclination.

Among all the test cases, the maximum difference between KS and HPOP theories absolute values of for 30 days in semi-major axis, eccentricity and inclination are found to be 0.838 km, 0.000015 and 0.000872 deg, respectively. Hence, from these all figures also, it is evident that the results obtained from KS theory are very well compared with HPOP results. The comparison shows that the KS method provides one of the best techniques for orbit prediction

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