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Research paper

On merging of resonant periodic orbits 4:3; 3:2 and 2:1 in Sun-Jupiter photo gravitational restricted three-body problem

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Abstract

We explore the merging of resonant periodic orbits in the frame work of planar circular restricted three body problem with the help of Poincaré surface of section. We have studied the effect of solar radiation pressure on 4:3, 3:2 and 2:1 periodic orbits. It is found that radiation pressure helps in merging these orbits (4:3 and 3.2 into 1:1 resonance and 2:1 into 1:1 resonance). At the time of merging these orbits become near-circular. The period and size of these orbits reduce with the increase in radiation pressure.

Keywords: *Restricted Three Body Problem; First-Order; Interior Resonance; Poincaré Surface of Section; Solar Radiation Pressure.*

1. Introduction

The restricted three- body problem (RTBP) is one of the most widely studied in celestial mechanics. Its application span the solar system dynamics, the lunar theory, the stellar dynamics etc. Beyond the classical technique of qualitative analysis, the modern techniques are used to explore the regions of the phase space that contain sensitive dependence of initial conditions. Jefferys [1] and Smith [2] made extensive studies on these regions by exploring large portion of the phase plane with Poincaré surface of section (PSS). As the classical model of the restricted three body problem (RTBP) does not account for some of the perturbing forces such as radiation pressure, oblateness and variations of the masses of the primaries, we wish to study the effect of these perturbing forces on the interior resonance periodic orbits. Poynting [3] and Robertson [4] showed that the effect produced by the radiation force on the dynamics of small body depend on its particular geometry, physical and physicochemical characteristics. Radzievskii [5] proposed a simplified theory and since then some of the notable research in the photo-gravitational restricted three body problem are by Perezhogin [6], Bhatnagar and Chawla [7] Schuerman [8], Simmons et al. [9] Roman [10], Kushvah and Ishwar [11] and Das et al.[12], Sharma [13, 22] included the oblateness of the more massive and small primary, respectively, in the photo-gravitational problem and studied the periodic solutions around the Lagrangian points. Further, Dutt and Sharma [14] studied the effect of the solar radiation pressure on the periodic orbits in the Sun-Mars system.
The study of resonance plays an important role in understanding the general properties of different dynamical systems.The earlier important works on the resonance in the dynamic evolution of the solar system are by Roy and Ovenden [15]. Useful review of the theory of resonance have been given by Greenberg [16] and Peale [17] .A detailed discussion on the theory of resonance is presented in Murray and Dermot [18]. Quarle et al. [19] have identify and classify the mean-motion resonance for the coplanar CRTBP for different mass ratio and recently Wang and Malhotra [20] study the high eccentricity regime of mean motion resonance in the CRTBP.
In this paper we study the restricted three body problem when the more massive primary is a source of radiation with its equatorial plane coincident with the plane of motion. The more massive primary is the Sun and smaller primary is the Jupiter. The method of Poincaré surface of section (PSS) is used to describe the nature and locations of periodic orbits.
The periodic orbits around the Sun with 4:3, 3:2 and 2:1 first-order interior resonances in the framework of Sun-Jupiter photo gravitational restricted three-body problem (PRTBP) are studied. Asteroids residing in the first-order mean motion resonances with Jupiter hold important information about the processes that set the final architecture of giant planets Brož and Vokrouhlicky [21]. It is known that the population of the asteroids exist in the Jovian first-order mean motion resonances 2:1(Hecuba-gap group), 3:2 (Hilda group) and 4:3(Thule group).The authors main results were an update of the observed 2:1, 3:2 and 4:3 resonant populations; discovery of two new objects in the 4:3 resonance and description of two asteroid families located inside the 3:2 group. In this paper, we have carried out a detailed study in PRTBP to find the effect of solar radiation pressure (ε) and Jacobian constant C on these obits in the Sun-Jupiter RTBP. It is found that the radiation pressure helps in merging these orbits (4:3 and 3.2) into 1:1 resonance and 2:1 into 1:1 resonance.

2. Planar circular photo gravitational restricted three-body problem

The motion of the third body is simulated by numerically integrating the planer restricted three body problem. This defines the motion of the third body on the plane of motion of the Sun and Jupiter. In the dimensionless synodic coordinate system with origin of the system positioned on the center of mass of the primaries, considering the more massive primary at the location (-μ,0) and smaller primary is at (1-μ,0) with its equatorial plane coincident with the plane of motion. Where $\mu = m_2/(m_1+m_2) \leq \frac{1}{2}$, (= 0.0009537284) is the mass ratio and m_1 and m_2 are the masses of the Sun and Jupiter respectively.
The effect of radiation pressure of a source can be expressed by a mass reduction factor q =1 - ε, where the radiation coefficient ε is the ratio of the force F_p which is caused by radiation to the force F_g which results from gravitation, i.e., $\varepsilon = F_p/F_g$. q is expressed in terms of particle radius 'a', density 'δ' and radiation pressure efficiency 'χ' (in CGS system) as

$$q = 1 - \frac{5.6 \times 10^{-5}}{a\delta}\chi \tag{2.1}$$

Knowing the mass and the luminosity of the radiating body, ε can be found for any given radius and density. Solar radiation pressure force F_p changes with distance by the same law of gravitational attraction force $F_g$and acts opposite to it. Thus, Sun's resulting force acting on the particle is Sharma [22]; Kalvouridis et al. [23]

$$F = F_g - F_p = (1 - F_p/F_g)\, F_g = (q)F_g \tag{2.2}$$

For q =1, there is no radiation effect, and for $0 < q \leq 1$, gravitational force exceeds radiation and we consider this case for our detailed study.
The planar equations of motion of the third body are (Bhatnagar and Chawla [24])

$$\ddot{x} - 2\dot{y} = \frac{\partial\Omega}{\partial x}, \tag{2.3}$$

$$\ddot{y} + 2\,\dot{x} = \frac{\partial\Omega}{\partial y}, \tag{2.4}$$

where

$$\Omega = \frac{1}{2}[(1-\mu)r_1{}^2 + \mu r_2^2] + \frac{q\,(1-\mu)}{r_1} + \frac{\mu}{r_2}, \tag{2.5}$$

$$r_1^2 = (x-\mu)^2 + y^2,$$

$$r_2^2 = (x+1-\mu)^2 + y^2.$$

The Jacobi integral is

$$\dot{x}^2 + \dot{y}^2 = 2\,\Omega - C. \tag{2.6}$$

3. Numerical results

The families of periodic orbits around the Sun in the Sun Jupiter system are studied using the Poincaré Surface of section method. We have constructed the PSS in the x, $\dot{x}$ plane. By defining the plane, say y = 0, in resulting three-dimensional space, the values of x and $\dot{x}$are plotted every time the particle has y = 0, whenever trajectory intersects the plane in a particular direction, say $\dot{y} > 0$. Poincare surface of section technique is good at determining the regular or chaotic nature of the trajectory. If there are smooth, well defined islands, then the trajectory is likely to be regular and the islands correspond to oscillation around a period orbit. As the curve shrink down to a point, the point represents a periodic orbit as par Kolmogorov- Arnold –Moser (KAM) theory. Any fuzzy distribution of points in the surface of section implies that the trajectory is chaotic.
The starting conditions for numerical integration were chosen as follows: for each value of Jacobian constant C, the value of x was selected so that $y = \dot{x} = 0$ and $\dot{y} > 0$. In order to generate the Poincaré surface of section, the equation of motion (2.3) and (2.4) are integrated using fourth-order Runge-Kutta-Gill method with integration step size Δt of 0.0005. Experimentation for the distance interval between two consecutive starting conditions was done and integration has been performed over different starting conditions in the range of Jacobi constant C between 0.5 and 2.9.

4. Resonance locations

We have generated the trajectories for different initial conditions for the Sun-Jupiter system at various locations for C = 2.95 and q = 1 given in Fig.1. The trajectory of the particle with starting value$x_0 = 0.3053$, $y_0 = 0.0$, $\dot{x}_0 = 0.0$ and $\dot{y}_0 > 0.0$ is determined from (2.6) and this trajectory is close to 2:1 resonance. The trajectory for C = 2.95 at x = 0.4990 is close to 3:2 interior resonance.It is observed that the trajectory for C = 2.95 at x = 0.5885 is close to 4:3 interior resonance.
To study the effect of radiation pressure of Sun, we consider the orbit having resonance 2:1 as shown in Fig. 2 for C = 2.95 and ε = 1 - q = 1- 0.99 = 0.01 at x = 0.3178 . The resonance of this periodic orbit is observed to become 1:1. The transformation of this periodic orbit with 2:1 interior resonance is shown in Figures 4 to 6 by increasing the radiation pressure ε from 0.01 to 0.11. It is also observed that the period of time of these orbits decreases with the increase in radiation pressure ε. Table 1 provides the initial locations of these orbits for Ɛ = 0.01 to 0.11. It may be noted that with the increase in radiation pressure ε, the orbits move towards Jupiter. Table 1 also contains the initial locations of 3:2 and 4:3 resonant orbits. These orbits also move towards Jupiter with increase in radiation pressure.

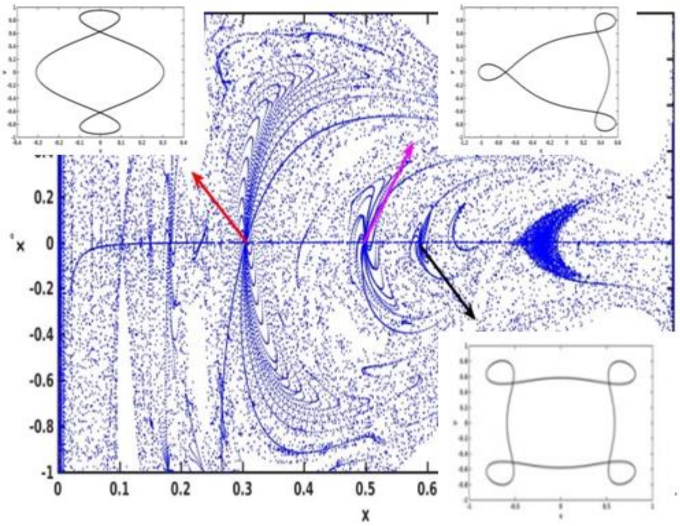

Fig. 1: PSS for Jacobi Constant C=2.95 and ε = 0.

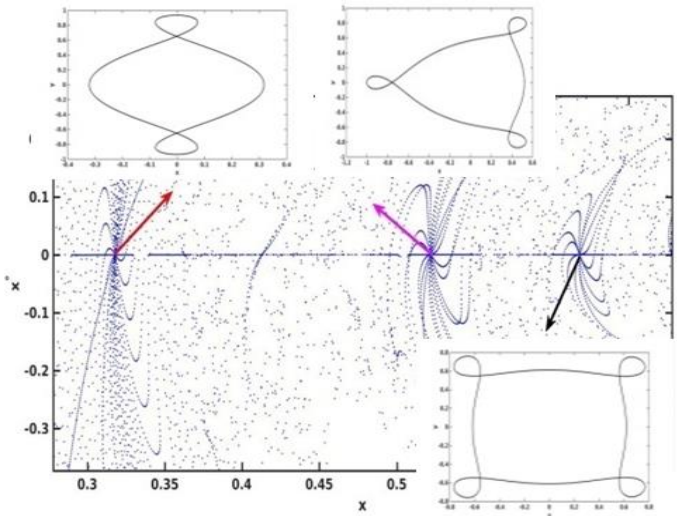

Fig. 2: PSS for Jacobi Constant C=2.95, ε =0.01.

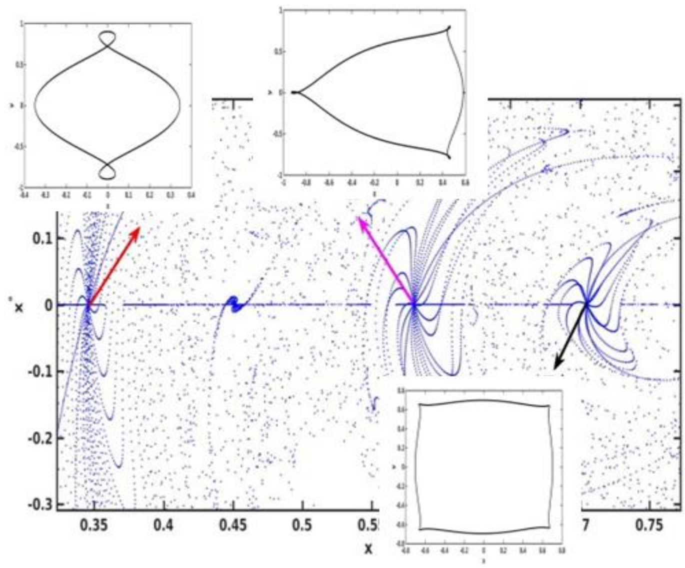

Fig. 3: PSS for Jacobi Constant C=2.95 and ε =0.03.

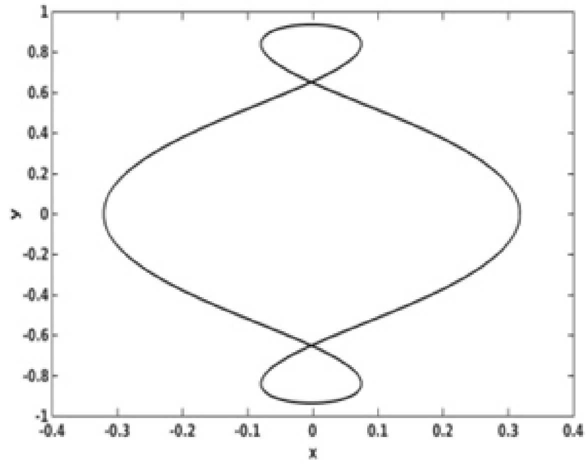

Fig. 4: C=2.95, ε = 0.01, x= 0.3178.

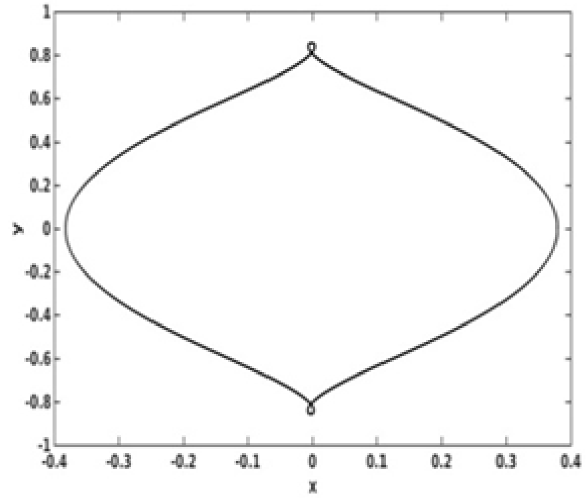

Fig. 5: C=2.95, ε =0.05, x= 0.3794.

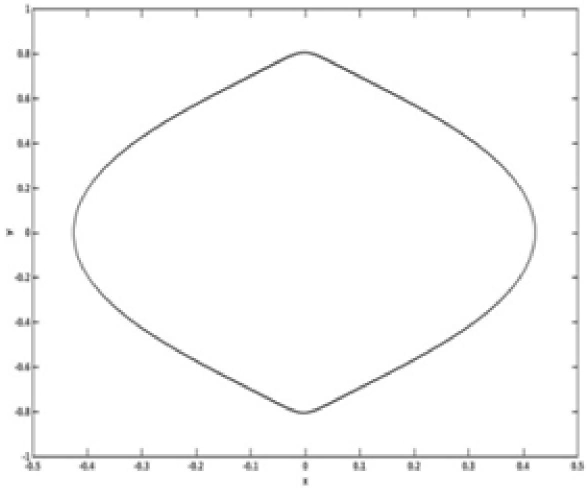

Fig. 6: C=2.95, ε =0.07, x= 0.4219.

Table 1: Initial Location of the Orbits

q	ε	Resonance		
		2:1	3:2	4:3
1	0	0.3052	0.4990	0.5885
0.99	0.01	0.3182	0.5221	0.6184
0.95	0.05	0.3794	0.5830	0.6844
0.93	0.07	0.4219	0.5841	0.6922

5. Evolution of Sun-centered orbits with radiation pressure

The Poincaré surface of section (PSS) method is used to find 2:1, 3:2 and 4:3 interior resonance periodic orbits around the Sun under the effect of the radiation pressure. The distance between the two consecutive starting conditions Δx and time step Δt in the numerical integration were suitably selected. PSS were generated for Jacobi constant C = 2.9 without the radiation pressure and presented in Figure 7. These orbits starting between x = 0.2743 and x = 0.4318 are shown in Fig. 7.They also shift towards Jupiter with increase in radiation pressure (ε). Their shape changes gradually to elliptic orbits. It is observed that with increase in ε, 4:3 resonant periodic orbit merges with 3:2 resonant periodic orbit at x = 0.6972 and the resonance of the merged periodic orbit becomes 1:1. This orbit shifts towards Sun with further increase in radiation pressure. As ε increases from 0.09 to 0.13, the periodic orbit with 2:1 interior resonance shift towards Jupiter and merges with the previous one at x = 0.5711 and become near 1:1 resonance. Table 2 provides a comparison of the time of the periodic orbits for ε = 0 to 0.13.

It may be seen from Figures 8 and 9 that as ε increases from 0.07 to 0.13, the orbit becomes gradually circular and smaller in size.

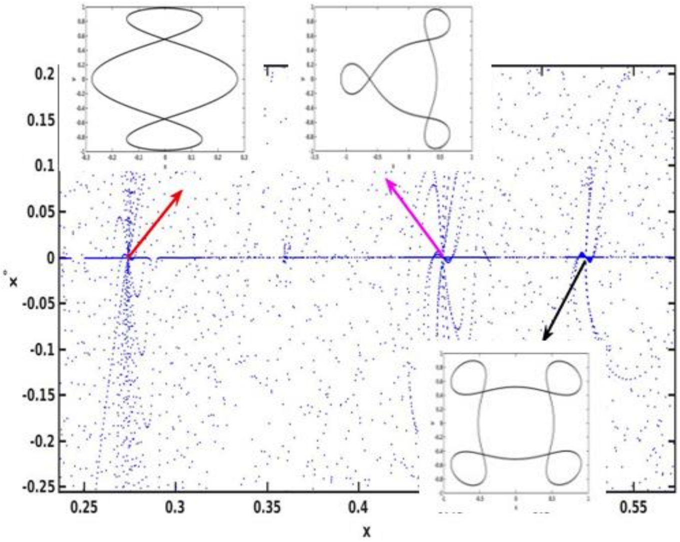

Fig. 7: PSS for Jacobi Constant C=2.9 and ε = 0.

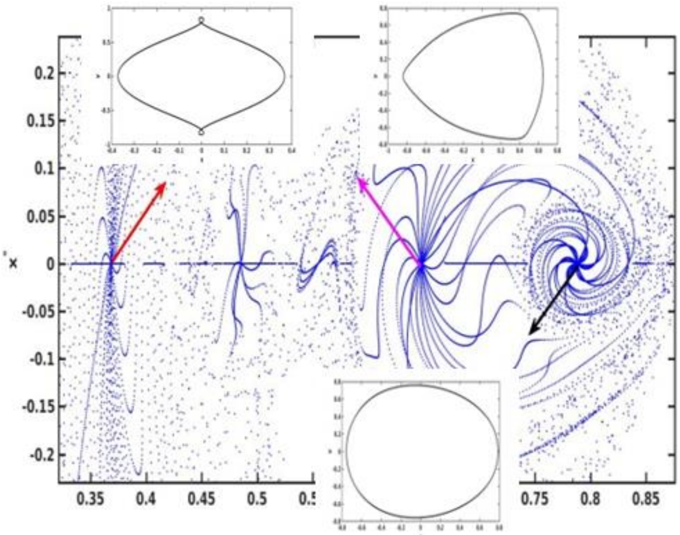

Fig. 8: PSS for Jacobi Constant C=2.9 and ε = 0.07.

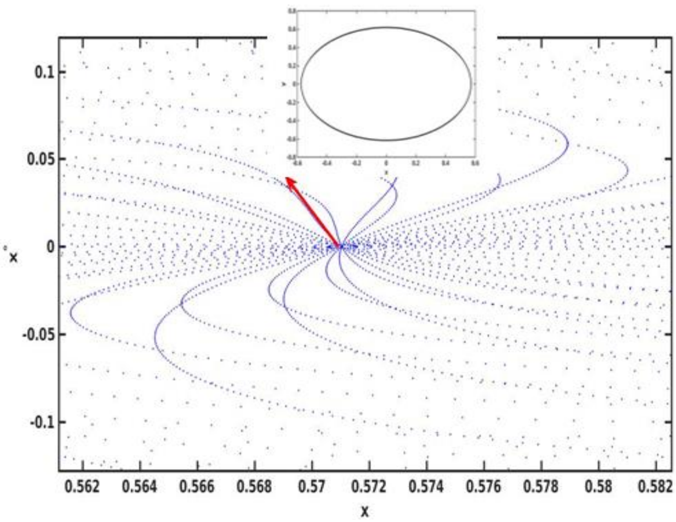

Fig. 9: PSS for Jacobi Constant C=2.9 and ε = 0.13.

Table 2: Initial Location and Time Period of the Orbit

Ɛ	Resonance 2:1 Location	Period	3:2 Location	Period	4:3 Location	Period
0	0.2743	6.2789	0.4460	12.563	0.4318	18.85
0.01	0.2849	6.2776	0.4630	12.545	0.4450	18.85
0.03	0.3085	6.2768	0.4505	12.543	0.4930	18.82
0.05	0.3364	6.2761	0.5606	12.542	0.5589	18.78
0.07	0.3692	6.2750	0.6481	12.540	0.6690	14.88
0.09	0.4098	6.2741	0.6972	9.7437	First Merge	
0.11	0.5417	6.2655	0.6472	7.4722		
0.13	0.5711	6.088	Second Merge			

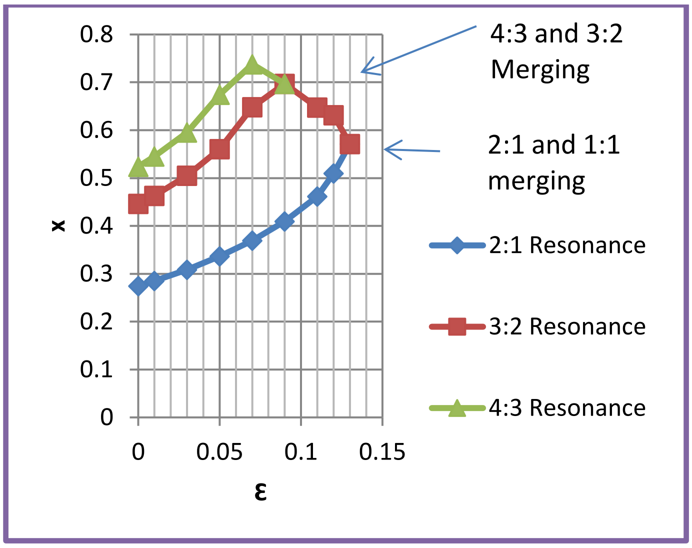

Fig. 10: Merging Locations for C=2.9.

6. Evolution of periodic orbit in Sun-Uranus system

A PSS is generated for Sun-Uranus system whose mass ratio less the mass ratio of Sun-Jupiter for C=2.9. In this case 4:3 resonant periodic orbit merges with 3:2 resonant periodic orbit at x = 0.6949 and the resonance of the merged periodic orbit becomes 1:1. This orbit shifts towards Sun with increase in radiation pressure. As ε increases from 0.09 to 0.13, the periodic orbit with 2:1 interior resonance shift towards Uranus and merges with the previous one at x = 0.5946 and become with near 1:1 resonance. Table 3 gives the location of periodic orbit for Ɛ = 0.0 to 0.13.

Table 3: Initial Location and Time Period of the Orbits

Ɛ	Resonance 2:1 Location	Period	3:2 Location	Period	4:3 Location	Period
0	0.2756	6.281	0.4464	12.561	0.5207	18.84
0.03	0.2310	6.280	0.5059	12.560	0.5940	18.82
0.05	0.3374	6.281	0.5606	12.550	0.6705	18.82
0.07	0.3702	6.280	0.6513	12.55	0.7712	14.73
0.09	0.4108	6.271	0.6949	9.700	First Merge	
0.13	0.5946	6.200	Second Merge			

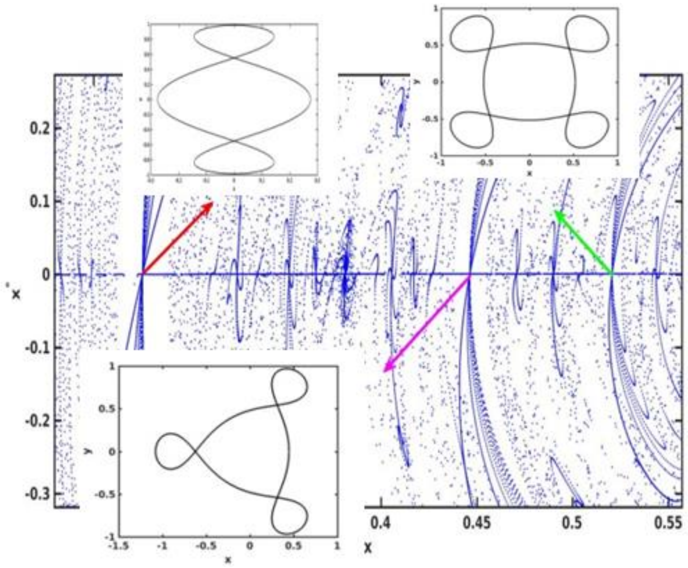

Fig. 11: PSS for Jacobi Constant C=2.9 and ε =0.

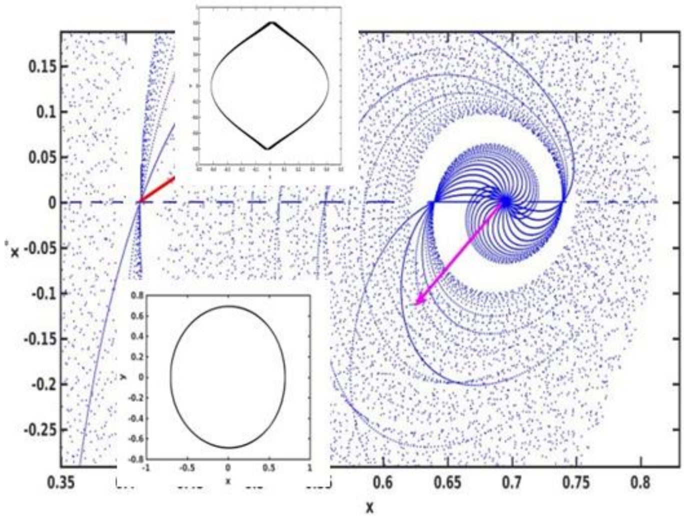

Fig. 12: PSS for Jacobi Constant C=2.9 and ε =0.09.

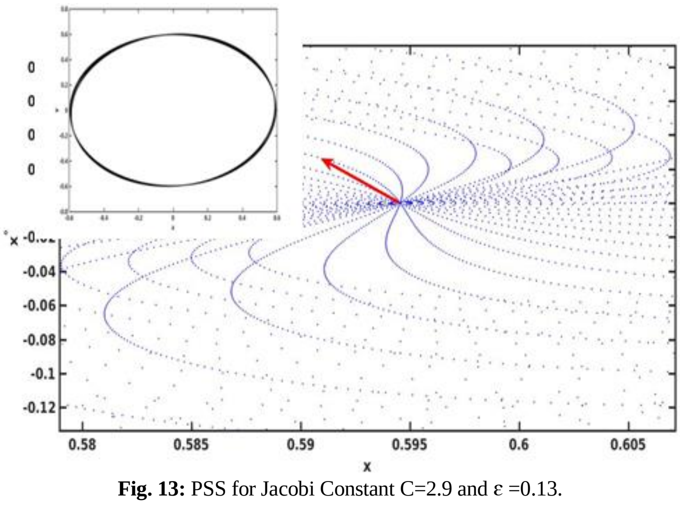

Fig. 13: PSS for Jacobi Constant C=2.9 and ε =0.13.

It is interesting to note that merging of 3:4 and 4:3 periodic orbits are found at almost the same location as obtained in the Sun-Jupiter system for C = 2.9. However, the merging of 2:1 and 1:1 occur at different locations. It is noticed that with decrease in mass ratio, the merged periodic orbit 1:1 moves towards the more massive primary.

7. Conclusions

The merging of resonant periodic orbits with 4:3, 3:2 and 2:1 first-order interior resonances into 1:1 resonance around the Sun in the Sun-Jupiter and Sun- Uranus systems in the frame work of planar circular photo gravitational restricted three-body problem are found with the help of Poincaré surface of section. The solar radiation pressure plays key role in generating these merging. At the time of merging, these orbits become near-circular. The period and size of these orbits reduce with the increase in the radiation pressure.

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