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Research paper

ON An Infra-α-Open Sets

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Abstract

In this paper, we define a new class of set in general topology called an infra-α-open set and we investigate fundamental properties by using this new class. The relation between infra-α-open set and other topological sets are studied. Moreover, In the light of this new definition, we also define some generalization of continuous mappings and discuss the relations between these new classes of mappings and other continuous mappings. Basic properties of these new mappings are studied and we apply these new classes to give characterization of connected space.

Keywords: *infra-α-open set; infra-α-closed set; infra-α-continuous mapping; infra-α-open mapping; infra-α-closed mapping; infra-α- irresolute; infra-α-connected.*

1. Introduction

In 2013, Missier and Rodrigo introduced new class of set in general topology called a α-open (supra-α-open) set but we believe he didn't succeed in defining that set. The definition given has no relation with the other set defined by him and other. Hence, there is no need to rely on it. But our definition in this paper, we think, is in connection with the other set, he has defined it or other and the converse relations between our definition and other are discussed by counter-examples. Moreover, new classes of mappings are introduced by using this definition and interesting results and basic properties are studied and investigated.

Throughout this paper (X,τ) or simply by X denote topological space on which no separation axioms are assumed unless explicitly stated and $f : X \to Y$ means a mapping f from a topological space X to a topological space Y. If u is a set and z is a point in X then $N(z)$, $Int\lambda$, $cl\ u$ and u^c denote respectively, the neighborhood system of z, the interior of u, the closure of u and complement of u.

Now we recall some of the basic definitions and results in topology.

Definition 1. *A set $\lambda \in X$ is called a $\alpha-$open [4] (resp. preopen [3], semi open [2] set if $\lambda \subseteq Int\ (\ cl(\ Int(\ \lambda)))$ (resp. $\lambda \subseteq Int\ (cl(\ \lambda))$, $\lambda \subseteq cl\ (Int(\ \lambda)))$. The collection of all α-open (resp. preopen, semi open) sets of X is denoted as $\alpha O(X)$ (resp. $PO(X)$, $SO(X)$).*

Definition 2. *A set $\mu \in X$ is called:*

- *semi*open (infra-semiopen) [6] if $\eta \subseteq \mu \subseteq Cl^*(\eta)$ where η is an open or equivalently $\mu \subseteq Cl^*(Int(\mu))$.*
- *semi*closed (infra-semiclosed) [6] if $Int^*(\eta) \subseteq \mu \subseteq \eta$ where where η is a closed or equivalently $Int^*(Cl(\mu)) \subseteq \mu$.*
- *pre*open (supra-preopen) [7] if $\mu \subseteq Int^*(Cl(\mu))$ and supra-preclosed (supra-preclosed) if $Cl^*(Int\mu))) \subseteq \mu$.*
- *α^*-open (supra-α-open) [5] if $\mu \subseteq Int^*(Cl(Int^*(\mu)))$ and α^*-closed (supra-α-closed) if $Cl^*(Int(Cl^*(\mu))) \subseteq \mu$.*

The family of all infra-semiopen, infra-semiclosed, supra-preopen, supra-preclosed, supra-α-open and supra-α-closed sets in X will be denoted as $ISO(X)$, $ISC(X)$, $SPO(X)$, $SPC(X)$, $S\alpha - O(X)$ and $S\alpha - C(X)$, respectively.

Definition 3. *A mapping $f : X \to Y$ is said to be:*

- *α-continuous [4] if inverse image of open set in Y, is a α-open set in X.*
- *semi continuous [2] if $f^{-1}(v)$ is a semiopen set in X for each open set v in Y.*
- *precontinuous [3] if $f^{-1}(v)$ is a preopen set in X for each open set v in Y.*

Definition 4. *[1] Let λ any set. Then,*
(i) $Cl^*\lambda = \cap\{\eta : \eta \supseteq\ \lambda,\ \eta$ is a generalized closed set of $X\}$ is called closure*.
(ii) $Int^*\lambda = \cup\{\mu : \mu \subseteq\ \lambda,\ \mu$ is a generalized open set of $X\}$ is called Interior*.

Lemma 1. *[1] Let λ any set. Then,*

- *$\lambda \subseteq Cl^*\lambda \subseteq Cl\lambda$.*
- *$Int\lambda \subseteq Int^*\lambda \subseteq \lambda$.*

2. Infra-α-open Set

Definition 5. *A subset λ of space X is called infra-α-open (infra-α-closed) set if $\lambda \subseteq Int\ Cl^*\ Int\ \lambda$ $(Cl\ Int^*\ Cl\ \lambda \subseteq \lambda)$. The class of all* *infra-α-open (infra-α-closed)sets in X will be denoted as $I\alpha - O(X)$ ($I\alpha - C(X)$).*

Definition 6. *For any set λ, we have,*

- $I\alpha - Cl\ \lambda = \cap\{\mu : \mu \supseteq \lambda,\ \mu$ *is an infra-α-closed set of* $X\}$ *is called an infra-α-closure.*
- $I\alpha - Int\ \lambda = \cup\{\mu : \mu \subseteq \lambda,\ \mu$ *is an infra-α-open set in* $X\}$ *is called an infra-α-Interior.*
- $IsCl\ \lambda = \cap\{\mu : \mu \supseteq \lambda,\ \mu$ *is an infra-semiclosed set of* $X\}$. *is called an infra-semi-closure*
- $IsInt\ \lambda = \cup\{\mu : \mu \subseteq \lambda,\ \mu$ *is an infra-semiopen set in* $X\}$. *is called an infra-semi-interior*

Theorem 1. *A set $\mu \in I\alpha - O(X)$ if and only if there exists an open set λ such that $\lambda \subseteq \mu \subseteq Int\ Cl^*\lambda$.*

Proof. Necessity: If $\mu \in I\alpha - O(X)$, then $\mu \subseteq Int\ Cl^*\ Int\mu$. Put $\lambda = Int\mu$, then λ is an open set and $\lambda \subseteq \mu \subseteq Int\ Cl^*\lambda$.
Sufficiency: Let λ be an open set such that $\lambda \subseteq \mu \subseteq Int\ Cl^*\lambda$, this implies that $Int\ Cl^*\lambda \subseteq Int\ Cl^*\ Int\mu$, then $\mu \subseteq Int\ Cl^*\ Int\mu$.■ .

Theorem 2. *A set $\lambda \in I\alpha - C(X)$ if and only if there exists a closed set μ such that $Cl\ Int^*\ \mu \subseteq \lambda \subseteq \mu$.*

Proof. Necessity: If $\lambda \in I\alpha - C(X)$, then $Cl\ Int^*\ Cl\ \lambda \subseteq \lambda$. Put $\mu = Cl\lambda$, then μ is a closed set and $Cl\ Int^*\ \mu \subseteq \lambda\ \subseteq \mu$.
Sufficiency: Let μ be a closed set such that $Cl\ Int^*\ \mu \subseteq \lambda\ \subseteq \mu$, this implies that $Cl\ Int^*\ Cl\lambda \subseteq Cl\ Int^*\ \mu$, then $Cl\ Int^*\ Cl\ \lambda \subseteq \lambda$.■

Theorem 3. *Let λ be a set of X. Then, the following properties are true:*

(a) $IsInt\lambda = \lambda \cap Cl^*Int\lambda$.
(b) $IsCl\lambda = \lambda \cup Int^*Cl\lambda$.

Proof. (a) We know that $IsInt$ is infra-semiopen,
then $IsInt(\lambda) \subseteq Cl^*Int(IsInt(\lambda)) \subseteq Cl^*Int(\lambda)$.
So, $IsInt(\lambda) \subseteq \lambda \cap Cl^*Int(\lambda) \to (1)$
We have $Int\lambda \subseteq \lambda \cap Cl^*Int(\lambda) \subseteq Cl^*Int(\lambda)$. By Definition 2 $\lambda \cap Cl^*Int(\lambda)$ is an infra-semiopen set and $\lambda \cap Cl^*Int(\lambda) \subseteq \lambda$,
then $\lambda \cap Cl^*Int(\lambda) \subseteq IsInt(\lambda) \to (2)$
From (1) and (2), we get $IsInt\lambda = \lambda \cap Cl^*Int\lambda$.■ .

Corollary 1. *Let λ be a set of X. Then, the following properties are true:*

(a) *If λ is a generalized closed set, then $IsInt\lambda = Cl^*Int\lambda$.*
(b) *If λ is a generalized open set, then $IsCl\lambda = Int^*Cl\lambda$.*

Proof. we will prove(b) and (a) is the same.
We know that, $Int^*\lambda \subseteq Int^*Cl\lambda$ but $Int^*\lambda = \lambda$, this implies that $\lambda \subseteq Int^*Cl\lambda$, then $IsCl\lambda = Int^*Cl\lambda$. ■

Theorem 4. *For any subset λ of a space X, the following implication $(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (iv)$ hold:*

(i) $\lambda \in I\alpha - C(X)$
(ii) $Cl^*\ Int\ \mu \subseteq \lambda \subseteq \mu$; *for closed set* μ;
(iii) $IsInt\ \mu \subseteq \lambda \subseteq \mu$; *for closed set* μ;;
(iv) $IsInt\ cl\ \lambda \subseteq \lambda$

Proof. It is clear from Lemma 1, Definition 5, Theorem 2 and Corollary 1. ■

Theorem 5. *For any subset λ of a space X, the following statements are hold:*

(i) *If* $\lambda \subseteq \mu \subseteq Int\ Cl^*\lambda$ *and* $\lambda \in I\alpha - O(X)$, *then* $\mu \in I\alpha - O(X)$.
(ii) *If* $Cl\ Int^*\lambda \subseteq \mu \subseteq \lambda$ *and* $\lambda \in I\alpha - C(X)$, *then* $\mu \in I\alpha - C(X)$.

Proof.

(i) Let $\lambda \in I\alpha - O(X)$, then there exists η an open set such that $\eta \subseteq \lambda \subseteq Int\ Cl^*\eta$, this implies that $\eta \subseteq \mu$ and $\lambda \subseteq Int\ Cl^*\eta$. Therefore, $Int\ Cl^*\lambda \subseteq Int\ Cl^*\eta$ and $\eta \subseteq \mu \subseteq Int\ Cl^*\eta$, then $\mu \in I\alpha - O(X)$

(ii) Easy to prove by using the same technique of proof (i). ■

Proposition 1. *Let λ and μ be the sets in X and $\lambda \subseteq \mu$. Then, the following statements hold:*

1. $I\alpha - Int(\lambda)$ *is the largest infra-α-open set contained in* λ.
2. $I\alpha - Int\ \lambda \subseteq \lambda$.
3. $I\alpha - Int\ \lambda \subseteq I\alpha - Int\ \mu$.
4. $I\alpha - Int(I\alpha - Int\ \lambda) = I\alpha - Int\ \lambda$.
5. $\lambda \in I\alpha - O(X) \Leftrightarrow I\alpha - Int\ \lambda = \lambda$.

Proposition 2. *Let λ and μ be the sets in X and $\lambda \subseteq \mu$. Then the following statements hold:*

1. $I\alpha - Cl(\lambda)$ *is the smallest infra-α-closed set containing* λ.
2. $\lambda \subseteq I\alpha - Cl(\lambda)$.
3. $I\alpha - Cl\ \lambda \subseteq I\alpha - Cl\ \mu$.
4. $I\alpha - Cl(I\alpha - Cl\ \lambda) = I\alpha - Cl\ \lambda$.
5. $\lambda \in I\alpha - C(X) \Leftrightarrow I\alpha - Cl\ \lambda = \lambda$.

Theorem 6. *Let λ be a set of X. Then, the following properties are true:*

(a) $(I\alpha - Int\ \lambda)^c = I\alpha - Cl\ \lambda$.
(b) $(I\alpha - Cl\ \lambda)^c = I\alpha - Int\ \lambda$.
(c) $I\alpha - Int\ \lambda \subseteq \lambda \cap Int\ Cl^*\ Int\ \lambda$.
(d) $I\alpha - Cl\ \lambda \supseteq \lambda \cup Cl\ Int^*\ Cl\ \lambda$.

Proof. We will prove only (a) and (d).

(a)
$$\begin{aligned}(I\alpha - Int\ \lambda)^c &= \big(\cup\{v : v \subseteq \lambda,\ v\ is\ an\ \ infra-\alpha-open\ set\ of\ X\}\big)^c\\ &= I\alpha - Cl\ \lambda.\end{aligned}$$

(d) Since $\lambda \subseteq I\alpha - Cl\ \lambda$ and $I\alpha - Cl\ \lambda$ is an infra-α-closed set. Hence, $Cl\ Int^*\ Cl(I\alpha - Cl\ \lambda) \subseteq I\alpha - Cl\ \lambda$. Then, $\alpha^*Cl\ \lambda \supseteq \lambda \cup Cl\ Int^*\ Cl\ \lambda$.■

Corollary 2. *Let λ be a set of X. Then, the following properties are true:*

(a) *If λ is an open set, then $I\alpha - Int\lambda \subseteq Int\ Cl^*\ Int\ \lambda$.*
(b) *If λ is a closed set, then $I\alpha - Cl\lambda \supseteq Cl\ Int^*\ Cl\ \lambda$.*

Theorem 7.

(a) *The arbitrary union of infra-α-open set is an infra-α-open set.*
(b) *The arbitrary intersection of infra-α-closed set is an infra-α-closed set.*

Proof.

(a) Let $\{\lambda_i\}$ be family of infra-α-open set. Then, for each i, $\lambda_i \subseteq Int\ Cl^*\ Int\ \lambda_i$ and $\cup\lambda_i \subseteq \cup(Int\ Cl^*\ Int\ \lambda_i) \subseteq Int\ Cl^*\ Int(\cup\lambda_i)$. Hence $\cup\lambda_i$ is an infra-α-open set.
(b) Obvious.■

Theorem 8. *Let λ be a set in X. Then,*
$Int^*\lambda \subseteq I\alpha - Int\ \lambda \subseteq \lambda \subseteq I\alpha - Cl\ \lambda \subseteq Cl^*\lambda$.

Proof. We know that $Int^*\lambda \subseteq \lambda$, this implies that $I\alpha - Int(Int^*\lambda) \subseteq I\alpha - Int\ \lambda$. Then, $I\alpha - Int(Int^*\lambda) = Int^*\lambda$ and so, $Int^*\lambda \subseteq I\alpha - Int\ \lambda \to (1)$.
Also, we know that $\lambda \subseteq Cl^*\lambda$, this implies that $I\alpha - Cl\ \lambda \subseteq I\alpha - Cl(Cl^*\lambda)$. Then, $I\alpha - Cl(Cl^*\lambda) = Cl^*\lambda$ and so, $I\alpha - Cl\ \lambda \subseteq Cl^*\lambda \to (2)$.
From (1) and (2), we obtain $Int^*\lambda \subseteq I\alpha - Int\ \lambda \subseteq \lambda \subseteq I\alpha - Cl\ \lambda \subseteq Cl^*\lambda$. ■

Theorem 9. *Let λ be a set of a topological space X. Then the following statements hold:*

(a) *If λ is an infra-α- open (infra-α- closed) set, then λ is a α-open (α-closed) set.*
(b) *If λ is an infra-α- open (infra-α- closed) set, then λ is a α^*-open (supra-α-open) $\big(\alpha^*$-closed (supra-α-closed)$\big)$ set.*

(c) *If λ is an infra-α- open (infra-α- closed) set, then λ is a pre* open(supra-preopen) $\big($pre* closed(supra-preclosed)$\big)$ set.*

(d) *If λ is an infra-α- open (infra-α- closed) set, then λ is a semi* open (infra-semiopen) $\big($semi* closed (infra-semiclosed)$\big)$ set.*

(e) *If λ is an open (closed) set, then λ is an infra-α- open (infra-α- closed) set.*

Proof. It is clear from Definition 1, Definition 2, Definition 5 and basic relations.■
The following "Implication Diagram 1" illustrates the relation of different classes of open sets.

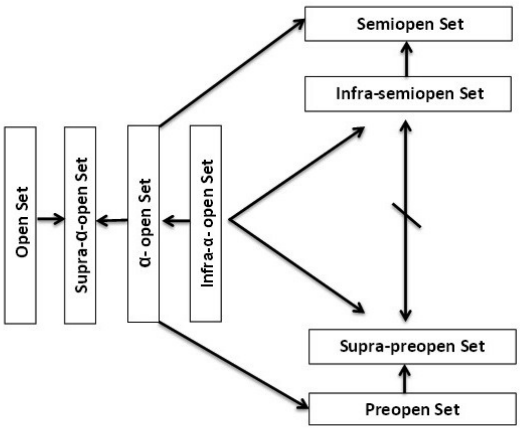

Diagram 1

Remark 1. *The following examples shows that the converses of these relations are not true in general.*

Example 1. *Consider the space (X,τ) where, $X=\{1,2,3,4\}$. Let A_1, A_2, A_3, A_4, A_5, A_6 and A_7 be sets of X defined as:*

$$A_1=\{1\} \quad A_2=\{2\} \quad A_3=\{1,2\} \quad A_4=\{1,2,3\}$$
$$A_5=\{1,4\} \quad A_6=\{2,4\} \quad A_7=\{1,2,4\}$$

Let $\tau=\{\phi,A_1,A_2,A_3,A_4,X\}$. We can see

- *A_5 is an infra-semiopen set which is not a supra-preopen set.*
- *A_6 is an infra-semiopen set but it is not an open set.*
- *A_4 is a supra-preopen set which is not an infra-semiopen set.*
- *A_7 is a supra-preopen set which is not an open set.*
- *A_7 is a supra-preopen set but it is not an infra-α-open set.*
- *A_6 is an infra-semiopen set which is not an infra-α-open set.*

Example 2. *Consider the space $(\mathbb{R},\tau_{co-fininte})$ where, $A=\mathbb{R}-]1,2[$. We can see A is an infra-α-open set which is not an open set.*

Example 3. *Consider the space X where, $X=\{1,2,3\}$. Let A_1 and A_2 be sets of X defined as:*

$$A_1=\{1\} \quad A_2=\{2,3\}$$

Let $\tau=\{\phi,A_1,X\}$. We can see

- *A_2 is a supra-α-open set which is not an infra-α-open set.*
- *A_2 is a supra-α-open set which is not an open set.*

Example 4. *Consider the space (X,τ) where, $X=\{1,2,3\}$. Let A_1 and A_2 be sets of X defined as:*

$$A_1=\{2\} \quad A_2=\{2,3\}$$

Let $\tau=\{\phi,A_1,X\}$. We can see

- *A_2 is a α-open set which is not an infra-α-open set.*
- *A_2 is a semiopen set which is not an infra-α-open set.*
- *A_2 is a preopen set which is not an infra-α-open set.*

3. infra-α- continuous Mapping

Definition 7. *A mapping $f:X\to Y$ is said to be:*
(i) infra-α- continuous if $f^{-1}(\mu)$ is an infra-α- open (infra-α- closed) set in X for each open (closed) set μ in Y.
(ii) infra-α- irresolute if $f^{-1}(\mu)$ is an infra-α- open infra-α- closed) set in X for each an infra-α- open (infra-α- closed) set μ in Y.

Theorem 10. *For a mapping $f:X\to Y$, the following statements are equivalent:*
(i) f is an infra-α- continuous;
(ii) For every $z\in X$ and every open set $\mu\in Y$ such that $f(z)\subseteq\mu$, there exists an infra-α- open set λ in X such that $z\in\lambda$ and $\lambda\subseteq f^{-1}(\mu)$;
(iii) For every $z\in X$ and every open set $\mu\in Y$ such that $f(z)\in\mu$, there exists an infra-α- open set $\lambda\in X$ such that $z\in\lambda$ and $f(\lambda)\subseteq\mu$;
(iv) The inverse image of each closed set in Y is an infra-α- closed;
(v) $Cl\ Int^*\ Cl(f^{-1}(\mu))\subseteq(f^{-1}(Cl\ \mu))$ for each μ in Y;
(vi) $f(Cl\ Int^*\ Cl(\lambda))\subseteq Cl\ f(\lambda)$ for each λ in X.

Proof.
$(i)\Rightarrow(ii)$. Consider $z\in X$ and every open set $\mu\in Y$ such that $f(z)\in\mu$, then there exists an open set $m\in Y$ such that $f(z)\in m\subseteq\mu$. Since f is an infra-α- continuous, $\lambda=f^{-1}(m)$ is an infra-α- open and we have
$z\in f^{-1}(f(z))\subseteq f^{-1}(m)\subseteq f^{-1}(\mu)$ or $z\in\lambda=f^{-1}(m)\subseteq f^{-1}(\mu)$.
$(ii)\Rightarrow(iii)$. Let $z\in X$ and every open set $\mu\in Y$ such that $f(z)\in\mu$, there exists an infra-α- open λ such that $z\in\lambda$ and $\lambda\subseteq f^{-1}(\mu)$. So, we have $z\in\lambda$ and $f(\lambda)\subseteq f^{-1}((f(\mu))\subseteq\mu$.
$(iii)\Rightarrow(i)$. Let $\mu\in Y$ and let us take $z\in f^{-1}(\mu)$. This shows that $f(z)\in f(f^{-1}(\mu))\subseteq\mu$. Since μ is an open set, then there exists an infra-α- open set λ such that $z\in\lambda$ and $f(\lambda)\subseteq\mu$. This shows that $z\in\lambda\subseteq f^{-1}(f(\lambda))\subseteq f^{-1}(\mu)$. It follows that $f^{-1}(\mu)$ is an infra-α- open set in X and hence f is an infra-α- continuous.
$(i)\Rightarrow(iv)$. Suppose μ be a closed in Y. This implies that $Y-\mu$ is open set. Hence $f^{-1}(Y-\mu)$ is an infra-α- open set in X. Thus, $f^{-1}(\mu)$ is an infra-α- closed set in X.
$(iv)\Rightarrow(v)$. Let $\mu\in Y$, then $f^{-1}(Cl\ \mu)$ is an infra-α- closed in X.
$(v)\Rightarrow(vi)$. Let $\lambda\in X$, put $\mu=f(\lambda)$ in v, then
$Cl\ Int^*\ Cl\ (f^{-1}(f(\lambda)))\subseteq f^{-1}(Cl(f(\lambda)))$. So,
$Cl\ Int^*\ Cl\ (\lambda)\ \subseteq f^{-1}(Cl(f(\lambda)))$. This gives
$f(Cl\ Int^*\ Cl\ \ (\lambda))\ \subseteq Cl\ f(\lambda)$.
$(vi)\Rightarrow(i)$. Let $\mu\in Y$. Put $\lambda=f^{-1}(\mu^c)$ in (vi), then $f(Cl\ Int^*\ Cl\ (f^{-1}(\mu^c)))\ \subseteq Clf(f^{-1}(\mu^c))\subseteq Cl(\mu^c)=\mu^c$.
That show that $Cl\ Int^*\ Cl\ (f^{-1}(\mu^c))\ \subseteq\ f^{-1}(\mu^c)$. That is $f^{-1}(\mu^c)$ is an infra-α- closed set in X, so f is an infra-α- continuous mapping. ■

Using the same arguments in Theorem 10 and using Propositions 1 and 2, one can prove the following theorem.

Theorem 11. *For a mapping $f:X\to Y$, the following statements are equivalent:*
(i) f is an infra-α-continuous;
(ii) The inverse image of each closed set in Y is an infra-α-closed;
(iii) $f(I\alpha-Cl(\lambda))\subseteq Cl(f(\lambda))$, for each set $\lambda\in X$;
(iv) $I\alpha-Cl(f^{-1}(\mu))\subseteq f^{-1}(Cl(\mu))$, for each set $\mu\in Y$;
(v) $f^{-1}(Int(\mu))\subseteq I\alpha-Cl(f^{-1}(\mu))$, for each set $\mu\in Y$.

Definition 8. *A mapping $f:X\to Y$ is said to be:*

- *supra-precontinuous if $f^{-1}(v)$ is a supra-preopen (supra-preclosed) set in X for each open (closed) set v in Y.*
- *infra-semicontinuous if $f^{-1}(v)$ is an infra-semiopen (infra-semiclosed) set in X for each open (closed) set v in Y.*
- *supra-α-continuous if $f^{-1}(v)$ is a supra-α-open (supra-α-closed) set in X for each open (closed) set v in Y.*

The "Implication Diagram 2 " illustrates the relations between different classes of continuous mappings.

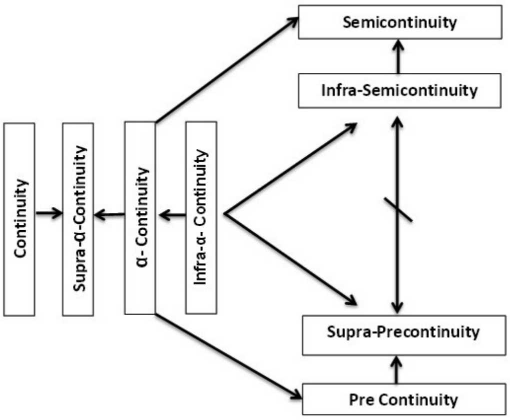

Diagram 2

Remark 2. *The converse of these relations need not be true in general as shown by the following example:*

Example 5. *Consider the space (X,τ_x) and (Y,τ_y) where, $Y = X = \{1,2,3,4\}$. Let A_1, A_2, A_3, A_4, A_5 and A_6 be sets of X defined as:*

$$A_1 = \{1\} \quad A_2 = \{2\} \quad A_3 = \{1,2\} \quad A_4 = \{1,2,3\} \quad A_5 = \{2,3,4\} \quad A_6 = \{2,4\}$$

Let $\tau_x = \{\phi,A_1,A_2,A_3,X\}$, $\tau_y = \{\phi,A_1,A_2,A_3,A_4,X\}$ and $f : (X,\tau_x) \to (Y,\tau_y)$ be an identity mappings. We can see

- *f is a supra-precontinuous mapping but not an infra-semicontinuous mapping.*
- *f is a supra-precontinuous mapping which is not a continuous mapping.*
- *f is a supra-precontinuous mapping which is not an infra-α-continuous mapping.*

Example 6. *Consider the example 5 but $\tau_y = \{\phi,A_1,A_2,A_3,A_6,X\}$. We can see*

- *f is an infra-semicontinuous mapping but it is not a supra-precontinuous mapping.*
- *f is an infra-semicontinuous mapping where as it is not a continuous mapping.*
- *f is an infra-semicontinuous mapping which is not an infra-α-continuous mapping.*

Example 7. *Consider the space (X,τ_x) and (Y,τ_y) where, $Y = X = \{1,2,3,4\}$. Let A_1, A_2 and A_3 be sets of X defined as:*

$$A_1 = \{1\} \quad A_2 = \{2\} \quad A_3 = \{2,3\}$$

Let $\tau_x = \{\phi,A_1,X\}$, $\tau_y = \{\phi,A_1,A_3,X\}$ and $f : (X,\tau_x) \to (Y,\tau_y)$ be an identity mappings. We can see

- *f is a supra-α-continuous mapping but not an infra-α-continuous mapping.*
- *f is a supra-α-continuous mapping which is not a continuous mapping.*

Example 8. *Consider the example 7 but $\tau_x = \{\phi,A_2,X\}$ and $\tau_y = \{\phi,A_2,A_3,X\}$. We can see*

- *f is a α-continuous mapping which is not an infra-α-continuous mapping.*
- *f is a semicontinuous mapping which is not an infra-α-continuous mapping.*
- *f is a precontinuous mapping which is not an infra-α-continuous mapping.*

Definition 9. *A mapping $f : X \to Y$ is said to be an infra-α-open (infra-α-closed) if $f(\lambda)$ is an infra-α-open (infra-α-closed) set in Y for each an open (closed) set λ in X.*

Theorem 12. *For a mapping $f : X \to Y$, the following statements are equivalent:*
(i) f is an infra-α-open;
(ii) $f(Int\lambda) \subseteq I\alpha - Int(f(\lambda))$, for each set $\lambda \in X$;
(iii) $Int(f^{-1}(A)) \subseteq f^{-1}(\alpha Int^*(A))$, for each set $A \in Y$;
(iv) $f^{-1}(I\alpha - Cl(\mu)) \subseteq Cl(f^{-1}(\mu))$, for each set $\mu \in Y$;
(v) $f(Int\lambda) \subseteq Int\ Cl^*\ Int\ (f(\lambda))$, for each set $\lambda \in X$;

Proof.
$(i) \Rightarrow (ii)$. Let f be an infra-α-open mapping and λ be a set in X, $f(Int(\lambda)) \subseteq f(\lambda)$. we have $I\alpha - Int(f(Int(\lambda))) \subseteq I\alpha - Int f(\lambda)$. Then, $f(Int(\lambda)) \subseteq I\alpha - Int f(\lambda)$.
$(ii) \Rightarrow (iii)$. Suppose $\mu \in Y$, then $f^{-1}(\mu)$ be a set in X. We put $f^{-1}(\mu) = \lambda$ in (ii), we get $f(Int(f^{-1}(\mu))) \subseteq I\alpha - Int(f(f^{-1}(\mu))) \subseteq I\alpha - Int(\mu)$. Then, $Int(f^{-1}(\mu)) \subseteq f^{-1}(I\alpha - Int(\mu))$.
$(iii) \Rightarrow (iv)$. Let $A \in Y$, then A^c also is a set in Y. In (iii) we put $A^c = A$, then we get $Int(f^{-1}(A^c)) \subseteq f^{-1}(I\alpha - Int(A^c))$. Then, $(Cl(f^{-1}(A)))^c \subseteq (f^{-1}(I\alpha - Cl(A)))^c$. Hence, $f^{-1}(I\alpha - Cl(A)) \subseteq Cl(f^{-1}(A))$.
$(iv) \Rightarrow (v)$. Let us consider λ be a set in X, then $(f(\lambda))^c$ is a set in Y. Using (iv), we get $f^{-1}(I\alpha - Cl((f(\lambda))^c)) \subseteq Cl(f^{-1}((f(\lambda))^c))$. This implies that $(f^{-1}(I\alpha - Int(f(\lambda))))^c \subseteq (Int(f^{-1}(f(\lambda))))^c$, then $Int(\lambda) \subseteq f^{-1}(I\alpha - Int(f(\lambda)))$ and $f(Int(\lambda)) \subseteq I\alpha - Int(f(\lambda)) \subseteq Int\ Cl^*\ Int\ (I\alpha - Int(f(\lambda)))$.
Hence, $f(Int\lambda) \subseteq Int\ Cl^*\ Int\ (f(\lambda))$.
$(v) \Rightarrow (i)$. Let λ be an open set in X.
By using (v), we have $f(\lambda) \subseteq Int\ Cl^*\ Int\ (f(\lambda))$, then f is an infra-α-open mapping..■

Corollary 3. *For the mapping $f : X \to Y$, the following statements are equivalent:*
(i) f is an infra-α-closed;
(ii) $f(I\alpha - Cl\mu) \subseteq Cl(f(\mu))$, for each set μ in Y;
(iii) $Int(f^{-1}(\mu)) \subseteq f^{-1}(\alpha Int^*(\mu))$, for each set μ in Y;
(iv) $Int(f^{-1}(\mu)) \subseteq f^{-1}(Int\ Cl^*\ Int\ (\mu))$, for each set μ in Y.

Now, we define an infra-α-connected as follows:

Definition 10. *A set $\lambda \in X$ is said to be an infra-α-connected if λ cannot written as the Union of two infra-α-separated sets.*

Theorem 13. *Let $f : X \to Y$ be an infra-α-continuous surjective mapping. If η is an infra-α-connected subset of X, then $f(\eta)$ is a connected in Y.*

Proof. Suppose $f(\eta)$ is not an infra-α-connected in Y. Then, there exists an infra-α-separated subsets λ and $\mu \in Y$ such that $f(\eta) = \lambda \cup \mu$.
Since f is an infra-α-continuous surjective mapping, $f^{-1}(\lambda)$ and $f^{-1}(\mu)$ are infra-α-open sets in X and $\eta = f^{-1}(f(\eta)) = f^{-1}(\lambda \cup \mu) = f^{-1}(\lambda) \cup f^{-1}(\mu)$.
It is clear that $f^{-1}(\lambda)$ and $f^{-1}(\mu)$ are infra-α- separated in X. Therefore, η is not an infra-α-connected in X, which is a contradiction!! Therefore $f(\eta)$ is an infra-α-connected. ■

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