Global Journal of Mathematical Analysis, 4 (2) (2016) 6-9

Global Journal of Mathematical Analysis

Website: www.sciencepubco.com/index.php/GJMA
doi: 10.14419/gjma.v4i2.5856
Research paper

A study of prey-predator model with harvesting on susceptible prey and predator

Dinesh Kumar Verma[1]*, V. H. Badshah[1], Suman Jain[2], Nayna Kadam[3]

[1]*School of Studies in Mathematics,Vikram University, Ujjain (M.P.)-INDIA*
[2]*Department of Mathematics,Govt. College, Kalapipal, Ujjian (M.P.)-INDIA*
[3]*Chameli Devi Group of Institution Indore (M.P.)-INDIA*
**Corresponding author E-mail: dineshverma_51@yahoo.com*

Abstract

In this paper, we study the prey predator model with susceptible prey and predator. Stability of the system is discussed in the present model. We analyzed the model in terms of catch rate coefficient.

Keywords:*Prey; Predator; Harvesting; SI Model; Equilibrium Point; Stability.*

1. Introduction

Studies of predator–prey interactions continue to be one of the most fascinating and important aspects of ecological research. The intense focus on this topic can be attributed to the central role of foraging in the lives of predators and their prey, and the importance of predation in driving population, community, and evolutionary dynamics.

More recently, behavioral ecologists have begun to investigate the population and ecosystem consequences of predators in modifying the behavior of their prey such as Lima and Dill [11], Lima [12], Brown and Kotler [3]. This can have profound consequences for prey populations and the dynamics of their communities analyzed by Werner and Peacor [16], Schmitz et al. [15], Preisser et al. [14], Heithaus et al. [8], Creel and Christanson [6].

Predator–prey studies, especially of prey choice by predators, are becoming increasingly important due to anthropogenic modification of ecosystems. For example, changes in energetic demands investigated by climate change could modify predator foraging needs and decisions, as could the introduction of exotic prey species or reductions in naturally important prey. Predators have more at stake while hunting than simply the risk of missing a meal if unsuccessful. Some potential prey may harm or even kill their predator, or the habitat in which particular prey are found may pose an injury or mortality risk to a predator. While the potential for poisonous prey to harm predators has been widely considered in studies of diet choice, as has foraging under the risk of predation, there has been less attention focused on foraged behaviors and decisions of predators hunting other types of dangerous prey. The ability of predators to recognize dangerous prey may vary widely depending on the novelty of such prey (e.g. exotic prey). Regardless, the decisions of when to attack or avoid such prey and the potential costs of doing so could be an important aspect of ecological dynamics.

There are numerous studies on the effects of harvesting on population growth. In the context of predator-prey interaction, some studies that treat the populations being harvested as a homogeneous resource include those of Brauer and Soudack[1,2], Coexistence region and global dynamics of a harvested predator-prey system discussed by Dai and Tang [7]. An ordinary differential equation model for a two-species predator-prey system with harvesting and stocking analyzed by Myerscough et al. [13]. On the combined harvesting of a prey-predator system obtained by Chaudhuri [4]. Optimal harvesting co-efficient control of steady state prey-predator diffusive Volterra-Lotka systems studied by Leung [9]. For a first look at the problem of harvesting from a Bio-economic or control theory point of view, see the works of Clark [5] and Levin et al. [10].

We analyzed a prey predator with vulnerable infected prey proposed by Wuhaib and Hasan [17]. In this paper we discuss the dynamics of a communicable disease in a prey-predator model with susceptible prey. A susceptible prey is considered by using SI model. The presence of both preys has affected predator population. We generalize a prey predator model by harvesting of the prey and predator under optimal conditions. The model is characterized by a pair of first order non-linear differential equations. The existence of the possible steady states along with their local stability is discussed.

2. The mathematical model

In this paper, the model equations for a two species prey-predator system are given by the following system of non-linear ordinary differential equations and generalize the result of [17].

$$\frac{dX}{dt} = rX\left(1-\frac{X}{K}\right) - PXY - q_1\varepsilon_1 X$$

$$\frac{dY}{dt} = PXY - \frac{\gamma YZ}{Z+\gamma\beta Y}, \qquad (2.1)$$

$$\frac{dZ}{dt} = \frac{e\gamma YZ}{Z+\gamma\beta Y} - dZ - q_2\varepsilon_2 Z.$$

Here X is the susceptible prey population, Y is the infected prey, Z is the predator, r is the growth rate, K is the carrying capacity, P is the incidence rate, λ is the total attack rate for predator, β is the handling time, e is the conversion efficiency and d is the death rate of predator, q_1, q_2 are catchobility coefficient and $\varepsilon_1, \varepsilon_2$ are effects applied to harvest the prey and predator species, $\frac{\gamma Y Z}{Z+\gamma\beta Y}$ and $\frac{e\gamma Y Z}{Z+\gamma\beta Y}$ are the Michaelis-Mention-Holling [13] functional and numerical responses. We next reduce the number of parameters by letting

$$x=\frac{X}{K}, y=\frac{Y}{K}, z=\frac{Z}{\gamma\beta K}, t=r\tau$$

We suppose that $k=\frac{PK}{r}, b=\frac{\gamma}{r}, c=\frac{e}{r\beta}, a=\frac{d}{r}$

Then system of equations (2.1) can be written as

$$x'(t)=x(1-x)-kxy-q_1\varepsilon_1 x,$$

$$y'(t)=kxy-b\frac{yz}{z+y}, \qquad (2.2)$$

$$z'(t)=c\frac{yz}{z+y}-\left(a+q_2\varepsilon_2\right)z.$$

Theorem. The solution of the system (2.2) is bounded.
Proof. We define $w(t)=x(t)+y(t)+z(t)$.
And let any positive number that $a>\alpha$, then

$$\dot{w}+\alpha w=\frac{dx}{dt}+\frac{dy}{dt}+\frac{dz}{dt}+\alpha x+\alpha y+\alpha z$$

$$\dot{w}+\alpha w=-x^2+\left(\left(1-q_1\varepsilon_1\right)+\alpha\right)x-\left(b-c\right)\frac{yz}{y+z}$$
$$-\left(\left(a+q_2\varepsilon_2\right)-\alpha\right)z+\alpha y.$$

If we take $b>c$ and assume the unhealthy prey is comparable to the healthy prey, we have

$$\dot{w}+\alpha w\le-\left(x^2+\left((1-q_1\varepsilon_1)+2\alpha\right)x+\left(\frac{((1-q_1\varepsilon_1)+2\alpha)}{2}\right)^2\right.$$
$$\left.+\left(\frac{((1-q_1\varepsilon_1)+2\alpha)}{2}\right)^2\right.$$

$$\dot{w}+\alpha w\le-\left(x+\frac{((1-q_1\varepsilon_1)+2\alpha)}{2}\right)^2+\left(\frac{((1-q_1\varepsilon_1)+2\alpha)}{2}\right)^2$$

$$\dot{w}+\alpha w\le+\left(\frac{((1-q_1\varepsilon_1)+2\alpha)}{2}\right)^2.$$

Suppose that $\left(\frac{((1-q_1\varepsilon_1)+2\alpha)}{2}\right)^2=v$

$$\dot{w}+\alpha w\le v$$

where

$$v=\frac{\left(1-q_1\varepsilon_1+2\alpha\right)^2}{2}.$$

Then

$$0<w\left(x.y,z\right)\le\frac{v}{\alpha}\left(1-e^{-\alpha t}\right)+e^{-\alpha t}\left(x.y,z\right)_{t=0}.$$

3. Equilibrium point and stability

Two equilibrium points are $E_0\left(0,0,0\right)$ and $E_1\left((1-q_1\varepsilon_1),0,0\right)$, the other equilibrium point are solution to the following system of equation

$$\left(1-q_1\varepsilon_1-x-ky\right)=0$$

$$kx-b\frac{z}{y+z}=0$$

$$c\frac{y}{y+z}-(a+q_2\varepsilon_2)=0.$$

We note that

$$x(1-q_1\varepsilon_1-x)=kxy=b\frac{yz}{z+y},$$

and $\quad \frac{yz}{z+y}=\frac{a+q_2\varepsilon_2}{c}z.$

This equilibrium is $\hat{E}_2(\hat{x},\hat{y},\hat{z})$

$$\hat{x}=\frac{b}{k}-\frac{b(a+q_2\varepsilon_2)}{ck}$$

$$\hat{y}=\frac{\left(1-q_1\varepsilon_1\right)-\hat{x}}{k}$$

$$\hat{z}=\left(\frac{c}{\left(a+q_2\varepsilon_2\right)}-1\right)\hat{y}.$$

This means that all populations exist with conditions

$$c>\left(a+q_2\varepsilon_2\right),\ \left(1-q_1\varepsilon_1\right)>\hat{x}.$$

Jacobean matrix of (2.2) is given by

$$J_{(x,y,z)}=\begin{bmatrix} 1-q_1\varepsilon_1-2x-ky & -kx & 0 \\ ky & kx-b\frac{z^2}{\left(y+z\right)^2} & -b\frac{y^2}{\left(y+z\right)^2} \\ 0 & \frac{cz^2}{\left(y+z\right)^2} & \frac{cy^2}{\left(y+z\right)^2}-\left(a+q_2\varepsilon_2\right) \end{bmatrix}$$

Now, Jacobean matrix of system (2.2) at $J_0(0,0,0)$ is

$$J_0=\begin{bmatrix} 1-q_1\varepsilon_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\left(a+q_2\varepsilon_2\right) \end{bmatrix}$$

The characteristic equation is

$$\begin{vmatrix} 1-q_1\varepsilon_1-\lambda & 0 & 0 \\ 0 & 0-\lambda & 0 \\ 0 & 0 & -(a+q_2\varepsilon_2)-\lambda \end{vmatrix}=0$$

$$\Rightarrow \quad (1-q_1\varepsilon_1-\lambda)(-\lambda)(-(a+q_2\varepsilon_2)-\lambda)=0 \quad (3.1)$$

The roots of (3.1) are $(1-q_1\varepsilon_1),0,-(a+q_2\varepsilon_2)$.

The $J_0(0,0,0)$ gives three Eigen values $(1-q_1\varepsilon_1),0,-(a+q_2\varepsilon_2)$, this equilibrium point is not asymptotically stable because second eigenvalue has zero real part.

The Jacobean matrix of system (2.2) at $J_1((1-q_1\varepsilon_1),0,0)$ is

$$J_1=\begin{bmatrix} 1-q_1\varepsilon_1-2 & -k & 0 \\ 0 & k & 0 \\ 0 & 0 & 0-(a+q_2\varepsilon_2) \end{bmatrix}$$

The characteristic equation is

$$\begin{vmatrix} -1-q_1\varepsilon_1-\lambda & -k & 0 \\ 0 & k-\lambda & 0 \\ 0 & 0 & -(a+q_2\varepsilon_2)-\lambda \end{vmatrix}=0$$

$$\Rightarrow \quad (-(1+q_1\varepsilon_1)-\lambda),(k-\lambda),(-(a+q_2\varepsilon_2)-\lambda). \quad (3.2)$$

The roots of (3.1) are $-(1+q_1\varepsilon_1),k,-(a+q_2\varepsilon_2)$.

The $J((1-q_1\varepsilon_1),0,0)$ gives three Eigen values $-(1+q_1\varepsilon_1),k,-(a+q_2\varepsilon_2)$, this equilibrium is not asymptotically stable because the second Eigen values is positive.

Now, Jacobean matrix of system (2.2) at J_2 is

$$J_2=\begin{bmatrix} -\hat{x} & b\left(\frac{a+a_2\varepsilon_2}{c}-1\right) & 0 \\ (1-q_1\varepsilon_1)-\hat{x} & b\left(1-\frac{a+a_2\varepsilon_2}{c}\right)\left(\frac{a+a_2\varepsilon_2}{c}\right) & -b\left(\frac{a+a_2\varepsilon_2}{c}\right)^2 \\ 0 & c\left((1-q_1\varepsilon_1)-\frac{a+a_2\varepsilon_2}{c}\right)^2 & -(a+a_2\varepsilon_2)\left(1-\frac{a+a_2\varepsilon_2}{c}\right) \end{bmatrix}$$

The characteristic equation is

$$\begin{vmatrix} -\hat{x}-\lambda & -k\hat{x} & 0 \\ (1-q_1\varepsilon_1)-\hat{x} & \frac{(a+q_2\varepsilon_2)k\hat{x}}{c}-\lambda & -b\left(\frac{a+q_2\varepsilon_2}{c}\right)^2 \\ 0 & c\frac{k^2\hat{x}^2}{b^2} & \frac{-(a+q_2\varepsilon_2)k\hat{x}}{b}-\lambda \end{vmatrix}=0$$

The characteristic equation of matrix J_2 is

$$\Rightarrow \quad \lambda^3-A\lambda^2+B\lambda+C=0.$$

Where

$$-A=a_{11}+a_{22}+a_{33}$$
$$=\hat{x}-\frac{(a+q_2\varepsilon_2)k\hat{x}}{c}+\frac{(a+q_2\varepsilon_2)}{b}k\hat{x}.$$

$A>0$ if $c>(a+q_2\varepsilon_2)$

$$B=a_{11}a_{33}+a_{22}a_{33}+a_{11}a_{22}-a_{23}a_{32}-a_{12}a_{21}$$

$$=\frac{(a+q_2\varepsilon_2)k\hat{x}^2}{b}-\frac{(a+q_2\varepsilon_2)k\hat{x}^2}{c}+k\hat{x}((1-q_1\varepsilon_1)-\hat{x}).$$

$$C=-a_{11}a_{22}a_{33}+a_{12}a_{21}a_{33}+a_{11}a_{23}a_{32}$$

$$C=\frac{(a+q_2\varepsilon_2)}{b}k^2\hat{x}^2((1-q_1\varepsilon_1)-\hat{x})>0$$

$AB-C>0$ If

$$\left[\left(\frac{a+q_2\varepsilon_2}{b}\right)\hat{x}^2+k\hat{x}\left((1-q_1\varepsilon_1)-\frac{(a+q_2\varepsilon_2)}{c}k\right)((1-q_1\varepsilon_1)-\hat{x})\right.$$
$$\left.+(a+q_2\varepsilon_2)k\hat{x}^2\left(\frac{1}{c}-\frac{1}{b}\right)^2\right]>\left(\frac{a+q_2\varepsilon_2}{c}\right)\hat{x}^2.$$

From the Routh-Hurwitz stability criterion J_2 is locally asymptotically stable if and only if $A>0, C>0$ and $AB>C$.

4. Conclusion

In this paper, the stability of the system is discussed by analyzing the equilibrium points. The existence of these populations are effected by the disease is also investigated. It is also seen that solutions are stable under some conditions. After observation we find that stability depends on catch rate coefficient.

References

[1] Brauer, F. and Soudack, A.C., Stability regions and transition phenomena for harvested predator-prey systems. J. Math. Biol. 7, (1979); 319-337. http://dx.doi.org/10.1007/BF00275152.

[2] Brauer, F. and Soudack, A.C., Stability regions in predator-prey systems with constant-rate prey harvesting. J. Math. Biol. 8, (1979); 55-71. http://dx.doi.org/10.1007/BF00280586.

[3] Brown, J. S. and Kotler, B. P., Hazardous duty pay and the foraging cost of predation. Ecology Letters 7, (2004); 999-1014.http://dx.doi.org/10.1111/j.1461-0248.2004.00661.x.

[4] Chaudhuri, K.S. and Ray, S.S., on the combined harvesting of a prey-predator system, J. Biol. Sys. 4. (1996); 373-389. http://dx.doi.org/10.1142/S0218339096000259.

[5] Clark, C.W., the Optimal Management of Renewable Resources. John Wiley ki Sons, MathemalicalBioeconomics, New York, (1979).

[6] Creel, S. and Christanson, D., Relationships between direct predation and risk effects. Trends in Ecology and Evolution 23, (2008); 194-201.http://dx.doi.org/10.1016/j.tree.2007.12.004.

[7] Dai, G. and Tang, M., Coexistence region and global dynamics of a harvested predator-prey system. SIAM J Appl. Math. 58, (1998); 193-210.http://dx.doi.org/10.1137/S0036139994275799.

[8] Heithaus, M. R., Frid, A., Wirsing, A. J. and Worm, B., Predicting ecological consequences of marine top predator declines. Trends in Ecology & Evolution 23, (2008); 202-210.http://dx.doi.org/10.1016/j.tree.2008.01.003.

[9] Leung, A., Optimal harvesting co-efficient control of steady state prey-predator diffusive Volterra-Lotka systems, Appl. Math. Optim. 31, (1995); 219.http://dx.doi.org/10.1007/BF01182789.

[10] Levin, S.A., Hallam, T.G. and Gross, J.L., Applied Mathematical Ecology, Springer-Verlag, (1989).http://dx.doi.org/10.1007/978-3-642-61317-3.

[11] Lima, S. L. and Dill, L. M., Behavioral decisions made under the risk of predation: a review and prospectus. Canadian Journal of Zoology 68, (1990); 619-640.http://dx.doi.org/10.1139/z90-092.

[12] Lima, S. L., Stress and decision making under the risk of predation: recent developments from behavioral, reproductive, and ecological

perspectives. Advances in the Study of Behavior 27, (1998); 215-290.http://dx.doi.org/10.1016/S0065-3454(08)60366-6.

[13] Myerscough, M. R., Gray, B.E., Hograth, W.L. and Norbury, J., An analysis of an ordinary differential equation model for a two-species predator-prey system with harvesting and stocking, .I. Math. Bzol. 30. (1992); 389-411. http://dx.doi.org/10.1007/bf00173294.

[14] Preisser, E., Bolnick, D. I. and Benard, M. F., Scared to death? The effects of intimidation and consumption in predator–prey interactions. Ecology 86, (2005); 501-509.http://dx.doi.org/10.1890/04-0719.

[15] Schmitz, O., Krivan, V. and Ovadia, O., Trophic cascades: the primacy of trait-mediated indirect interactions. Ecology Letters 7, (2004); 153-163.http://dx.doi.org/10.1111/j.1461-0248.2003.00560.x.

[16] Werner, E. E. and Peacor, S.D., A review of trait-mediated indirect interactions in ecological communities. Ecology 84, (2003); 1083-1100.http://dx.doi.org/10.1890/0012-9658(2003)084[1083:AROTII]2.0.CO;2.

[17] Wuhaib, S. and A., Hasan, Y. A., A prey predator model with vulnerable infected prey. Applied Mathematical Sciences 6, 107, (2012); 5333-5348.